Frequency-domain analysis of debt service in a macro-finance model for the euro area

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Abstract

This paper illustrates how a parsimonious macro-finance model can be exploited to investigate the frequency-domain properties of debt service implied by various financing srategies. This orginal approach is valuable to public debt managers seeking to assess the fiscal-hedging properties of the financing strategies they implement. The model is estimated on euro-area data over the period 1999-2009. At business-cycle frequencies, the variance of interest payments is lower when nominal long-term bonds are issued. From a budget-smoothing perspective, debt service variability plays a major role, but pro- or counter-cyclicality of debt service also matters. In this respect, the results suggest that while interest payments associated with medium- to long-term nominal bonds are negatively correlated with real activity, those associated with inflation-linked bonds and short-term nominal bonds tend to be pro-cyclical.

JEL Codes: C51, E32, E43, G12, H63.

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1 Introduction

Beyond trading purposes, encompassing derivative pricing and interest-rate exposure hedging, yield-curve modeling has been extensively used for policymaking. Apart from the central banker, another natural user of yield-curve modeling is the public debt manager, who has to decide continually what kind of instrument should be issued. Obviously, a good understanding of yield-curve dynamics is necessary when it comes to managing significant volumes of negotiable debt using fixed-income instruments.[1]

In this paper, I build on the recent developments in macro-finance modeling to develop a framework aimed at investigating the dynamic properties of debt servicing. In particular, this framework

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[1] See e.g. Bolder (2006) [10], (2003) [11], (2006) [12], Bernaschi et al. (2007) [8], Anthony et al. (2008) [4] or Hörngren et al. (2008) [54] for an overview of modeling tools and approaches used by public debt managers.

makes it straightforward to analyze the pro- or counter-cyclicality of interest payments that are implied by various financing strategies. As suggested by Dai and Philippon (2005) [23], once infra-year fluctuations are removed, primary-deficit variability is mainly accounted for by inflation and real-activity shocks. Consequently, analyzing the comovements between debt service and such macroeconomic variables is key if debt management is aimed at hedging against fiscal shocks. This kind of budget-smoothing objective is consistent with the general concept of fiscal insurance that also encompasses optimal tax smoothing or debt stabilisation and that recognizes that the role of debt management is to support fiscal policy (see Faraglia et al., 2008a [36]). In particular, according to the tax-smoothing literature, the design of public debt should seek to minimize changes in tax rates, which would otherwise be needed to meet unexpected changes in financing needs.[2] Building on such a framework, Lucas and Stockey (1983) [62], Chari et al. (1994) [17], Barro (1997) [6] or Angeletos (2002) [3] assert that ideal debt instruments would be negatively indexed to public spending and positively indexed to output. However, while some debt management offices have qualitatively taken fiscal insurance into consideration at some point –notably when introducing a new class of funding instruments like ILBs (see Coeuré, 2004 [21] or Dudley, 2007 [29])– those principles do not constitute a primary concern for public debt managers (Wolswijk and de Haan, 2004 [72]). This can be accounted for by the difficulty in turning these conceptual objectives into plain and consensual quantitative measures or by the fact that available attempts to exhibit "optimal" debt portfolio result in pratically unreachable debt structures, with asset positions that are large multiples of GDP (see Faraglia et al., 2008b 37 or Buera and Nicolini, 2004 14). While the approach developed in this paper is not normative (the analysis does not extend as far as the derivation of an optimal debt portfolio, which would notably need to define government preferences), it is aimed at facilitating the debt managers' taking into account of basic fiscal-insurance principles by providing them with a tool to assess the business-cycle properties of debt servicing implied by various financing strategies.

The framework proposed in this paper is guided by recent research suggesting that a joint macro-finance modeling strategy provides a comprehensive explanation of movements in the term-structure of interest rates.[3] Pioneered by Ang and Piazzesi (2003) [1], this modeling builds on the more general affine term structure models (ATSM), that were popularized by Duffie and Kan (1996) [31], whose formalization encompasses earlier models due to Vasicek (1977) [71], Cox, Ingersoll, and Ross (1985) [22], and Longstaff and Schwartz (1992) [61]. Other cornerstones regarding ATSM

[2] See e.g. Missale (1997) [63] and (1999) [64] for an in-depth presentation of tax smoothing and its implications in terms of public debt management.

[3] Diebold et al. (2005) [28] provide a comprehensive view of macro-finance modeling.

include e.g. Dai and Singleton (2000) [24] who give a detailed analysis of the affine models in a generalized formulation as well as, more recently, Joslin, Singleton and Zhu (2011) [57]. ATSM are factor models, so only a small number of sources of variation –four in the present model, inspired by Rudebusch and Wu (2008)– underlie the pricing of the entire term structure of interest rates. Besides, these models impose the standard no-arbitrage restriction from finance, which ensures that, after accounting for risk, the dynamic evolution of yields over time and across state of nature is consistent with the cross-sectional shape of the yield curve at any point in time. The role of macroeconomic variables in no-arbitrage affine model is explored by several papers. In particular, Piazzesi (2005) [68] shows that, relative to standard latent-factor models, using macroeconomic information can substantially lower pricing errors. Besides, Hördahl et al. (2006) [53] have also shown that their macro-finance model has superior forecasting power for yields at all maturities.[4] Beyond such properties, two major advantages of this type of model should be stressed when it comes to public debt management. First, modeling simultaneously interest rates and real activity is necessary if one wants to investigate the debt service properties within the business cycle. Second, once the price of risk is specified, it is potentially possible to price any products whose cashflows depend on the factors that enter the model. For instance, the prices of ILBs of any maturity can be modeled as soon as inflation enters the model. This turns out to be particularly relevant here since inflation-indexed debt represents a significant part of public debt in many countries.[5]

While not intensively used in the recent literature, a technique that is useful in analyzing data generated by econometric models is spectral analysis.[6] Spectral analysis makes it possible to conduct time series analysis in the frequency domain, where a stationary series is thought of as being made up of sine and cosine waves of different frequencies and amplitudes. In a univariate case, one is interested in determining how much of the total variance of the series is determined by each frequency component, which is provided by the spectral density function. In a multivariate setup, spectral analysis provides a description of linear relationships between time series at different frequencies. To the extent that the model boils down to a vector auto-regression model, it is straightforward to assess the frequency domain properties of any linear combinations of the (lagged-)variables. This is exploited so as to investigate the implications of some financing strategies on the business cyclical behavior of debt charges.

[4] See also Jardet et al. (2009) [55] for an investigation of the impact of econometric specifications –and notably the treatment of nearly non-stationarity of interest rates– on forecast performances of these models.

[5] At the end of 2008, the share of inflation-indexed debt in total bond outstanding was equal to 14% in the US, to 25% in the United Kingdom, to 25% in Sweden, to 17% in France and to 7% in Italy (OECD, 2009 [67]).

[6] Spectral analysis was initially applied to engineering and physical science data where large data sets are generated by experiments, and was imported to economic time series data much later (see Harvey, 1975 [48], Naylor et al., 1969 [65]).

The results suggest that the choice of the financing strategy strongly affects the properties of debt charges. Overall, debt charges are more volatile when short-term bonds are issued, which has a twofold explanation. First, long-term interest rates present a lower variance than short-term rates. Second, lower amounts need to be renewed at each period when the maturity of issued bonds increases, which tends to smooth the average interest rate. Besides, the shorter the maturity of issued (nominal) bonds, the higher share of debt-service variance is explained by business-cycle components, whose periodicity is comprised between 1.5 and 8 years.[7] Because of inflation volatility, issuing ILBs implies more variable debt service than when nominal bonds of the same maturity are issued. However, the difference between debt-servicing volatility is lower when infra-year fluctuations are extracted. In addition, it appears that debt charges associated with ILBs are more in phase with real activity than debt service resulting from the issuance of nominal bonds with maturities higher than 2 years, which gives ILBs a greater potential for budget smoothing proposes.

The paper is organized as follows. Section 2 describes the model. Section 3 presents the data, as well as the estimation procedure and results. Section 4 briefly presents spectral analysis and shows an application to public debt management. Section 5 concludes.

2 Model

In the model, inspired from Rudebusch and Wu (2008) [69], four sources of variation underlie the pricing of the entire term structure of interest rates. The factors are closely linked to a reduced-form macro-model which incorporates explicitly some standard channels of transmission of inflationary shocks and of monetary policy. Monthly inflation, denoted with $\widetilde{\pi}_t$ (with $\widetilde{\pi}_t = ln(P_t/P_{t-1})$, where P_t is the price index for month t), can be due to transitory ($\varepsilon_{\pi,t}$) or more persistent ($\varepsilon_{L,t}$) inflation shocks that have a direct impact on prices. Inflation is also affected by demand shocks ($\varepsilon_{y,t}$), which increase output above potential and create excess demand, denoted with y_t. Meanwhile, monetary policy can affect inflation via stimuli or restrictions of aggregate demand, by modifying the real interest rates through monetary-policy surprises ($\varepsilon_{S,t}$). The model includes an unobservable factor corresponding to a medium-term inflation rate L_t, that Rudebusch and Wu (2008) [69] interpret as the inflation objective of the central bank. While broadly following the lines of Rudebusch and Wu's (2008) [69] model, the specifications depart from theirs. In particular, the model does not

[7] Business cycles commonly refers to the components of a time series that passes through an ideal highpass (or bandpass) filter. Hodrick and Prescott (1997) [50] define the business cycle in terms of periodic components lasting 8 years or less. Baxter and King (1999) [7] define it in terms of components whose periodicities range from 1.5 to 8 years.

include forward-looking components.[8]

In the model, the one-period nominal interest-rate $i_{1,t}$ is set by the central bank and breaks down into three components

$$i_{1,t} = \delta_0 + L_t + S_t. \tag{1}$$

The first component δ_0 is a constant steady-state real interest rate, L_t corresponds to a time-varying medium-term inflation rate and S_t is a cyclically responsive component. The latter is given by a Taylor-type reaction function

$$S_t = \rho_S S_{t-1} + (1 - \rho_S)\left[g_y y_{t-1} + g_\pi(\pi_{t-1} - L_{t-1})\right] + \varepsilon_{S,t} \tag{2}$$

where π_t represents year-on-year inflation. The process followed by medium-term inflation reads

$$L_t = \rho_L L_{t-1} + (1 - \rho_L)\chi\pi_{t-1} + \varepsilon_{L,t}, \tag{3}$$

meaning that this factor is the sum of the exponential smoothing of inflation and of an autoregressive process of order one. The parameter χ ($\chi \in [0,1[$) is added in order to make the model stationary.[9] As regards monthly inflation $\widetilde{\pi}_t$, its dynamics take the form of an aggregate supply equation, or "Phillips curve", relating consumer-price inflation to its own lags, the medium-term inflation and excess demand –also termed with real activity in the following–, according to

$$\widetilde{\pi}_t = L_t + \alpha_\pi(\widetilde{\pi}_{t-1} - L_{t-1}) + \alpha_y y_{t-1} + \varepsilon_{\pi,t}. \tag{4}$$

The investment-saving (IS) curve relates the excess demand variable to its own lags and the real interest rate.

$$y_t = \beta_y(L)y_{t-1} - \beta_r\left(i_{1,t-1} - E_{t-1}(\widetilde{\pi}_t)\right) + \varepsilon_{y,t}. \tag{5}$$

Equations (2) to (5) constitute a small-sized structural macroeconomic model with its own dynamics. Using state-space vocabulary, equations (1), (4) and (5) constitute the measurement equations and equations (2) and (3) constitute the transition equations.[10] If the different variables entering these equations –and some of their lags– are stacked in a vector F_t, this model can read

[8] Regarding real activity, previous literature points to a relatively limited degree of forward-lookingness in the Eurozone IS curve (see Hördahl and Tristani, 2007 [52] or Goodhart and Hofmann, 2005 [42]). As regards inflation, the medium-term component L_t is aimed at capturing the inflation expectations.

