How to make a time series sing like a choir? Extracting embedded frequencies from economic and financial time series using EMD

Patrick M. Crowley

TAMUCC

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Introduction

- New FD technique developed by researchers at NASA
- Most economists blissfully unaware of Empirical Mode Decomposition (EMD) and Hilbert-Huang Transform (HHT)
- Adaptive data method
- Purely empirical

Introduction

- Very few applications in economics -
- Huang and Shen (2005) to mortgage interest rates;
- Zhang, Lai and Wang (2007) to energy prices; and
- Crowley and Schildt (2012) to output and consumption and coincident indicators

- Wavelet analysis usually superior to spectral analysis due to global and local stationarity problems
- Still problems with wavelet analysis though:
   i) still linearly generated;
   ii) placement problem - dyadic ranges with DWT and variants
   iii) overlap and spurious observations with CWT - frequency resolution quite problematic with CWT
   iv) usually only symmetric wavelet functions available "off the shelf" with CWT
- Advantage of EMD/HHT is that it is "A posteriori adaptive" and it is applied only in the time domain

	Spectral	Time-varying spectral	DWT	CWT	HHT/EMD
Basis?	A priori	A priori	A priori	A priori	A posteriori
Domain?	Frequency	Frequency through time	Time-frequency	Time-frequency	Time
Stationary?	Yes	Yes within each window	No	No	No
Linearly generated?	Yes	Yes	Yes	Yes	No
Mathematical underpinning?	Yes	Yes	Yes	Yes	No, empirical
Asymmetric cycles?	No	No	Yes	Yes	Yes

Summary of frequency domain methods

Key papers

1. Huang Shen Long et al (1998)
2. Huang and Shen (2005)
3. Wu and Huang (2008)

- Norden Huang no longer at NASA - see http://www.youtube.com/watch?v=YcV1B5ZzsvE
- Recent conference at http://Idaa.fio.org.cn/Program.pdf
- Recent advance has been the introduction of EEMD or Ensemble EMD.
- Also new journal (Adaptive Data Analysis)

Methodology

2-step procedure

Approach: identify the intrinsic oscillatory modes by their characteristic time scales in the data empirically, and then decompose the data accordingly.

1. Do EMD to obtain intrinsic mode functions (IMFs); and
2. use the Hilbert spectrum or Direct quadrature method to obtain estimate of instantaneous frequency for each IMF.

Step by step:

- i) identify maxima and minima of $x(t)$
- ii) generate upper and lower envelopes with cubic spline interpolation $e_{\min}(t)$ and $e_{\max}(t)$.
- iii) calculate mean of upper and lower envelopes:

$$m(t) = (e_{\max}(t) + e_{\min}(t))/2 \tag{1}$$

- this process is shown in figure 1.

Methodology

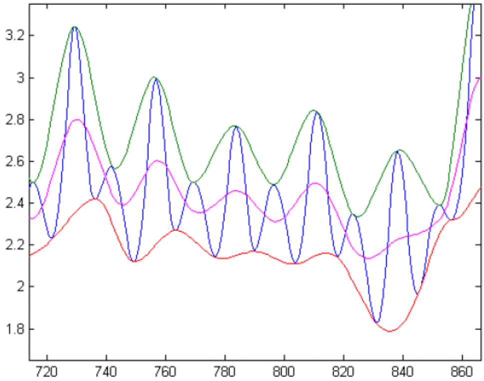

Figure: The spline-envelope process under EMD for a hypothetical series

Methodology

iv) the mean is then subtracted from the series to yield a difference variable, $d(t)$:

$$d(t) = x(t) - m(t) \tag{2}$$

v) if the stopping criterion (SC):

$$\sum_{t=1}^{T} \frac{\left[d_j(t) - d_{j+1}(t)\right]^2}{d_j^2(t)} < SC \tag{3}$$

is met, where $d_j(t)$ is the result from the jth iteration, then denote $d(t)$ as the ith IMF and replace $x(t)$ with the residual

$$r(t) = x(t) - d(t) \tag{4}$$

Methodology

vi) if the stopping criterion it is not an IMF, replace $x(t)$ with $d(t)$.

vii) repeat steps i) to v) until residual $r_n(t)$ has at most only one local extremum or becomes a monotonic function from which no more IMFs can be extracted.

The EMD process can also be illustrated by a diagrammatic flow chart.The resultant decomposition of the series can be written as:

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{5}$$

where $c_j(t)$ represents the jth IMF.

