

Figuring Out the Fed - Beliefs about Policymakers and Gains from Transparency

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Abstract

In this paper, I use a Markov Chain Monte Carlo algorithm to estimate a macroeconomic model of private-sector behavior that does not feature rational expectations and, instead, leaves firms and households uncertain about how monetary policy is set. In particular, the private sector is endowed with two competing views of monetary policymaking, optimal monetary policy under discretion and under commitment. Firms and households use Bayes' law on a rolling data sample to distinguish between those two models.

I then use this setup to study the evolution of beliefs about the Federal Reserve since 1960 and the possible gains from transparency (i.e. convincing the public that the Federal Reserve is committed to a certain policy).

Using data on inflation, output, and interest rates, I find that the Federal Reserve's actions in the 1960s and 1970s lead the private sector to quickly view the Federal Reserve as a discretionary policymaker, while the policy actions of the 1980s induced an increase in the probability of the Federal Reserve acting under commitment that is slower and more volatile than what is often believed.

Employing a series of counterfactuals, I show that there would have been substantial gains from transparency in the US since 1960, especially in terms of inflation, but also in terms of output.

Finally, I show how much influence the Federal Reserve had on beliefs at different points in the last 40 years and try to clarify why that influence varied substantially across time.

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1 Introduction

From 1960 to 2005 there have been five Federal Reserve chairmen, each with possibly different views about monetary policy and faced with differing degrees of political pressure and different economic environments, from the oil price shocks of the 1970s to the prosperity of the 1990s. The Federal Funds rate has varied greatly within this period, from 2 % to 18 % annualized, showing a great degree of variation in the Federal Reserve's main policy instrument. Figure (1) displays those large movements in economic conditions and, in particular, the Federal Funds rate. The alternating grey and white bars represent the terms in office of the five chairmen during this sample.

FIGURE 1 HERE

Given these large changes over time it seems natural to ask how the private sector's view of Federal Reserve policy has evolved. This paper is concerned with calculating objects that can be interpreted exactly as representing the private sector's view of monetary policy since 1960, and inferring the effects of changes in these beliefs on macroeconomic outcomes, namely output and inflation. In particular, I will ask what the *gains from transparency* are: how would these outcomes have changed if the private sector's beliefs about monetary policy coincided with the actual policy conduct of the Federal Reserve?

A standard New Keynesian model (or any rational expectations model featuring monetary policy for that matter) posits a policy rule that is both the rule the central bank in that model follows and the policy rule that firms and households use to form beliefs about the path of future interest rates. I remove the assumption of rational expectations of firms and households from such a New Keynesian model. Instead, the private sector is uncertain about how monetary policy is set and uses Bayes' law on a rolling data sample to update the model probabilities on two models of monetary policymaking. The models (i.e. monetary policy rules) that the private sector is endowed with are solutions to optimal policy problems, one under discretion and one under commitment. Firms and households do know the preferences of the hypothetical central banks within each model, so the statistical problem that private agents face each period is to discriminate between two models with known parameter values.

The preferences of the hypothetical central banks in the two models are allowed to be different from each other. I estimate these parameters jointly with the other parameters governing private-sector behavior using a Metropolis-Hastings algorithm to provide both a well-fitting model and estimates of statistical uncertainty for the policy experiments in later sections.

Using data on inflation, output, and interest rates, I find that the private sector in the 1960s firmly believed that the Federal Reserve was a discretionary policymaker, a finding in line with anecdotal evidence. Only in 1980 were policy actions of the Volcker Federal Reserve able to significantly move the private sector's beliefs toward a central bank that acts under commitment. However, this shift is less pronounced than what is often believed.

I calculate a series of counterfactuals, varying both the path of nominal interest rates and the private sector's beliefs about the Federal Reserve. These

counterfactuals lead to the main finding of this paper: transparency (i.e. convincing the public that the Fed tries to implement an optimal policy plan under commitment) would have led to a lower and less volatile path for inflation in the 1970s across a number of specifications for the actual path of nominal interest rates, but these assumptions regarding the path of nominal interest rates have a pronounced influence on output.

I also compute the private sector's expectations of inflation and the output gap, and find that these expectations are reasonable when compared to both actual outcomes and survey measures of expectations. This paper thus provides evidence that models of optimal policymaking under different timing assumptions can be helpful to understand the evolution of output and inflation in the US since 1960¹.

Having calculated the private sector's beliefs throughout the second half of the last century, I then use another series of counterfactuals to assess how much the Federal Reserve could have changed the private sector's beliefs throughout that period.