[9] With if χ equal to one, preliminiary estimations implied non-anchored inflation expectations. While this feature can be desired to account for some structural breakdown in the data, our estimation period (1999-2009) is supposed to make the estimation relatively immune to it.

[10] For a general description of state-space models and Kalman filtering techniques, see e.g. Hamilton (1994) [46] or Kim and Nelson (1999) [66].

$$F_t = \Psi F_{t-1} + \Sigma \varepsilon_t \tag{6}$$

where the stochastic shocks ε_t are *i.i.d.* over time and have a standard normal distribution (see Annex A).

Assets whose one-period-ahead returns are random can be priced once a stochastic discount factor –or equivalently, once the risk-neutral dynamics of the factors– is specified. The existence and uniqueness of a stochastic discount factor, or pricing kernel, is implied by the assumption of no arbitrage.[11] Following Duffie and Kan (1996) [31], Dai and Singleton (2002) [25] and Ang and Piazzesi (2003) [1], among others, I assume that the stochastic discount factor (or pricing kernel) is conditionally log-normal with functional form

$$m_{t+1} = \exp\left[-\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1} - i_{1,t}\right] \tag{7}$$

where, partly for the sake of tractability, the price of risk is assumed to be a linear combination of the factors

$$\begin{aligned} \Lambda_t &= \begin{bmatrix} \Lambda_{\pi,t}, & \Lambda_{y,t}, & \Lambda_{L,t}, & \Lambda_{S,t} \end{bmatrix} \\ &= \lambda_0 + \lambda_1 F_t. \end{aligned} \tag{8}$$

Let $b_{j,t}$ and $i_{j,t}$ denote respectively the price and the yield to maturity of a nominal j-period zero-coupon bond. In this framework, the logarithm of $b_{j,t}$ is given by a linear combination of the factors

$$\ln b_{j,t} = \overline{A}_j + \overline{B}_j' F_t.$$

Equivalently, if $A_j = -\overline{A}_j/j$ and $B_j = -\overline{B}_j/j$, the –continuously compounded– yields are given by:

$$i_{j,t} = A_j + B_j' F_t. \tag{9}$$

Matrices A_j and B_j can be calculated numerically by solving a series of linear difference equations (see Annex B). Formally

$$\begin{cases} \overline{A}_j = \overline{A}_{j-1} - \delta_0 + \frac{1}{2}\overline{B}_{j-1}'\Sigma\Sigma'\overline{B}_{j-1} - \overline{B}_{j-1}'\Sigma\lambda_0 \\ \overline{B}_j' = -\delta_1' + \overline{B}_{j-1}'\Psi - \overline{B}_{j-1}'\Sigma\lambda_1 \end{cases} \tag{10}$$

[11] More precisely, existence and uniqueness of the stochastic discount facor is implied by three assumptions: existence and uniqueness of a price, linearity and continuity of a price and absence of arbitrage opportunity (see Hansen and Richard (1987) 47 or Bertholon et al. (2008) 9.

with $\begin{cases} \overline{A}_1 = -\delta_0 \\ \overline{B}_1 = -\delta_1 \end{cases}$

It is important to note that the pricing kernel allows one to price any security. In particular, denoting with $b^r_{j,t}$ the price of a real bond that provides us with the payoff P_{t+j}/P_t in period $t+j$, this price is given by[12]

$$\begin{aligned} b^r_{1,t} &= E_t\left(m_{t+1}\frac{P_{t+1}}{P_t}\right) \\ &= E_t\left(m_{t+1}\exp(\widetilde{\pi}_{t+1})\right) \\ &= \exp\left(-\delta_0 + \tfrac{1}{2}\Gamma\Sigma\Sigma'\Gamma' - \Gamma\Sigma\lambda_0 + (-\delta_1' + \Gamma\Psi - \Gamma\Sigma\lambda_1)F_t\right) \end{aligned} \tag{11}$$

where the vector Γ is such that $\Gamma F_t = \widetilde{\pi}_t$. Then, remarking that $b^r_{j,t} = E_t\left(m_{t+1}b^r_{j-1,t+1}P_{t+1}/P_t\right)$, it can be shown that $\overline{A}^r_j$ and $\overline{B}^r_j$ are recursively obtained by

$$\begin{cases} \overline{A}^r_j = \overline{A}^r_{j-1} - \delta_0 + \frac{1}{2}(\overline{B}^{r\prime}_{j-1} + \Gamma)\Sigma\Sigma'(\overline{B}^r_{j-1} + \Gamma') - (\overline{B}^{r\prime}_{j-1} + \Gamma)\Sigma\lambda_0 \\ \overline{B}^{r\prime}_j = -\delta_1' + (\overline{B}^{r\prime}_{j-1} + \Gamma)\Psi - (\overline{B}^{r\prime}_{j-1} + \Gamma)\Sigma\lambda_1 \end{cases} \tag{12}$$

with (from equation 11), $\begin{cases} \overline{A}^r_1 = -\delta_0 + \frac{1}{2}\Gamma\Sigma\Sigma'\Gamma' - \Gamma\Sigma\lambda_0 \\ \overline{B}^r_1 = -\delta_1 + \Psi'\Gamma' - \lambda_1'\Sigma'\Gamma'. \end{cases}$

These iterative equations define the real-term structure of interest: if $A^r_j = -\overline{A}^r_j/j$ and $B^r_j = -\overline{B}^r_j/j$, the yield to maturity of a j-period inflation-linked zero-coupon bond is given by

$$i^r_{j,t} = A^r_j + B^{r\prime}_j F_t. \tag{13}$$

All variables in the model follow Gaussian processes, including yields. It would have been possible to allow for conditional heteroskedasticity, for example, using auto-regressive gamma processes (which is a discretized version of the CIR processes, see Gourieroux and Jasiak, 2006 [43]). However, while the fit of the data would then be improved, this would add mathematical complexity and increase the over-parameterization risks underlined by Kim (2008) [59]. Following Ang and Piazzesi (2003) [1], Ang et al. (2006) [2] and Campbell and Viceira (2001) [16], I expect this to be a sufficient first approximation of the joint dynamics of the yield curve and macroeconomic variables. In addition, this is consistent with the limited length of the estimation period (see next Section).

[12] Since the nominal one-month rate is equal to $\delta_0 + \delta_1' F_t$, it comes from equation (11) that the difference between the nominal and real short rates includes the one-period inflation expectation ($\Gamma\Psi F_t$), the inflation risk premium ($-\Gamma\Sigma(\lambda_0 + \lambda_1 F_t)$) and a convexity term $\frac{1}{2}\Gamma\Sigma\Sigma'\Gamma'$.

3 Data and estimation

3.1 Data

The data are monthly and cover the period from January 1999 to June 2009, except for the real yields that begin in 2004 (see section 3.2 for details about the treatment of missing observations).[13]

[Insert Figure 1 about here]

Real activity is represented by the first principal component of a set of 5 business confidence indicators corresponding to quanta of European Commission short-term qualitative surveys (industrial confidence, construction confidence, retail trade confidence, service confidence and consumer confidence). On average across the variables, 75% of the variance is explained by the first principal component. As regards inflation, the choice of the series is guided by the index that is used for inflation-linked products, that is, the HICP excluding tobacco (HICPxT, see e.g. Garcia and van Rixtel, 2007 [41]). The inflation series is demeaned and seasonally adjusted using the multiplicative Census X12 procedure. In order to price nominal and real bonds, I am required to use a volatile one-period (i.e. one-month) rate of inflation (see equation 11). It is worth noting here that modeling two kinds of inflation (a monthly inflation $\widetilde{\pi}_t$ and a medium-term inflation L_t) partially alleviates the "spanning" criticisms addressed by Kim (2008) [59]. According to Kim, the presence of a short-run component that is not related to yield curve movements may undermine the validity of models using raw inflation as a state variable, since much of the "spanned" component of inflation –i.e. the part of inflation that is effectively related to the yield curve– is about the trend component. Breaking down inflation into a short- and a medium-term component (L_t) makes it technically possible to use only the spanned component of inflation to explain yield-curve deformations. Following D'Amico et al. (2008) [26] and Hördahl (2008) [51], I include survey data amongst the observations. This is aimed at overcoming the underestimation of the variability of long-term expectations that arises when dealing with small samples (see Orphanides and Kim, 2005 [60]). As explained by Hördahl (2008) [51], when including information from survey data, the parameter configurations that imply model expectations that deviate from survey expectations are penalised in the estimation. Specifically, one-, two- and five-year ahead expectations for the rate of inflation are taken from the quarterly ECB Survey of Professional Forecasters.[14]

[13] Macroeconomic data are taken from Eurostat, survey data from the ECB and yield data come from Bloomberg (inflation saps) and Datastream (TEC indices).

[14] The forecast horizons are approximately one-month shorter than 1, 2 and 5 years due to deadlines for questionnaire responses that are usually at the end of the first month of a quarter (see Bowles et al., 2007 [13]).

Yields are derived from end-of-month French yield curves and European inflation swaps. French yields are seen as a proxy for AAA-rated euro area central government bond.[15] The nominal zero-coupon yields are bootstrapped from a coupon yield curve based on CNO TEC indices. The TEC_n index corresponds to a hypothetical n-year yield obtained by interpolation of hte two benchmark bonds with maturities closest to n years (see Favero et al., 2000 [38]). A cubic spline is first applied to the TEC_n indices to get a full par-yield curve required by the bootstrapping procedure. The maturities of the nominal zero-coupon used in the estimation are as follows: 1 month, 3 months, 6 months, 1 year, 2 years, 3 years, 5 years, 7 years and 10 years.

Real yields are obtained as the difference between nominal yields and inflation swap rates. The latter can indeed be seen as inflation break-even rates. Because of inflation lags inherent in inflation swaps, the data are treated in order to exclude the known part of inflation that is included in the break-even.[16] For Eurozone inflation products, the inflation index is the 3-month lagged HICPxT. Consequently, the rise in these consumer prices between month $m - 3$ and m is extracted from the inflation swap rate. The procedure hence implicitly suggests that the price index of month m is known at the end of m, which is not the case in practice.[17] The resulting error is assumed to be taken into account by the measurement error of the state-space model. Note that the HICPxT that is extracted is seasonally-adjusted: to the extent that inflation swaps should not in principle be affected by seasonality (because they refer to full-year maturities), extracting a 3-month non-seasonally-adjusted inflation from it would indeed induce seasonality in the remaining break-even.[18] An implication of this procedure is that the maturities of the resulting zero-coupon break-evens are no longer integer numbers of years (but respectively 9 months, 21 months, 57 months and 117 months for 1-year, 2-year, 5-year ad 10-year inflation swaps). In order to compute real rates, nominal rates of the same maturities are therefore needed. The latter rates are obtained by applying a cubic spline on the above-mentioned nominal zero-coupon yield curve.