Methodlogy

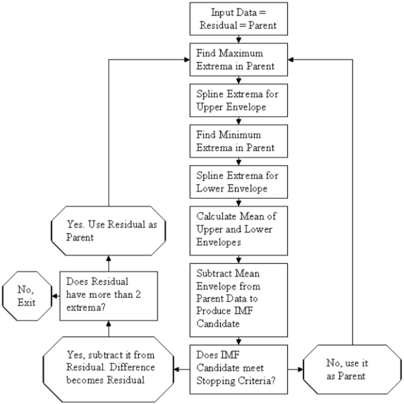

Figure: Flow chart of EMD sifting process

Methodlogy

The Hilbert spectrum lends itself directly to the task of estimating instantaneous frequency, thus allowing the researcher to account for all types of frequency modulation. In mathematical terms, for any function $x(t)$ of L^p class, its Hilbert transform $y(t)$ is:

$$y(t) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau \quad (6)$$

where P is the Cauchy principal value of the singular integral. The Hilbert transform $y(t)$ of any real-valued function $x(t)$ will yield the analytic function:

$$z(t) = x(t) + iy(t) = a(t) \exp\left[i\phi(t)\right] \quad (7)$$

where $i = \sqrt{-1}$, $a(t)$ represents the amplitude and $\phi(t)$ the phase $(\phi(t) = \arg(x(t)))$. $a(t)$ is then given by

$$a(t) = \left(x^2 + y^2\right)^{1/2} \quad (8)$$

and:

$$\phi(t) = \tan^{-1}\left[\frac{y}{x}\right] \tag{9}$$

Instantaneous frequency, ω, then is given by:

$$\omega = \frac{d\phi}{dt} \tag{10}$$

EMD/HHT is fully adaptive in that it can detect "intra-wave" modulations as well as "inter-wave" modulations

3 major problems:

1. End effects - extra data can be added to reduce this
2. Mode mixing - using an ensemble approach can mitigate this
3. Frequency resolution - Hilbert transform replaced by direct quadrature method