This study is closely related to work by Bianchi (2009) and Liu, Waggoner & Zha (2009), where the private sector also considers two possible models of policymaking. However, in those models the private sector does not face a learning problem. Rather, the private sector in these models knows the type of central bank in power today, but considers the possibility of a future change in the policy rule, a feature absent in this paper². Hence, the approach taken by these papers and the approach taken here are complementary. The approach I take appears to be more convenient for addressing the possible gains from transparency. Another key difference between these papers and the work presented here is the set of monetary policy models that the private sector is endowed with. I endow the private sector with models derived from optimal policy problems under different timing (or commitment) assumptions while the papers mentioned before follow most of the literature on New Keynesian models and use Taylor-type rules³.

Roberds (1987), Schaumburg & Tambalotti (2007), Debortoli & Nunes (2007) and Debortoli & Nunes (2008) use a framework where a policymaker can re-optimize (and thus renege on prior commitments) at random points in time. Hence, their approach could give a possible explanation for changes in private sector beliefs about committed vs discretionary policymakers if that approach was embedded in a learning framework.⁴ However, similarly to the papers mentioned in the previous paragraph, firms and households in this strand of

¹Other papers that have looked at the empirical implications of optimal policymaking include Ireland (1999) and Ruge-Murcia (2003), who find that models in the spirit of Barro & Gordon (1983) are helpful for understanding US economic outcomes. Those papers focus on discretionary policymaking and do not feature private-sector learning.

²The learning algorithm employed by the agents will be set up in such a way that they can detect changes in the model generating the interest rate rather quickly, thus offering some 'implicit insurance' against changes in the model governing monetary policy.

³For an in-depth discussion of the class of models used by Bianchi (2009) and Liu et al. (2009), see Farmer, Zha & Waggoner (2009).

⁴Debortoli & Nunes (2008) also allow for changes in preferences when the policymaker can renege on its previous promises. I find in my estimated model that the two hypothetical central banks that the private sector considers have substantially different preferences.

the literature do not face a learning problem because they can observe when a reoptimization takes place.

Technically this paper contributes to the growing literature on the estimation of learning models in macroeconomics by presenting a likelihood-based approach that allows the econometrician to leave unspecified the "true" data-generating process for the aspects of the economy about which the economic agents are uncertain. Instead, the econometrician can focus on the perceived law of motion of the agents. This approach is embedded in a Markov Chain Monte Carlo algorithm to calculate posterior distributions of statistics of interest.

2 The Model

The private sector in the economy described in this section holds beliefs about monetary policy that evolve over time, as described in the second subsection. These beliefs influence not only the agents' expectations of future short-term nominal interest rates, but also their views about steady state inflation and, as a consequence, the steady state nominal interest rate. The uncertainty about how monetary policy is set is the only uncertainty (besides uncertainty about future exogenous variables) that the private sector faces. In particular, firms and households know all parameter values of both monetary policy models; however they do not know which of the two models generates the nominal interest rate.

2.1 Private Sector Behavior Conditional on Beliefs

Conditional on the perceived steady state of inflation and one-step-ahead expectations of inflation π and (log) output deviations from trend y , current period values of those variables are determined by a New Keynesian Phillips Curve with full indexation to past inflation and the representative household's Euler equation:

$$\pi_t - \bar{\pi}_t = \frac{\beta}{1 + \beta} E_t(\pi_{t+1} - \bar{\pi}_t) + \frac{1}{1 + \beta} (\pi_{t-1} - \bar{\pi}_t) + \frac{\kappa}{1 + \beta} y_t - z_t \quad (1)$$

$$y_t = -\sigma^{-1} (i_t - \bar{i}_t - E_t(\pi_{t+1} - \bar{\pi}_t)) + E_t y_{t+1} + g_t \quad (2)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \quad (3)$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \quad (4)$$

Variants of these equations under rational expectations (and thus a constant perceived steady state, which equals the true steady state) can be derived as an approximation to the equilibrium conditions of a non-linear representative agent model with monopolistically competitive firms. One model that leads to the equations given here can be found in Del Negro & Schorfheide (2004), with the exception that Del Negro & Schorfheide (2004) do not use an indexation scheme for prices, leading to a New Keynesian Phillips curve that does not include a lagged inflation term. A price setting scheme that leads to the Phillips curve described here can be found in Christiano, Eichenbaum & Evans (2005).