3.2 Estimation procedure

As Ang and Piazzesi (2003) [1], I use a two-step estimation procedure. In the first step, the macro-model parameters are estimated by maximizing the log-likelihood obtained by applying the Kalman

[15] Since the end of 2006, the ECB publishes daily yield curves for average AAA-rated euro area central government bonds: over the period common with our sample (from January 2007 to June 2009), the correlation of the 10-year yields (TEC10 on the one hand and 10-year ECB-estimated yield on the other) is 0.99 and the standard deviation is 4 bp.

[16] Evans (1998) [35], D'Amico et al. (2007) [26] or Kandel et al. (1996) [58] use a method for correcting from the indexation lag. However, these papers abstract from the impact of seasonality.

[17] While a Flash Estimate is released by Eurostat by the end of the month, the final index is released two weeks later.

[18] Ejsing et al. (2007) [32] provide a comprehensive view of HICPxT seasonality and its implication on break-even measurement.

filter on system (6), enlarged with survey-data measurement equations (11-month, 23-month and 59-month inflation expectations). Measurement errors regarding survey data are assumed to be normally, identically and independently distributed. Starting values for the numerical optimization –conducted using Scilab, employing the quasi-Newton algorithm– are based on ordinary least square regressions, using an exponential smoothing of inflation in place of L_t. In order to obtain both convergence and plausible estimates, three parameters have been calibrated. First, the inflation parameter entering the Taylor rule is taken equal to 0.5, which corresponds to its original value (see Taylor, 1993 [70]). Second, the two parameters defining the dynamics of medium-term inflation (equation 3) –namely the autoregressive parameter ρ_L and the parameter χ , that allows for inflation anchoring– are respectively taken equal to 0.95 and 0.5.[19]

In a second step, the state-space model is enlarged by adding nominal and real yields amongst the observed variables, their dynamics being given by equations (9) and (13). Annex A gives the state-space form of the complete model. The prices of risk and the standard deviations of the yield measurement errors are then estimated while holding all pre-estimated parameters fixed. In this second step, the standard deviations of survey-data measurement errors are enlarged ($\times 2$) in order not to constraint too much the estimation of the prices of risk. This approach can be seen as a very simple attempt of dealing with possible discrepancies between the information sets of survey respondents and financial market participants (see Christensen et al., 2008 [20]). To avoid the implication of the model that arbitrarily-chosen bond returns are driven by the factors only, I assume that all bond yields are measured with error.[20] The errors in yields are normally distributed, serially uncorrelated and uncorrelated across bonds. Following a common practice in the specification of macro-finance models, I set the coefficients of the market price of risk (λ_1 matrix) that load on lagged macro variables to zero (e.g. Ang and Piazzesi, 2003 [1] and Hördahl et al., 2006 [53]), leaving twenty price-of-risk parameters to be estimated.

The facts that (a) real yields are only available from 2004 onwards and that (b) the survey data are at the quarterly frequency give rise to a missing-data problem. This problem is alleviated by Kalman-filter techniques. For each period, the Kalman filter calculates a prediction of the state variables and computes the covariance matrix of the errors (prediction step). For these calculations, only the dynamic properties of the state variables are used, which do not depend on the number of observable variables. The filter then incorporates the new information given by the vector of

[19] Robustness tests suggest that the overall results are fairly insensitive to the choice of these calibrated values.

[20] See e.g. Jegadeesh and Pennacchi (1996) [56] or De Jong and Santa-Clara (1999) [27]. An alternative approach consists in assuming that some of the yields are assumed without error (e.g. Chen and Scott, 1993 [18], Duffee, 2002 [30], or Ang and Piazzesi, 2003 [1]).

observable variables (updating step), which leads to optimal estimates of the state vector and of the covariance matrix. What is key is that the number of observations can vary with time. Of course, the greater the number of observations available to update the filter, the better the accuracy of the estimation.[21]

3.3 Estimation results

Parameter estimates of the first and second steps of the estimation are reported in Table 1. The standard errors of the parameter estimates are based on the information matrix computed using Engle and Watson's (1981) [33] formula. All of the estimates are reasonable with respect to sign and size. The estimate of ρ_S, equal to 0.95, indicates a significant degree of interest rate smoothing by the central bank, which is in line with previous results in the literature. As illustrated in Figure 2, the model achieves to reproduce the main correlations observed in the historical data.

[Insert Table 1 about here]

[Insert Figure 2 about here]

The forecast error variance beakdown is presented in Table 2. It is worth noting that, in comparison with nominal yields, a larger share of real yield variations is explained by the demand shock shock$\varepsilon_{y,t}$ at all horizons. On the contrary, while a significant share of the unconditional variance of long-term nominal yields is accounted for by persistent inflation shock $\varepsilon_{L,t}$, the contribution of this shock to long-term real yield varitions is far lower. Apart from the short interest rate, only a slight share of variances is accounted for by monetary surprises $\varepsilon_{S,t}$.

[Insert Table 2 about here]

Figure 3 presents the impulse responses of the macroeconomic variables, selected bond yields and break-even rates to the four shocks of the model. Although the matrix Ψ contains only stable roots, several of them come in complex conjugate pairs, which results in the fact that some of the impulse responses will not take a direct way back to zero but will cross the zero line before dying out. The impulse responses suggest that the negative effect on the output gap and inflation of a surprise in the policy interest rate reaches its highest about 1.5 year after the policy tightening. As expected, a demand shock results in a rise in inflation and in policy interest rates. Following those shocks, the longer-term interest rates move in the same direction as the short-term interest rate, but while the yield curve tends to flatten in response to a medium-term inflation shock and a medium-term

[21] See e.g. Harvey and Pierse (1984) [49], Burmeister et al. (1986) [15] or Feldhütter and Lando (2008) [39].

inflation shock, it slopes upward following a short-term inflation shock. A demand shock results in a short-lived steepening of the yield curve, followed by a flattening due to rising short-term interest rates. The rigt-hand side plots of Figure 3 compare the responses of the 5-year break-even rate with those that would be obtained if all prices of risk were equal to zero. The difference between these two response functions correspond to the inflation risk premium responses. It turns out that the inflation risk premium reacts positively to both short-and medium-term inflation shocks as well as to demand shocks.

[Insert Figure 3 about here]

The estimates of inflation risk premiums over the estimation period are reported in the upper-left panel of Figure 4. Owing to the affine structure of the model, the inflation risk premiums are also affine functions of the state vectors. A 95% confidence band is also reported. This confidence band takes into account the uncertainty associated with the second estimation step as well as with Kalman-filtering uncertainty.[22] Its narrowness is mainly due to the fact that significant sources of uncertainty are not taken into account by this confidence interval.[23] Calibrated parameters (g_π, ρ_L and χ) are indeed taken as certain, as well as the estimated parameters resulting from the first estimation step. Inflation risk premiums included in hypothetical 5-year zero-coupon bonds were on average equal to 50 bp during the period 1999-2009.[24] Lower panels of Figure 4 show the unconditional term structure of nominal and real zero-coupon yields (left panel) and the unconditional term-structure of inflation premiums and term premiums (right panel).

[Insert Figure 4 about here]

4 A frequency domain application to debt management

4.1 The approach

The previous model depicts a stochastic framework that parsimoniously describes the joint dynamics of macroeconomic and financial variables. Such a framework is rich enough to analyze many aspects of monetary or fiscal policy. In this section, I focus on debt management and I propose an analysis of financing-strategy performances in the frequency domain. Although not often used in the recent

[22] The two kinds of uncertainty are jointly taken into account following Hamilton (1986) [45].

[23] Taking these sources of uncertainty into account would raise major computation issues since second-step estimates are conditional on the first-step and calibrated ones (the joint distribution of all parameters is not directly available).

[24] Note that owing to the data used for the estimation, these risk premiums are associated with inflation swap rates rather than sovereign ILBs. As stressed by Ejsing et al. (2007) [32], the fixed leg of inflation swaps is on average higher than the breakeven extracted from sovereign bonds because the former are more flexible than ILBs to create an inflation-hedging portfolio (before 2007, the spread was approximately equal to 10 bp but has often reached several tens of basis points over the last two years).

literature, spectral analysis proves to be useful in analyzing data generated by econometric models (see e.g. Naylor et al., 1969 [65] Harvey, 1975 [48], Forni and Reichlin, 2001 [40], Hallett and Richter, 2004 [44] or Assenmacher-Wesche and Gerlach, 2008 [5]). For the purpose of describing the behavior of a stochastic variable over time, the information content of spectral analysis is indeed greater than raw second-order moments. More precisely, spectral analysis breaks down the overall covariance into components at different frequencies (see Annex C for a formalized presentation of the spectral analysis tools used here). The approach leads to a comprehensive view of the variable (co-)dynamics and hence makes it possible to conveniently compare some implications of alternative economic policies in a given econometric model. Besides, spectral analysis makes it easy to to filter out the fluctuations associated with some frequencies.

In the frequency domain, a time series is viewed as a weighted sum of many cosine or sine functions of time with different periodicities. The spectral density, being a function of frequency ω, measures the importance of that frequency as a component of the time series. The spectral density function is obtained by way of the auto-covariance function of a times series (that is readily available as soon as the model can be written as a vector auto-regression) and a similar operation on the cross-covariance is carried out to obtain a cross-spectral density function that shows the relations between the cyclical movements of two time series. The cross-spectral density is a complex function whose real and imaginary parts are respectively called cospectrum and quadrature spectrum. Evaluated at ω, the cospectrum is proportional to the portion of the covariance between two variables that is attributable to cycles with frequency ω. It is convenient to describe the cross-spectral function in polar coordinate form, with a gain $R(\omega)$ and a radian angle $\varphi(\omega)$. The latter function is also termed as the phase-difference cross-spectral density. For a given couple of series, it shows that the first one lags behind the second time series by $\varphi(\omega)/\omega$ periods –or by $\varphi(\omega)/2\pi$ cycles– in relation to the cyclical component of frequency ω. The gain $R(\omega)$ measures the covariance between the periodic components of frequency ω in the two time series, once their phase-difference is ironed out.[25]

4.2 Modeling the financing strategies

At this stage, the model depicts the joint dynamics of the nominal and real term-structure of interest rates, inflation and real activity. As soon as the frequency-domain representations of these variables are known, it is straightforward to carry out the frequency analysis of linear combinations of these variables and their lags (see Annex C). This is exploited here in order to analyze the frequency

[25] Cycles of frequency ω may be quite important for both time series individually but yet fail to produce much contemporaneous covariance between the variables because at any given date the two series are in a different phase of the cycle.

domain properties of debt servicing.

Assume that one has to fund an amount D of debt. If (a) one chooses to fund it on a monthly basis and (b) the debt stock is not fed back by interest payments, then the debt service of the rolling strategy is proportional to $i_{1,t-1}$. Next, suppose that funding is based on 3-month bills and that future redemptions are evenly spread over the quarter –so that one third of total debt outstanding has to be rolled over every month–, then accrued interest payments are proportional to $1/3 \times (i_{3,t-1} + i_{3,t-2} + i_{3,t-3})$. As a rule, if nominal n-month bonds are used to fund the debt, interest payments are proportional to $1/n \times (i_{n,t-1} + \ldots + i_{n,t-n})$.