Illustrative examples
DJIA

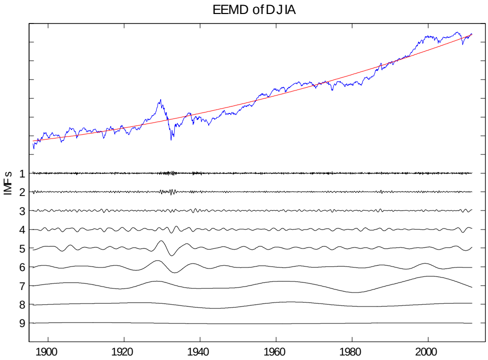

Figure: IMFs for DJIA

Illustrative examples
DJIA

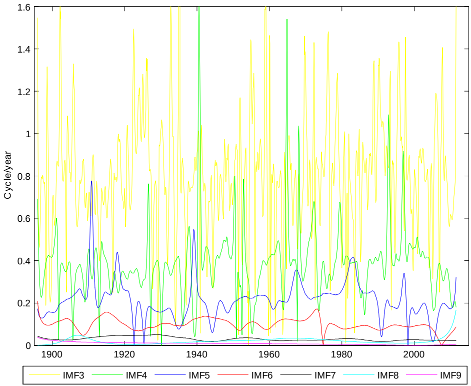

Figure: Instantaneous frequencies for DJIA IMFs

Illustrative examples
DJIA

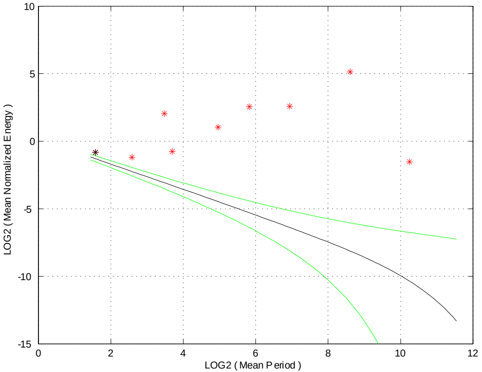

Figure: Significance test of DJIA IMFs against white noise

Illustrative examples

DJIA

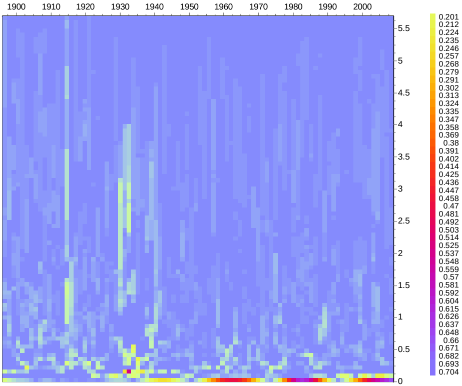

Figure: Hilbert spectrum for DJIA IMFs

Illustrative examples
DJIA

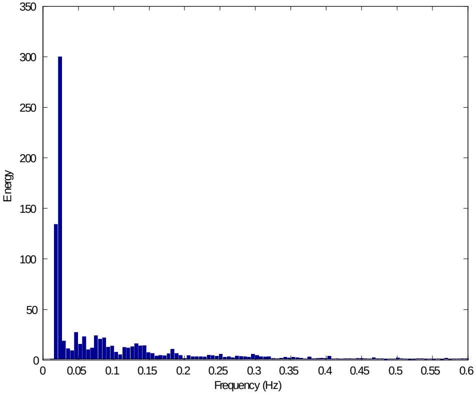

Figure: Marginal Hilbert power spectrum for DJIA IMFs

Illustrative examples

US industrial production

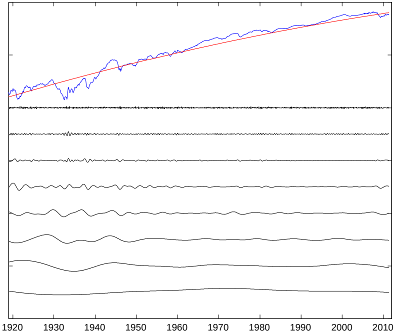

Figure: IMFs for US industrial production

Illustrative examples

US industrial production

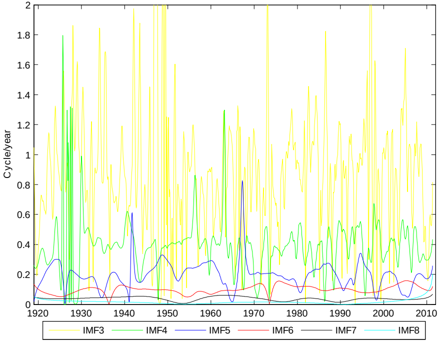

Figure: Instantaneous frequencies for IMFs from US industrial production

Illustrative examples

US industrial production

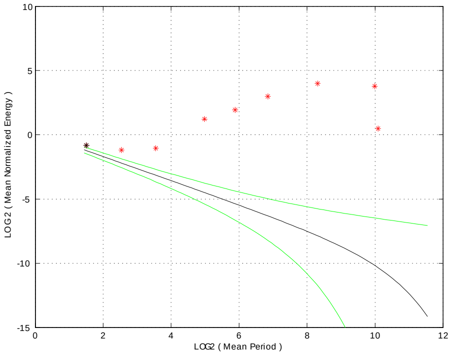

Figure: Significance test of US industrial production IMFs against white noise

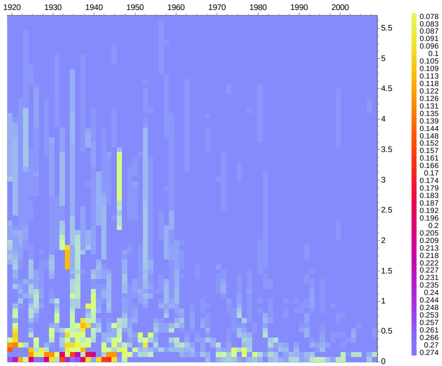

Figure: Hilbert spectrum for IMFs of US industrial production

Illustrative examples

US industrial production

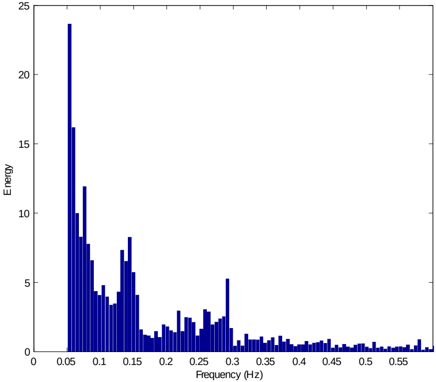

Figure: Marginal Hilbert power spectrum for IMFs of US industrial production

Illustrative examples
UK M0

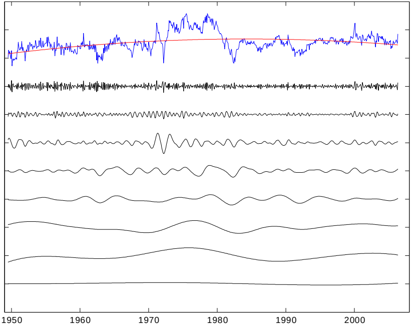

Figure: IMFs for UK M0

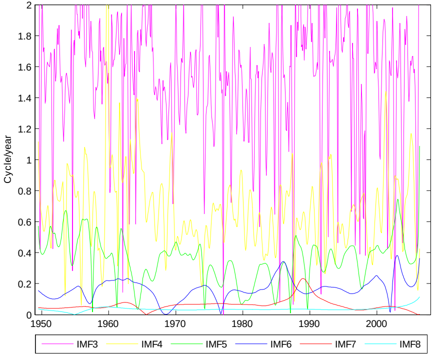

Figure: Frequency of IMFs for UK M0

Illustrative examples
UK M0

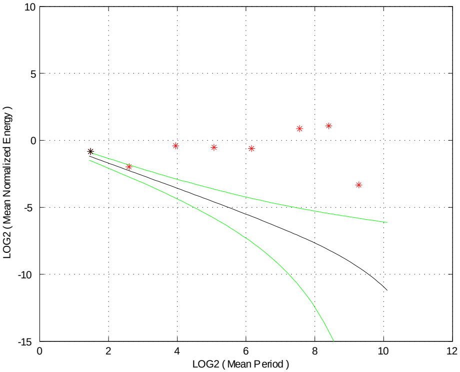

Figure: IMF significance for UK M0 vs white noise

Conclusions

- EMD/HHT is a new FD technique that has not gained much traction in economics or finance yet
- Clearly advantages though in using a purely empirical method particularly when "intra-wave" rather than "inter-wave" modulation is evident
- Problems with decision criteria for number of IMFs and also for mode-mixing
- New emerging technology that is readily available to economists - see the links in paper