The exogenous shocks z_t and g_t can represent a wide variety of stochastic disturbances hitting the economy, depending on the exact set-up of the underlying non-linear model. For the purposes of this paper the exact interpretation of these shocks is not crucial (a similar approach has been taken by, among others, Bianchi (2009) and Lubik & Schorfheide (2004)). Both exogenous shocks follow AR(1) processes, where the innovations ε_t^z and ε_t^g follow a bivariate normal distribution with variances σ_z^2 and σ_g^2 and a contemporaneous correlation coefficient of ρ_{gz} . This correlation is necessary if we want to allow for the possibility of structural shocks in the underlying non-linear economy that feed into both z_t and g_t , such as in Lubik & Schorfheide (2004).

κ governs the slope of the New Keynesian Phillips curve. Its value is inversely related to degree of price stickiness in the underlying non-linear economy. $\bar{\pi}_t$ and \bar{i}_t denote the perceived steady state variables of inflation and the nominal interest rate, where the perceived steady state value of the latter is given by the sum of perceived steady state inflation $\bar{\pi}_t$ and the steady state real interest rate r . The steady state real interest rate r is known by the agents and is independent of their views regarding monetary policy. Note that because of the specific form of the New Keynesian Phillips curve assumed here $\bar{\pi}_t$ actually drops out of (1). Furthermore, because of the relationship between the perceived steady states of inflation and nominal interest rates, one could rewrite (2) as a function of r instead of $\bar{\pi}_t$ and \bar{i}_t . Thus, changes in perceived steady states from period to period only influence output and inflation via $E_t(\pi_{t+1})$ and $E_t(y_{t+1})$.

Remark 1 *The output gap y_t is defined as the log ratio of output and the efficient level of output*⁵. *The "cost-push" shock z_t drives a time-varying wedge between the efficient and natural levels of output (i.e., the level of output under flexible prices). For a further discussion of this issue see Woodford (2003). It turns out, conveniently for the empirical application below, that this output gap seems to behave very similar to standard measures of the output gap such as deviations from an HP-filtered trend, as found by Justiniano & Primiceri (2008).*

2.2 Belief Formation - Calculating Model Probabilities

The private sector is endowed with two models of monetary policymaking⁶. Each model is a description of how a fictitious central bank in that model would set the nominal interest rate given a set of variables that are observable to the private sector (the members of that set will depend on the model at hand). The policy prescriptions of these two models will be denoted by

$$i_t^c = f^c(X_t^c) \tag{5}$$

$$i_t^d = f^d(X_t^d) \tag{6}$$

⁵Depending on the steady state of the non-linear underlying model this could be true only up to a constant. One can however always find an underlying non-linear model such that the statement is exactly true, which is the case if the steady state is efficient. For a full discussion of this issue see Woodford (2003).

⁶I will refer to those two models as "submodels" throughout the paper.

where c and d denote the two models and X^c and X^d denote the observable variables governing policy in each model. How the private sector comes up with these two policy rules is the subject of the next two subsections. Given these two policy rules, the private sector calculates the likelihoods l_t^c and l_t^d of the observed interest rate data at time t given each model and the sequence of relevant right-hand side variables. Therefore, the private sector only learns about the two models via the observed interest rates. Because I will assume throughout that the state variables for each policy model, X_t^c and X_t^d , can be observed by the private sector, the private sector could use (5) and (6) to immediately infer which of the two models, if any, is correct when it observes the interest rate i_t at period t . To make the agents' learning problem more interesting, I will assume that the agents know that both policy models are imperfect descriptions of monetary policymaking. The private sector realizes that each period there will be a difference between the two models' policy prescriptions and the observed nominal interest rate. The private sector assumes that the differences between each models' prescription and the observed interest rate follows the same distribution, as can be seen below.

$$i_t = i_t^j + \nu_t^j, j \in \{c, d\} \quad (7)$$

$$\nu_t^j \sim N(0, \sigma_\nu^2) \forall j, t$$

Equation (7) is then used to form the likelihoods l_t^c and l_t^d each period. Deviations of prescribed interest rates are penalized equally across models, as both error terms are assumed to follow the same distribution when the private sector calculates the likelihoods. This setup is open to other interpretations as well. The error terms could be interpreted as the private sector realizing that the nominal interest rate cannot be perfectly controlled by the central bank, for example. To economize on notation, I define p_t^j and $E_t^j(\cdot)$ as follows:

$$p_t^j \equiv p(\text{model } j | \mathcal{I}_t) \quad (8)$$

$$E_t^j(\cdot) \equiv E(\cdot | \mathcal{I}_t, \text{model } j), j \in \{c, d\} \quad (9)$$

where \mathcal{I}_t is the information set at time t .