Turning more specifically to public debt management, let denote with D_t the debt outstanding at the end of month t: it includes the issuances of month t (denoted with I_t) but excludes those bonds that fall due in month t). Potential output is denoted with GDP_t^* and is assumed to grow at a constant positive pace of $g\%$.

Defining a financing strategy consists in determining what kinds of bonds are issued at each period in order to face the financing needs of government (see e.g. Bolder, 2003 [11]). For instance, issuing nominal n-period bonds constitutes a financing strategy. More generally, a financing strategy that consists in issuing at each period a constant fraction, defined by weights w_p (with $p \in \{1, \ldots, q\}$), of τ_p-period bonds results in the following debt service (in percentage of potential GDP):[26]

$$\eta_t = \frac{1}{GDP_t^*} \sum_{p=1}^{q} \sum_{j=1}^{\tau_p} \vartheta_{\tau_p,t-j} w_p I_{t-j}.$$

with $\vartheta_{\tau_p,t-j} = i_{\tau_p,t-j}$ if class-p bonds are nominal τ_p-period bonds and $\vartheta_{\tau_p,t-j} = r_{\tau_p,t-j} + \widetilde{\pi}_t$ if class-p bonds are τ_p-period ILBs. Let further assume that issuances I_t grow also at the $g\%$ pace.[27] Then, the ratio I_t/GDP_t^* is constant and, denoting it by γ, last equation reads

$$\begin{aligned} \eta_t &= \frac{I_t}{GDP_t^*} \sum_{p=1}^{q} w_p \sum_{j=1}^{\tau_p} \frac{\vartheta_{\tau_p,t-j}}{(1+g)^j} \\ &= \gamma \sum_{p=1}^{q} w_p \sum_{j=1}^{\tau_p} \frac{\vartheta_{\tau_p,t-j}}{(1+g)^j}. \end{aligned} \tag{14}$$

Therefore, in this context, accrued interest payments in percentage of potential GDP –and

[26] The next equation implicitly assumes that interest payments are accounted for on an accrual basis. Such an accounting treatment is consistent with the reference framework of the European System of Integrated Economic Account (ESA95, Eurostat, 2002 [34]).

[27] If issuances grow at a constant pace which is lower (respectively larger) than $g\%$, the debt-to-potential-GDP converges to zero (respectively explodes). One can also show that the debt-to-potential-GDP ratio is constant when the growth of I_t is of $g\%$,

in percentage of the debt outstanding– takes the form of a weighted moving average of yields (and of inflation if ILBs are involved in the financing strategy). Consequently, debt service is an affine function of the factors, which makes the computation of its frequency-domain representations immediate. [28]

4.3 Results

In the following, I investigate the frequency-domain features of debt servicing that result from the implementation of some particular financing strategies in the simplified economy presented above. This is done without normative objective, that is, I do not look for the implications of the model in terms of optimal debt structure. Doing so would require defining the government's preferences, notably in terms of cost–risk trade-off, and to expand the modeling of taxes and public expenditures, which is beyond the scope of this paper. However, since this framework makes it possible to analyze the comovements of debt charges and business cycles, the results can be linked to the tax smoothing literature. Specifically, to the extent that primary deficit fluctuations are countercyclical, I argue that those financing strategies that result in larger debt charges during expansion periods –and vice versa– have potential to meet tax smoothing objectives.

The spectral density functions of inflation and real activity are shown in the upper plots of Figure 5. The spectral densities are presented for frequencies ranging from 0 to $\pi/12$, that correspond respectively to infinite-period cycles and to 2-year-period cycles.[29] The spectrum of real activity presents a peak for cycles with periods of about 9 years. This peak also emerges for inflation, albeit only as a local optimum. Inflation indeed appears to be mainly driven by low-frequency cycles. As expected, while low-frequency components of monthly and year-on-year inflation are identical, monthly inflation is more affected by higher-frequency components.

[Insert Figure 5 about here]

The lower part of Figure 5 shows the spectral densities of (individual) debt charges associated with selected financing strategies. The strategies that lead to the bottom-left spectral densities involve only nominal constant-maturity zero-coupon. Three maturities are considered: 2 years, 5 years and 7 years. The fact that the 2-year spectral density curve is on average above the others indicates that the unconditional variance of debt charges is more important when shorter-maturity bonds are issued (this can also be read in the upper pat of Table 3). Besides, it appears that a

[28] This aspect is not negligible since experiments have shown that computing those frequency-domain representations using empirical formulae –based on Monte-Carlo-simulated debt service– is computational intensive.

[29] If the frequency of a cycle is ω, its period is given by $2\pi/\omega$.

large share of the debt service variance is accounted for by business-cycle components –defined as those components with periods comprised between 2 and 8 years– in the case of the 2-year financing strategy. As a rule, the lower the maturity of bonds issued, the larger the share of debt service variations explained by business-cycle components (see Table 3).[30] The bottom-right plot of Figure 5 shows the spectral densities of debt service implied by strategies involving 10-year zero-coupon bonds. A first strategy involves nominal bonds only, a second strategy uses ILBs only and the two kinds of bonds (30% of ILBs and 70% of nominal bonds) are mixed in the case of a third strategy. At all frequencies, debt service is more variable when ILBs are issued. Besides, the share of the debt-servicing variance explained by high-frequencies components tends to be higher when funding is based on ILBs (see the upper part of Table 3). Once those components whose period is lower than a year are removed, the differences in variances between nominal and inflation-linked bonds are less dramatic.[31]

[Insert Figure 6 about here]

The analysis is completed by measuring the comovements between, on the one hand, debt service associated with some financing strategies and, on the other hand, real activity. To that end, cospectrum (upper-left plot), quadrature spectrum (upper-right spot), gain (bottom-left plot) and phase (bottom-right plot) of interest payments implied by four different strategies are reported in Figure 6. The strategies still consist in issuing only one type of bond at each period: 10-year nominal bonds for the first strategy, 10-year ILBs for the second, 2-year nominal bonds for the third and 6-month bills for the fourth strategy.

[Insert Table 3 about here]

The gain plots indicate that once the phase differences are ironed out, the most important covariance between debt service and real activity is obtained with the 6-month strategy and, to a slightly lesser extent, with the 2-year strategy, followed by the 10-year indexed strategy. It is worth noting that these differences mainly stem from the differences observed in the quadrature spectrum (recall that the gain is the modulus of the vector whose coordinates are the cospectrum and the quadrature spectrum).

The upper-left panel of Figure 6 plots the cospectrum, that depicts the covariance between the variables when phase differences are not withdrawn. At business-cycle frequencies –i.e. with cycles

[30] While such a result partly stems from the gain shape of the weighted-moving-average filter implicitely applied to interest rates (see equation 14), counterfactual experiments –consisting in systematically replacing long-term yields by the short-term ones inequation (14)– showed that specific frequency-domain properties of the yields matter to the quantitative results.

[31] See the line named "Excl. infra-year [frequencies]" in Table 3.

between 1.5 and 8 years–, while the correlation between real activity and debt charges is negative when nominal bonds with maturities larger than 2 years are issued, it is positive if bills –with maturities lower than 1 year– or ILBs are issued (see also Table 3 for intermediate maturities not reported in Figure 6). This can be accounted for by the lead-lag relationships between debt charges and real activity, as represented by the phase plot (bottom-right plot in Figure 6). At business-cycle frequencies, debt charges implied by the issuance of nominal bonds follow real activity with a delay of more than a quarter of cycle, which results in the contra-cyclicality of these debt charges[32]. On the contrary, when issuing bills or ILBs, the phase difference is lower, which tends to make debt charges pro-cyclical. As shown in Table 3, this is particularly marked at business-cycle frequencies: when focusing on the components of interest payments with periods ranging from 1.5 to 8 years, the correlations between real activity and interest payments are equal to 0.17 and -0.18 for strategies that are respectively based on 10-year ILBs and 10-year nominal bonds.

5 Conclusion

In this paper, I present an approach aimed at assessing the business-cycle properties of debt service implied by various funding strategies. To the extent that the primary deficit is significantly linked to real-activity fluctuations, such an assessment is necessary to test if a financing strategy is consistent with fiscal-hedging principles. To this respect, this approach may contribute to narrow the significant gap between theory and practice regarding debt management.

The analysis is conducted in the frequency domain, the affine structure of the model –estimated over the last decade on euro-area data– making it straightforward to compute the spectral properties of any linear combination of the variables (and their lags). This is exploited to assess the business-cycle behavior of debt charges implied by financing strategies based on the issuance of any-maturity, nominal or inflation-linked bonds.

The results suggest that when nominal short-term bonds are issued, a large share of debt-servicing variance is accounted for by components at business-cycle frequencies. When nominal longer-term bonds are issued, debt charges present a lower unconditional variance and are relatively more driven by low-frequency components. In comparison with nominal bonds, inflation-linked bonds imply more volatile debt charges because of inflation volatility. However, in the latter case, debt charges are also more in phase with real activity: whereas the correlation between interest payments and real activity is negative when nominal medium- to long-term bonds are issued, the

[32] As a rule, if the lead (or lag) is smaller than a quarter of a cycle (i.e. if $|\varphi/2\pi| < 1/4$), the correlation between the two variables is positive. The correlation is null if the lead (or lag) is equal to a quarter of cycle and negative otherwise.

correlation is positive for inflation-linked bonds. This last result is the most pronounced at business-cycle frequencies.

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A State-space form

Let denote with X_t and Z_t the respective vectors of observable and unobservable variables of the complete model. The first elements of X_t correspond to monthly inflation, real activity, short term interest rate and some of their lags. This first part of X_t, which is denoted by X_t^*, is completed by (a) survey data of inflation expectations and (b) observed nominal yields and real yields, gathered respectively in vectors X_t^{SPF} and X_t^i and X_t^r. The complete state-space model is given by the following measurement and transition equations:

$$\begin{aligned} X_t &= \mu + \Theta_X X_{t-1} + G Z_t + M \upsilon_t \\ Z_t &= \Theta_Z X_{t-1} + H Z_{t-1} + N \xi_t \end{aligned}$$

Since X_t contains the observed interest rates, the matrices Θ_X, Θ_Z, G and H depend on the matrices A, B, A^r and B^r, whose computation is based on the knowledge of F_t's dynamics, where $F_t = [X_t^* \; Z_t]'$ (see Annex B). The dynamics of X_t^{SPF} is also based on F_t's one. As a result, a first step consists in writing a smaller state-state model depicting the dynamics of F_t only. This model reads:

$$\begin{aligned} X_t^* &= \Theta_X^* X_{t-1}^* + G^* Z_t + M^* \upsilon_t \\ Z_t &= \Theta_Z^* X_{t-1}^* + H Z_{t-1} + N \xi_t. \end{aligned}$$

Substituting $\Theta_Z^* X_{t-1}^* + H Z_{t-1} + N \xi_t$ for Z_t in the first of the last two equations, the dynamics of F_t is given by

$$F_t = \Psi F_{t-1} + \Sigma \varepsilon_t$$

where

$$\Psi = \begin{bmatrix} \Theta_X^* + G^* \Theta_Z & G^* H \\ \Theta_Z^* & H \end{bmatrix}, \; \varepsilon_t = \begin{bmatrix} \upsilon_t \\ \xi_t \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} M^* & G^* N \\ 0 & N \end{bmatrix}.$$