Given prior model probabilities p^c and p^d , the private sector then uses a Quasi-Bayesian approach to attach probabilities to each model each period by calculating model probabilities the following way:

$$p_t^c = \frac{(\prod_{i=t-t^*}^t l_i^c) p^c}{\sum_{j=c,d} (\prod_{i=t-t^*}^t l_i^j) p^j} \quad (10)$$

For a fixed t^* , this approach is Quasi-Bayesian because it only applies Bayes' Law to a rolling sample of observations instead of the entire available sample⁷.

⁷Two comments about this setup are in order:

1. If the available sample includes less than $t^* + 1$ datapoints the private sector will use all available data until the sample becomes large enough so that the restriction to start the model probability calculation at time $t - t^*$ becomes meaningful.
2. This restriction is similar in spirit to constant gain least squares learning, which is used

If, instead, we set $t - t^*$ equal to period 1 of the entire data sample for each t , the equation given above is equal to the more common recursive representation of Bayes' law:

$$p_t^c = \frac{p_{t-1}^c l_t^c}{\sum_{j=c,d} p_{t-1}^j l_t^j} \quad (11)$$

Allowing the private sector not to use all past data leads to a situation where model probabilities adjust more quickly to new evidence in favor of one of the submodels. This is the 'implicit insurance' against changes in the model actually generating monetary policy mentioned in the introduction.

2.3 Optimal Policy Under Commitment

The policy rule $f^c(X_t^c)$ is derived as the solution to an optimal policy problem where the hypothetical central bank solves the optimal policy problem described below once, and then commits to that policy rule forever. The hypothetical policymaker in this submodel minimizes the objective function (12) subject to constraints (1)-(4) under the additional assumption that the expectations in the objective function and the constraints are formed conditional on this model being true, i.e. not taking into account the private sector's algorithm for forming expectations over two models of monetary policymaking. This simplifying assumption allows me to use a standard algorithm (see Soderlind (1999) or Backus & Driffill (1986)) to solve this optimal policy problem, which will make estimation of the model feasible.

$$E_0 \sum_{t=0}^{\infty} \beta^t [(\pi_t - \pi^c)^2 + \lambda^c (y_t)^2 + \lambda_i^c (i_t - i^c)^2] \quad (12)$$

As is standard in the solution of optimal policy problems under commitment, lagged Lagrange multipliers on the constraints that feature forward-looking expectations ($\lambda_{NKPC,t-1}$ on (1) and $\lambda_{IS,t-1}$ on (2)) become state variables that influence the optimal choice of the nominal interest rate in this model each period. These lagged Lagrange multipliers represent the influence of past commitments on current policy actions. Thus X_t^c is given by

$$X_t^c = \begin{pmatrix} \pi_{t-1} \\ z_t \\ g_t \\ \lambda_{NKPC,t-1} \\ \lambda_{IS,t-1} \end{pmatrix} \quad (13)$$

The preference parameters π^c , λ^c and λ_i^c will be estimated later. On the other hand, the interest rate target i^c is not a free parameter, but is instead set equal to $r + \pi^c$.

in the majority of the literature on learning in macroeconomics. That approach puts greater weight on more recent observations versus observations that are further in the past. For more details on this approach see, for example, Evans & Honkapohja (2001).

2.3.1 How the Private Sector Calculates the Lagrange Multipliers

To calculate the interest rate prescribed by the optimal policy problem under commitment, households and firms need to know the values for the multipliers $\lambda_{NKPC,t}$ and $\lambda_{IS,t}$. The solution to the optimal policy problem under commitment delivers a VAR representation for X_t^c :

$$X_t^c = AX_{t-1}^c + B\varepsilon_t \quad (14)$$

where ε_t is a vector of consisting of ε_t^z and ε_t^g . Given initial values for the Lagrange multipliers and other state variables, the private sector uses the observed shocks ε_t every period to update the Lagrange multipliers and calculate the optimal interest rate under commitment.