It remains to specify the composition of the different vectors and matrices:[33]

$$X_t = \begin{bmatrix} X_t^* & X_t^{SPF} & X_t^i & X_t^r \end{bmatrix} \text{ with}$$

$$\begin{aligned} X_t^* &= \begin{bmatrix} \widetilde{\pi}_t & \ldots & \widetilde{\pi}_{t-11} & y_t & \ldots & y_{t+1-p_y} & i_{1,t} \end{bmatrix} \\ X_t^{SPF} &= \begin{bmatrix} E_t\left(\frac{1}{10}\sum_{i=1}^{i=10}\widetilde{\pi}_{t+i}\right) & E_t\left(\frac{1}{22}\sum_{i=1}^{i=22}\widetilde{\pi}_{t+i}\right) & E_t\left(\frac{1}{58}\sum_{i=1}^{i=58}\widetilde{\pi}_{t+i}\right) \end{bmatrix} \\ X_t^i &= \begin{bmatrix} i_{\tau_2,t} & \ldots & i_{\tau_n,t} \end{bmatrix} \\ X_t^r &= \begin{bmatrix} r_{\varsigma_1,t} & \ldots & r_{\varsigma_q,t} \end{bmatrix} \\ Z_t &= \begin{bmatrix} L_t & S_t & L_{t-1} & S_{t-1} & L_{t-2} & L_{t-3} & L_{t-4} \end{bmatrix} \end{aligned}$$

$$G^* = \begin{bmatrix} 1 & 0 & -\alpha_{\pi,1} & 0 & -\alpha_{\pi,2} & -\alpha_{\pi,3} & -\alpha_{\pi,4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_r(\rho_L - \alpha_{\pi,1}) & 0 & -\beta_r\alpha_{\pi,2} & -\beta_r\alpha_{\pi,3} & -\beta_r\alpha_{\pi,4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

[33] Matrices G^{SPF}, Θ_X^{SPF} and M^{SPF}–that appear below in the definition of respectively G, Θ_X and M– are derived from the VAR representation of $F_t's$ dynamics. More precisely, the derivation of these matrices is based on an extensive use of $E_t(\widetilde{\pi}_{t+i}) = \Gamma\Psi^i F_t$ where $\Gamma = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$, since the first element of F_t is the monthly inflation).

$$\Theta_X^* = \begin{bmatrix} \alpha_\pi' & \alpha_y & 0 & \mathbf{0} & 0 \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \beta_r\left(\chi\frac{1-\rho_L}{12}+\alpha_\pi'\right) & \beta_{y,1}+\beta_r\alpha_y & \cdots & \beta_{y,p_y} & -\beta_r \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M^* = \begin{bmatrix} \sigma_\pi & 0 \\ 0 & \vdots \\ \vdots & \\ & 0 \\ & \sigma_y \\ \vdots & 0 \\ 0 & 0 \end{bmatrix} H = \begin{bmatrix} \rho_L & 0 & 0 & 0 & 0 & 0 & 0 \\ -(1-\rho_S)g_\pi & \rho_S & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Theta_Z^* = \begin{bmatrix} (1-\rho_L)\chi/12 & \cdots & (1-\rho_L)\chi/12 & 0 & \cdots & & 0 \\ (1-\rho_S)g_\pi/12 & \cdots & (1-\rho_S)g_\pi/12 & (1-\rho_S)g_y & 0 & \cdots & 0 \\ 0 & \cdots & & & & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & & & & \cdots & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} \sigma_L & 0 \\ 0 & \sigma_S \\ \multicolumn{2}{c}{\mathbf{0}} \end{bmatrix}$$

$$G = \begin{bmatrix} G^* \\ G^{SPF} \\ B_X G^* + B_Z \\ B_X^r G^* + B_Z^r \end{bmatrix}, \mu = \begin{bmatrix} 0 \\ 0 \\ A \\ A^r \end{bmatrix}, \Theta_X = \begin{bmatrix} \Theta_X \\ \Theta_X^{SPF} \\ B_X\Theta_X \\ B_X^r\Theta_X \end{bmatrix}, M = \begin{bmatrix} M^* \\ M^{SPF} \\ B_X M^* \\ B_X^r M^* \end{bmatrix}$$

B Computation of the matrices A and B

By definition, in period $t+1$, a zero-coupon nominal bond that was bought $b_{j,t}$ in period t has a price equal to $b_{j-1,t+1}$. The pricing kernel m_{t+1} is such that

$$b_{j,t} = E_t\left(m_{t+1} b_{j-1,t+1}\right)$$

Assume that there exist some $\overline{A}_j$ and $\overline{B}_j$ matrices such that $\ln b_{j,t}^r = \overline{A}_j + \overline{B}_j F_t$ for any state F_t, any period t and any maturity j. Then the last equation writes

$$\begin{aligned}
1 &= E_t\left(m_{t+1}\exp\left[\overline{A}_{j-1}+\overline{B}'_{j-1}F_{t+1}-\overline{A}_j-\overline{B}'_jF_t\right]\right) \\
&= E_t\left(\exp\left[-\frac{1}{2}\Lambda_t'\Lambda_t-\Lambda_t'\varepsilon_{t+1}-\delta_0-\delta_1'F_t+\overline{A}_{j-1}+\overline{B}'_{j-1}F_{t+1}-\overline{A}_j-\overline{B}'_jF_t\right]\right) \\
&= \exp\left[\overline{A}_{j-1}-\overline{A}_j-\overline{B}'_jF_t+\overline{B}'_{j-1}\Psi F_t-\frac{1}{2}\Lambda_t'\Lambda_t-\delta_0-\delta_1'F_t\right] \\
&\quad\times\exp\left[\frac{1}{2}(\overline{B}'_{j-1}\Sigma-\Lambda_t')(\overline{B}'_{j-1}\Sigma-\Lambda_t')'\right] \\
&= \exp(\overline{A}_{j-1}-\overline{A}_j-\overline{B}'_jF_t-\delta_0-\delta_1'F_t+\overline{B}'_{j-1}\Psi F_t+ \\
&\quad \frac{1}{2}\overline{B}'_{j-1}\Sigma\Sigma'\overline{B}_{j-1}-\overline{B}'_{j-1}\Sigma\left(\lambda_0+\lambda_1F_t\right))
\end{aligned}$$

which gives

$$\overline{A}_{j-1}-\overline{A}_j-\delta_0+\frac{1}{2}\overline{B}'_{j-1}\Sigma\Sigma'\overline{B}_{j-1}-\overline{B}'_{j-1}\Sigma\lambda_0=\left(\overline{B}'_j+\delta'_1-\overline{B}'_{j-1}\Psi+\overline{B}'_{j-1}\Sigma\lambda_1\right)F_t.$$

Equations of system (10) result from the fact that the last equation must be satisfied for any period t and state F. Similar calculations yield to system (12).

C Spectral Analysis

When using spectral analysis, one assumes that the fluctuations of the underlying process are produced by a large number of elementary cycles of different frequencies, and that the contribution of each cycle is constant throughout the sample. Accordingly, the spectral density, being a function of frequency, measures the importance of the cosine function of that frequency as a component of a time series. For a general presentation of spectral analysis and its applications, see Hamilton (1994) [46] and Chow (1975) [19].

The spectral density function can be obtained by way of the auto-covariance function, which are readily available as soon as the model can be written as a vector auto-regression model with stable roots. Specifically, for a n_V-dimensional covariance-stationary process V_t, whose mean is given by $\overline{V}$, the spectral density function –or population spectrum–, which associates an $n_V \times n_V$ matrix of complex numbers with the real scalar ω, is given by

$$s_V(\omega)=\frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\Gamma_k e^{-i\omega k}$$

where $\Gamma_k^V=E\left[\left(V_t-\overline{V}\right)\left(V_{t-k}-\overline{V}\right)'\right]$. Thus, if V_t follows a VAR(1) process

$$V_t=\Phi V_{t-1}+\Omega\varepsilon_t,$$

then, given that $\Gamma_{-k}^V=\Gamma_k^{V\prime}$ and that gamma $\Gamma_k^V=\Phi^k\Gamma_0^V$, it comes

$$s_V(\omega)=\frac{1}{2\pi}\Gamma_0^V+\frac{1}{2\pi}\sum_{k=1}^{\infty}(\Phi^k\Gamma_0^V e^{-i\omega k}+\Gamma_0^V\Phi^{k\prime}e^{i\omega k})$$

where (since $V_t=\Omega\varepsilon_t+\Phi\Omega\varepsilon_{t-1}+\Phi^2\Omega\varepsilon_{t-2}+\ldots+\Phi^k\Omega\varepsilon_{t-k}+\ldots$)

$$\Gamma_0^V=\Omega\Omega'+\Phi\Omega\Omega'\Phi'+\Phi^2\Omega\Omega'\Phi^{2\prime}+\ldots+\Phi^k\Omega\Omega'\Phi^{k\prime}+\ldots$$

On the diagonal of the population spectrum matrix $s_V(\omega)$, one finds the spectral density functions of the variables that constitute the vector V_t. These functions are real-valued periodic functions of ω. Intuitively, for a given frequency ω, their values correspond to the contribution of frequency-ω cycle to the variance of the variables in V_t. The off-diagonal elements of $s_V(\omega)$ are complex conjugate of each other. The real part of the latter elements are known as the cospectrum and the imaginary part is known as the quadrature spectrum. The cospectrum $c_V(\omega)$ evaluated at ω is proportional to the portion of the covariance between two variables that is attributable to cycles with frequency ω. However, the cospectrum only looks for evidence of in-phase cycles. The quadrature spectrum $q_V(\omega)$ then complete the picture by looking for evidence of out-of-phase cycle. A dual representation of both the cospectrum and the quadrature spectrum is provided by the gain $R(\omega)$ and phase measures $\varphi(\omega)$. The former corresponds to the modulus of the complex elements in the population spectrum and the latter corresponds to their phases.

Denoting with L the lag operator and with $\{H_k\}_{k=-\infty}^{\infty}$ an absolutely summable sequence of $n_W \times n_V$ matrices, let W_t denote a n_W-dimensional vector process given by

$$W_t=H(L)V_t=\sum_{i=-\infty}^{\infty}H_iV_{t-i}.$$

Tab. 1: Parameter estimates

α_1	α_y	σ_π $\times 10^3$		β_1	β_4	β_r	σ_y $\times 10^3$
0.21	0.043	1.52		1.14	-0.14	0.06	0.35
(0.05)	(0.013)	(0.1)		(0.04)	(0.04)	(0.03)	(0.02)

ρ_S	g_π	g_y	σ_S $\times 10^3$		ρ_L	χ	σ_L $\times 10^3$
0.95	0.50	0.83	0.117		0.95	0.50	0.050
(0.018)	(-)	(0.24)	(0.009)		(-)	(-)	(0.01)

	λ_0	λ_1			
		π_t	y_t	L_t	S_t
ε_t^π	-0.01	0	-83	-55	-59
	(-0.022)	(2.4)	(0)	(-1.9)	(-11.3)
ε_t^y	-0.69	-169	436	-319	73
	(-0.063)	(0)	(3.3)	(-4.3)	(1.1)
ε_t^L	0.11	-168	-32	-226	-20
	(0.036)	(-0.2)	(-7.2)	(-1.3)	(-1.5)
ε_t^S	-0.13	-48	49	-138	60
	(-0.029)	(-7.6)	(7.8)	(-0.3)	(3)

σ_{3mth} $\times 10^4$	σ_{6mth} $\times 10^4$	σ_{1yr} $\times 10^4$	σ_{2yr} $\times 10^4$	σ_{3yr} $\times 10^4$	σ_{5yr} $\times 10^4$	σ_{7yr} $\times 10^4$	σ_{10yr} $\times 10^4$
0.78	1.21	2.00	2.35	1.83	1.27	0.84	2.17
(0.05)	(0.08)	(0.13)	(0.15)	(0.12)	(0.09)	(0.08)	(0.15)

σ^r_{1yr} $\times 10^4$	σ^r_{2yr} $\times 10^4$	σ^r_{5yr} $\times 10^4$	σ^r_{10yr} $\times 10^4$		σ^{SPF}_{1yr} $\times 10^4$	σ^{SPF}_{2yr} $\times 10^4$	σ^{SPF}_{5yr} $\times 10^4$
4.82	4.35	2.91	2.20		0.88	0.60	0.38
(0.44)	(0.37)	(0.26)	(0.2)		(0.13)	(0.07)	(0.05)

Note: The estimated parameters define the Taylor rule (equation 2), the medium-term inflation dynamics (equation 3), the Phillips curve (equation 4), the investment-saving curve (equation 5) and the price of risk specification (equation 8). Brackets indicate the asymptotic standard errors, which are based on the information matrix calculated using Engle and Watson's (1981) [33] formula. The σ_i's, σ_i^r's and σ_i^{SPF}'s refer to the standard deviation of the measurement errors for the nominal yields, the real yields and the SPF inflation expectations, respectively.

Then, the population spectrum of W_t is related to the population spectrum of V_t according to (see Hamilton, 1994 [46])

$$s_W(\omega) = \left[H(e^{-i\omega})\right] s_V(\omega) \left[H(e^{i\omega})\right]'.$$

Tab. 2: Variance decomposition

	Forecast horizon					
	1 month	6 months	1 year	2 years	5 years	∞
Y-o-y inflation	15	46	70	74	78	79
ε^{π}	1.00	0.99	0.97	0.91	0.86	0.85
ε^{y}	0.00	0.00	0.01	0.04	0.05	0.06
ε^{L}	0.00	0.01	0.02	0.06	0.09	0.09
ε^{S}	0.00	0.00	0.00	0.00	0.00	0.00
Real activity	3	11	18	23	25	27
ε^{π}	0.00	0.00	0.00	0.00	0.01	0.01
ε^{y}	1.00	0.99	0.99	0.96	0.94	0.93
ε^{L}	0.00	0.00	0.00	0.00	0.00	0.00
ε^{S}	0.00	0.00	0.01	0.03	0.06	0.06
1-mth yield	15	36	66	126	174	186
ε^{π}	0.00	0.03	0.09	0.08	0.05	0.05
ε^{y}	0.00	0.15	0.51	0.80	0.85	0.86
ε^{L}	0.15	0.12	0.06	0.03	0.02	0.02
ε^{S}	0.85	0.69	0.33	0.09	0.08	0.08
1-yr nom. Yield	29	54	76	126	156	165
ε^{π}	0.10	0.16	0.16	0.11	0.08	0.08
ε^{y}	0.05	0.32	0.57	0.77	0.80	0.81
ε^{L}	0.05	0.08	0.06	0.04	0.03	0.03
ε^{S}	0.12	0.19	0.11	0.04	0.06	0.06
5-yr nom. Yield	26	54	66	93	107	114
ε^{π}	0.40	0.42	0.40	0.34	0.31	0.30
ε^{y}	0.07	0.19	0.29	0.42	0.43	0.44
ε^{L}	0.17	0.27	0.25	0.21	0.21	0.21
ε^{S}	0.02	0.02	0.01	0.01	0.03	0.03
10-yr nom. Yield	38	66	85	107	126	131
ε^{π}	0.33	0.41	0.43	0.41	0.40	0.39
ε^{y}	0.04	0.11	0.17	0.24	0.24	0.25
ε^{L}	0.16	0.30	0.30	0.29	0.30	0.30
ε^{S}	0.00	0.00	0.00	0.00	0.02	0.02
5-yr real Yield	35	38	38	54	66	66
ε^{π}	0.01	0.04	0.07	0.08	0.07	0.06
ε^{y}	0.00	0.06	0.19	0.44	0.55	0.58
ε^{L}	0.00	0.01	0.02	0.02	0.02	0.02
ε^{S}	0.01	0.03	0.04	0.02	0.04	0.04
10-yr real Yield	27	31	35	38	54	54
ε^{π}	0.04	0.13	0.18	0.20	0.19	0.19
ε^{y}	0.01	0.06	0.15	0.31	0.37	0.39
ε^{L}	0.02	0.07	0.10	0.11	0.12	0.12
ε^{S}	0.00	0.01	0.01	0.01	0.02	0.02

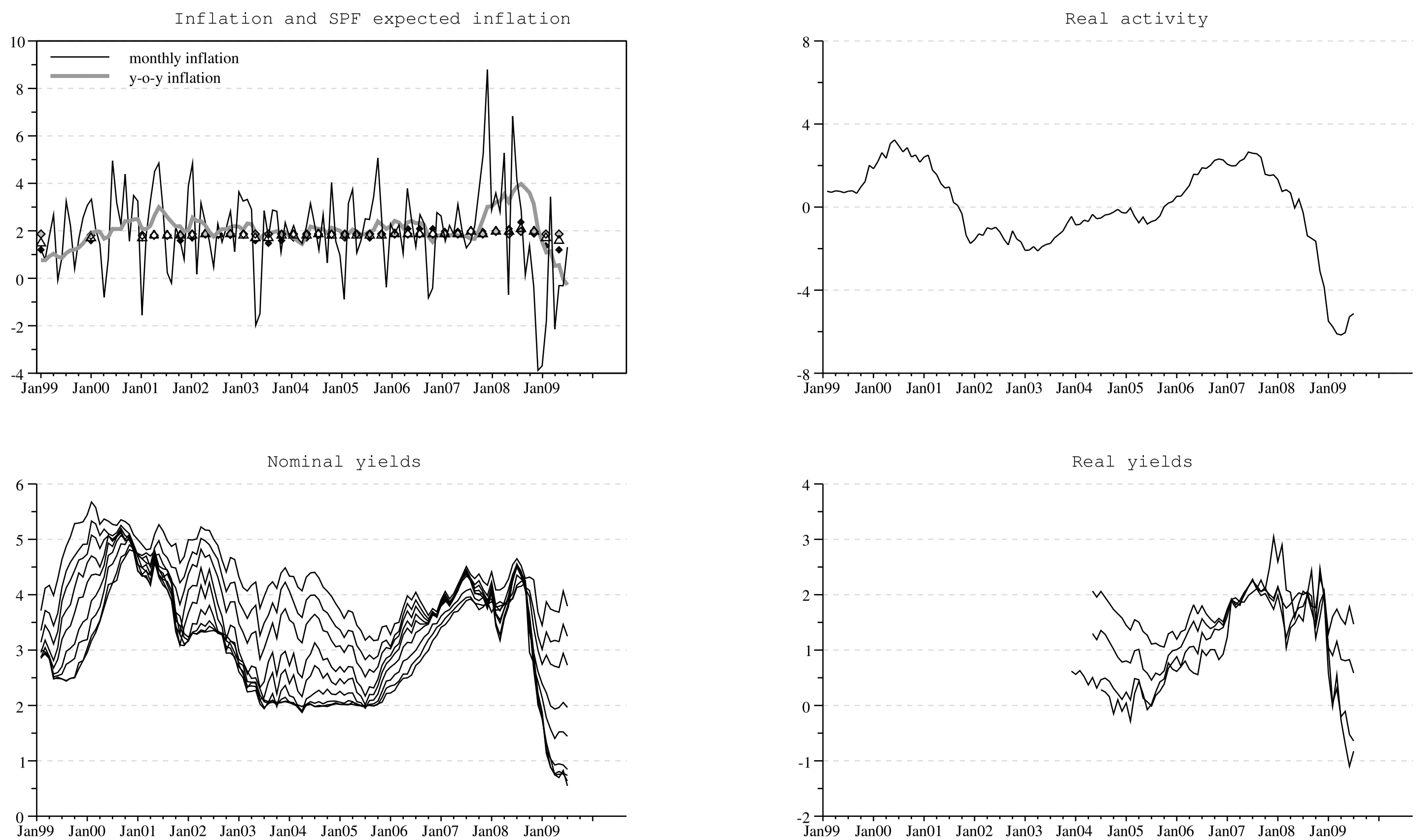
Note: This table presents the contribution of the shocks ε^{π}, ε^{y}, ε^{L} and ε^{S} to the h-period ahead forecast variance of different variables. For each variable, the standard deviations are given in the first line and the variance breakdown is reported the four subsequent lines. As regards inflation and interest rates, standard deviations are expressed in basis points per year. Some variance breakdowns do not sum to one: the remaining share is explained by the measurement errors.

Tab. 3: Spectral decomposition

Bonds issued:	Nominal					Indexed	
	6-mth	1-yr	2-yr	5-yr	10-yr	5-yr	10-yr
A- Variance decomposition of debt service							
Frequencies	**Standard deviation of debt service** (in bp)						
All	185	167	132	93	97	202	201
Business-cycle	96	84	58	16	9	66	67
Excl. infra-year	181	163	127	85	84	115	111
Cycle's length	**Variance decomposition**						
> 8 yrs	0.68	0.69	0.73	0.79	0.71	0.11	0.09
1.5 yr << 8 yrs	0.27	0.25	0.19	0.03	0.01	0.11	0.11
1 yr << 1.5 yr	0.01	0.01	0.01	0.02	0.03	0.10	0.11
< 1 yr	0.05	0.05	0.07	0.17	0.25	0.68	0.69
B- Covariance decomposition (debt service – real activity)							
Frequencies	**Covariance** ($\times 10^7$)						
All	21.0	16.0	6.0	-1.0	0.1	3.0	4.0
Business-cycle	5.0	3.0	-1.0	-1.0	-0.2	0.9	2.0
Excl. infra-year	20.0	15.0	6.0	-2.0	-0.3	3.0	4.0
Frequencies	**Correlation**						
All	0.51	0.43	0.22	-0.07	0.00	0.06	0.09
Business-cycle	0.34	0.21	-0.12	-0.57	-0.18	0.09	0.17
Excl. infra-year	0.51	0.43	0.21	-0.10	-0.01	0.10	0.16
Cycle's length	**Covariance decomposition**						
> 8 yrs	0.71	0.75	1.17	1.00	-0.68	0.69	0.50
1.5 yr << 8 yrs	0.24	0.19	-0.17	1.00	-2.33	0.31	0.50
1 yr << 1.5 yr	0.00	0.00	0.00	0.00	0.39	0.00	0.00
< 1 yr	0.05	0.06	0.00	-1.00	3.62	0.00	0.00

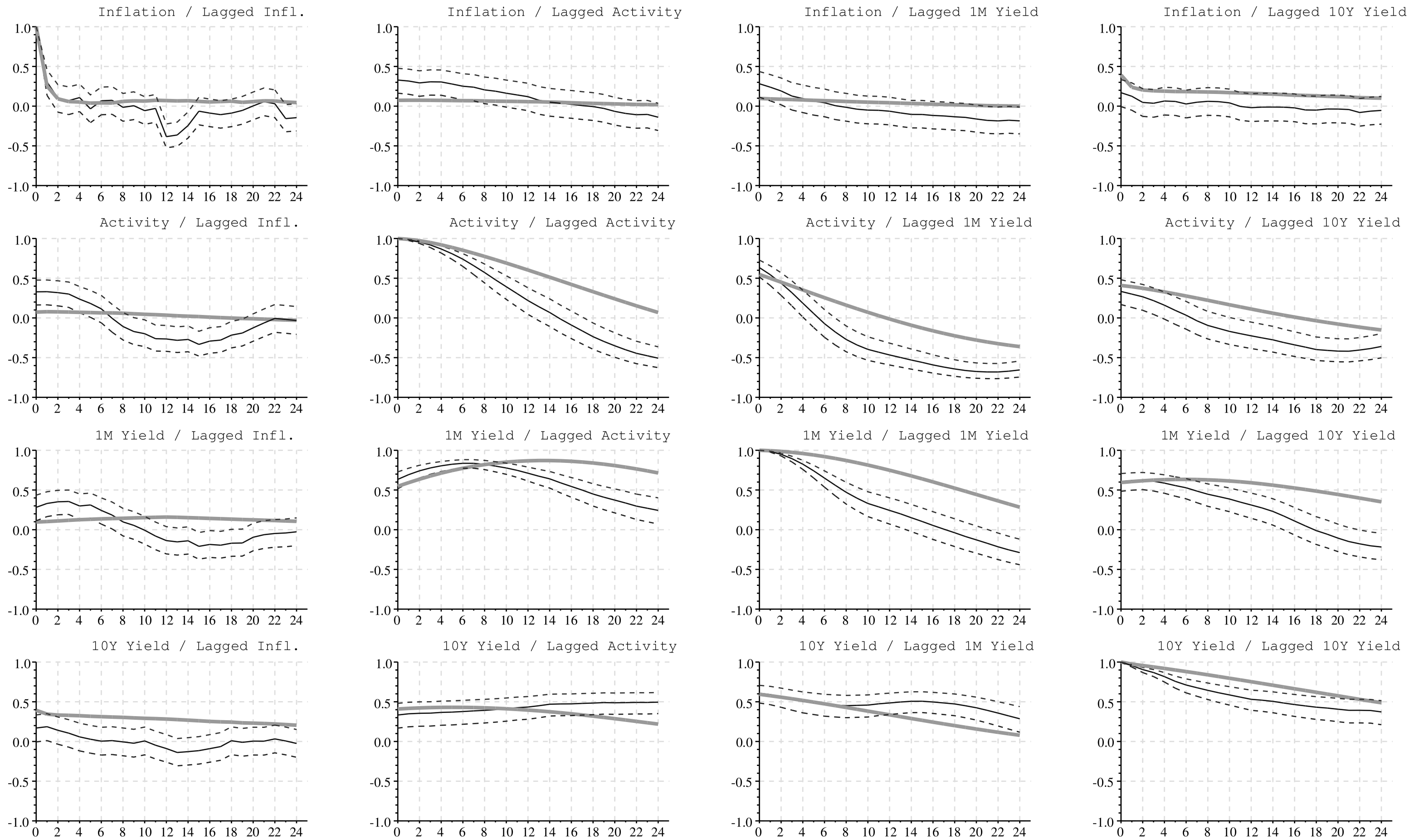
Note: Business-cycle frequencies correspond to cycles with periods ranging from 1.5 to 8 years (see Baxter and King, 1999 [7]). The standard deviations are expressed in basis points per year (debt service can be considered here as a weighted average rate). While the upper part (part A) of the table deals with debt service variability, the lower part (part B) depicts the covariances and correlations between debt service and real activity.

Fig. 1: Estimation data



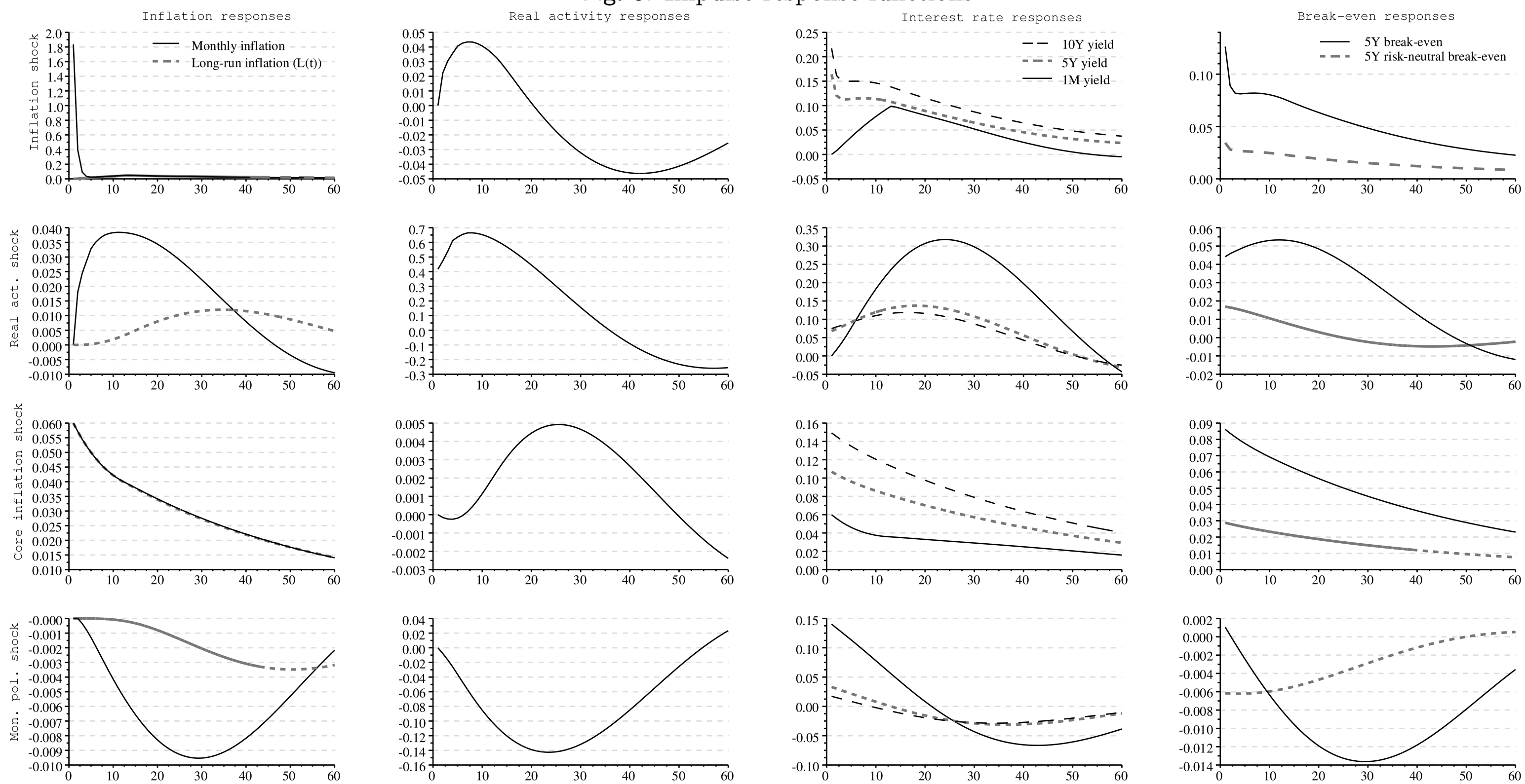
Note: The upper-left panel plots inflation (monthly and year-on-year, both annualized) together with the ECB Survey of Professional Forecasters (1-year ahead inflation: black diamonds; 2-year ahead inflation: white triangles; 5-year ahead inflation: grey diamonds). The yields presented in the bottom-left plot include, from the lowest to the highest line (with occasional cross-overs when the yield curves are inverted): 1-, 3-, 6-month, 1-, 2-, 3-, 5-, 7- and 10-year zero-coupon yields. On the bottom-right plot, from the "lowest" to the "highest" line: 1-, 2-, 5- and 10-year real yield. Inflation and interest rates are expressed in percent per year.

Fig. 2: Simulated vs. historical cross-correlations



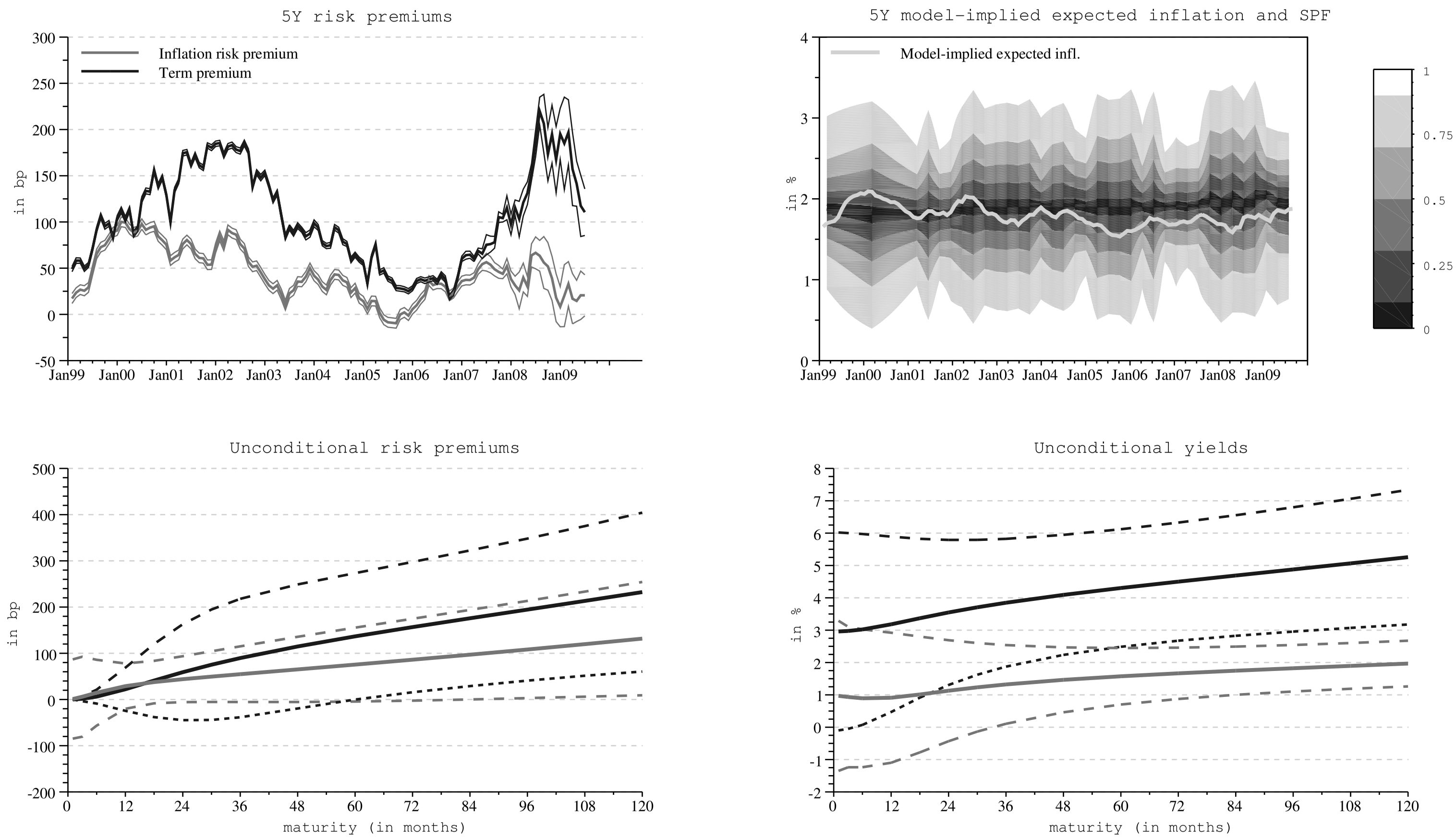
Note: Each plot presents the (lagged) cross-correlation between a couple of variables among the following: monthly inflation, real activity, 1-month yield and 10-year nominal yield. Thick grey lines represent the correlations resulting from the model. Dotted lines are 95% confidence interval of the historical (1999-2009) correlations, the latter being represented by the thin solid lines. The confidence interval is based on the Fisher's z transformation.

Fig. 3: Impulse response functions



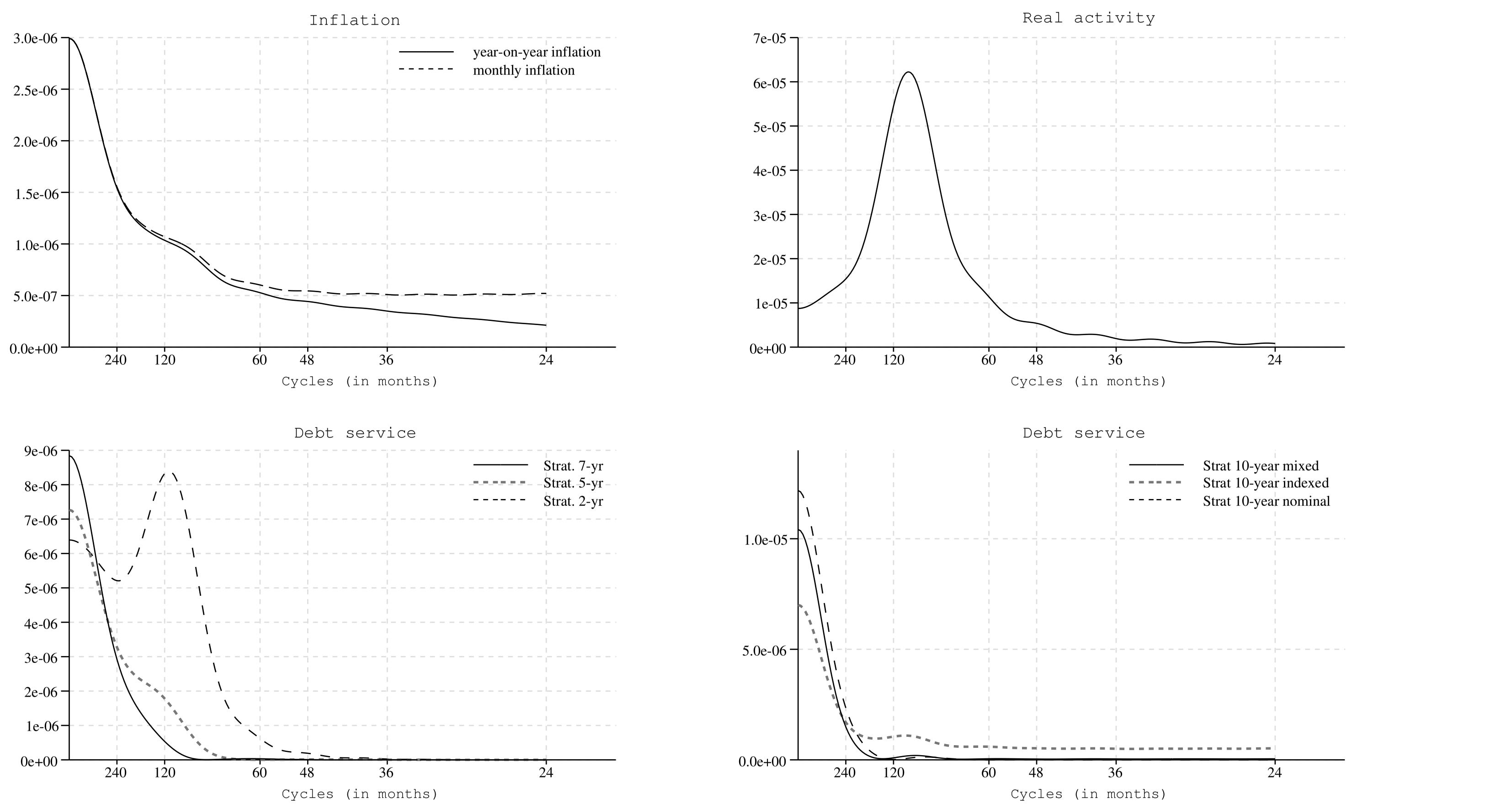
Note: Impulse responses are from a one standard deviation shock (ε^{π}, ε^{y}, ε^{L} and ε^{S}). Inflation and interest rates are annualized. The time scale is in months.

Fig. 4: Yield and risk premiums term structures



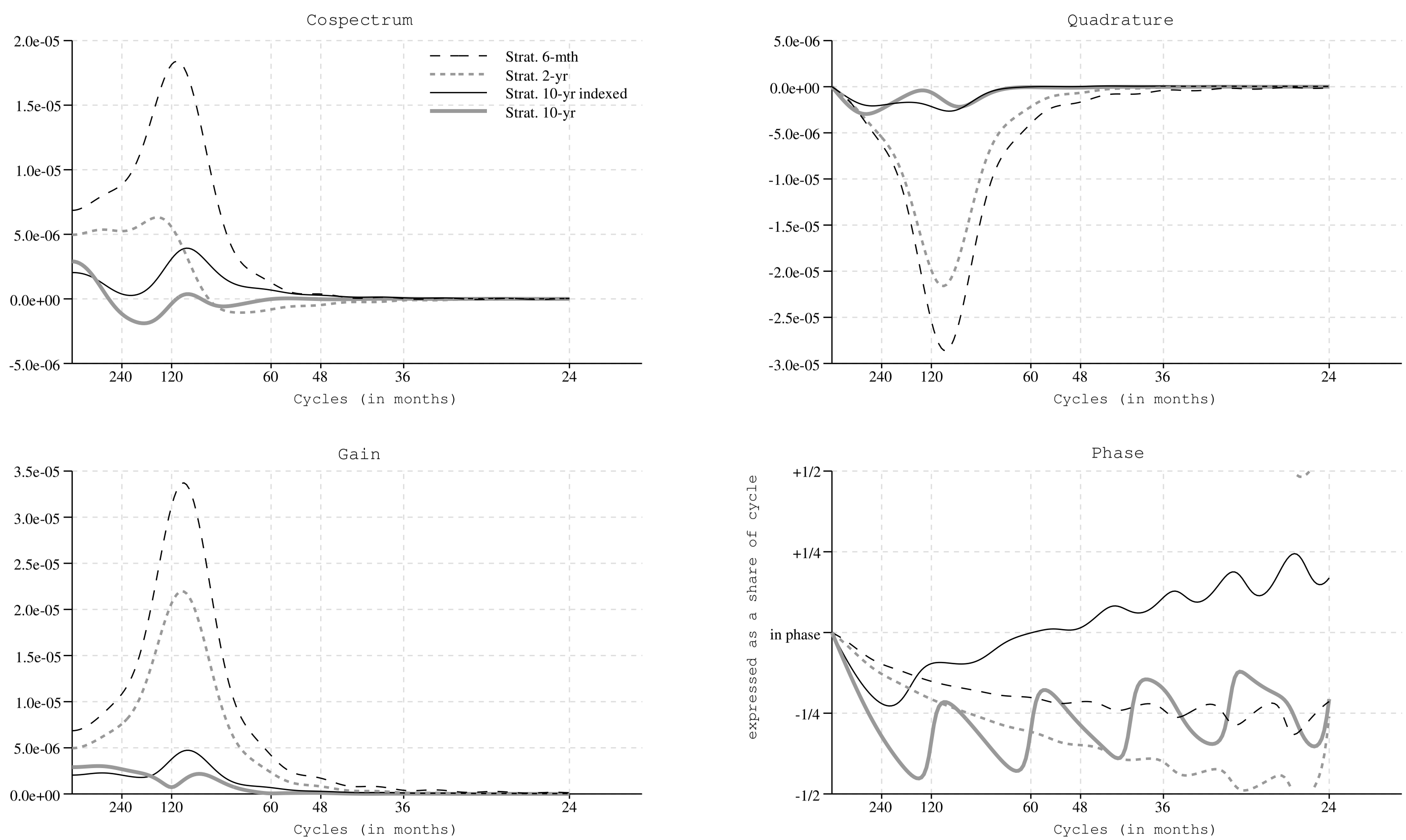
Note: The upper-left panel presents the 5-year inflation risk premium together with the 5-year term premium (in basis points, annualized). The thin lines delimits the 95% confidence interval (accounting for the Kalman-filter uncertainty as well as the uncertainty surrounding the price-of-risk estimates). The upper-right panel compares the 5-year model-implied inflation expectation with the SPF (Survey of Professional Forecasters of the ECB, see Bowles et al., 2007 [13]): the SPF confidence interval is based on an approximation –for each quarter– of the SPF distribution by a mixture of two normal distributions (the grey areas correspond to the 90% confidence interval reported by the average professional forecasters questionned by the ECB). The bottom panels present the unconditional term structures of the inflation risk premiums and term premiums (left panel) and of the yield curve (right panel). The dashed lines delimit the unconditional 95% confidence intervals for the respective yields and premiums.

Fig. 5: Population spectrums



Note: The lower plots give the population spectrum of the debt charges that are implied by basic financing strategies that consist in issuing the same kind of bonds at each period. The issued bonds differ in maturities (2 to 10 years) and type (nominal or indexed). On the lower-left panel, only nominal-bond strategies are considered. The "mixed" strategy of the lower-right panel consists of financing 30% of the needs with 10-year ILBs and 70% with 10-year nominal bonds.

Fig. 6: Cross spectrums (debt service / real activity)



Note: The lower plots give the population spectrum of the debt charges that are implied by basic financing strategies that consist in issuing the same kind of bonds at each period. The issued bonds differ in maturities (2 to 10 years) and type (nominal or indexed). On the lower-left panel, only nominal-bond strategies are considered. The "mixed" strategy of the lower-right panel consists of financing 30% of the needs with 10-year ILBs and 70% with 10-year nominal bonds.