2.4 Optimal Policy Under Discretion

$f^d(X_t^d)$ is derived as the solution to an optimal policy problem under discretion, i.e., it is part of a Markov Perfect equilibrium. Each period the hypothetical central bank in this submodel takes as given future policy and private-sector behavior when choosing the nominal interest rate to minimize an expected discounted quadratic loss function. Hence, the solution to this optimal policy problem describes the behavior of an opportunistic central bank that reoptimizes every period instead of honoring past commitments. $f^d(X_t^d)$ is the policy rule that arises in a situation where the policy that the hypothetical central bank expects to be followed in the future coincides with its own response to the state variables given that policy. The state variables for this problem are given by

$$X_t^d = \begin{pmatrix} \pi_{t-1} \\ z_t \\ g_t \end{pmatrix} \quad (15)$$

Consequently this hypothetical central bank solves the Bellman equation (16) subject to constraints (1)-(4).

$$V(X_t^d) = \min_{i_t} (\pi_t - \pi^d)^2 + \lambda^d (y_t)^2 + \lambda_i^d (i_t - i^d)^2 + \beta E_t V(X_{t+1}^d) \quad (16)$$

As in the previous section, I assume that the expectations that are calculated when solving this policy problem do not take into account the private sector's learning problem, again allowing me to use a standard algorithm (linear quadratic value function iteration in this case) to solve for the optimal policy. The solution algorithm is described in great detail in Soderlind (1999) and Backus & Driffill (1986). Similarly to the commitment case, the preference parameters π^d , λ^d and λ_i^d will be estimated, while the interest rate target i^d is equal to $r + \pi^d$.

Remark 2 *Both hypothetical central banks have quadratic preferences over deviations of inflation from a target value (which could be different across these two hypothetical policymakers), deviations of the interest rate from a target value (which will depend on the respective inflation target) and deviations of output from trend, or equivalently deviations of the output gap from zero. Given the*

fact that both central banks do not have a non-zero target for the output gap the traditional average inflation bias of Kydland & Prescott (1977) is not at work here. However, a stabilization bias of discretionary policy is present: the committed central bank can smooth the effect of shocks more efficiently over multiple periods, as it can credibly commit to such a response. The only endogenous state variable that allows for history dependence in the case of discretion is the lagged interest rate.

As the two hypothetical central banks are allowed to have different inflation targets, it would be hard to distinguish empirically the effect of different inflation targets vs. an average inflation bias (i.e., the result of a non-zero target for the output gap). Furthermore, it is convenient to be able to interpret directly the inflation targets as the long-run average inflation levels within each of the submodels.

2.5 Calculating Expectations (and Perceived Steady States)

One-period-ahead expectations of the output gap and inflation are calculated using the model probabilities derived in the previous section and equilibrium laws of motion for the relevant variables that are part of the solution of the commitment and discretion optimal policy problems. In particular, I assume that

$$E_t(\pi_{t+1}) = p_{t-1}^c E_{t-1}^c(\pi_{t+1}) + p_{t-1}^d E_{t-1}^d(\pi_{t+1}) \quad (17)$$

$$E_t(y_{t+1}) = p_{t-1}^c E_{t-1}^c(y_{t+1}) + p_{t-1}^d E_{t-1}^d(y_{t+1}) \quad (18)$$

The probabilities are calculated using the Quasi-Bayesian approach described earlier, while the expectations are calculated using the solution to the two optimal policy problems.

Expectations only depend on observables dated $t - 1$ or earlier to avoid having to solve a non-linear fixed point problem when estimating the model⁸. The perceived steady state of inflation is calculated as follows:

$$\bar{\pi}_t = p_{t-1}^c \pi^c + (1 - p_{t-1}^c) \pi^d \quad (19)$$

3 Estimation

I estimate the parameters of the model (except for the discount factor, which I calibrate) using a Bayesian approach. To do so, I need to combine the likelihood

⁸While this sort of assumption is common in the literature on learning in macroeconomics, it is used here for a slightly different reason: it is not the agents in this model who would have to solve a non-linear fixed point problem, as the expectations in (1) and (2) would only be functions of contemporaneous exogenous variables z_t and g_t and predetermined endogenous variables π_{t-1} , $\lambda_{NKPC,t-1}$ and $\lambda_{IS,t-1}$. Rather, the fixed point problem would be one of jointly solving for z_t , g_t and expectations consistent with those values when trying to calculate the likelihood of the model (see section 3.1). If the expectations appearing in equations (17) and (18) depend on date t state variables of the two policy problems (including z_t and g_t) then by equations (1) and (2) z_t and g_t would have to be jointly determined with model probabilities and conditional expectations.

of the data with a prior distribution of the model's parameters:

$$\underbrace{p(\Theta|y^T, \pi^T, I, i^T)}_{\text{posterior}} \propto \underbrace{p(\Theta)}_{\text{prior}} \underbrace{p(y^T, \pi^T|\Theta, I, i^T)}_{\text{likelihood}} \quad (20)$$

Θ denotes the vector of parameters, x^T the sample of size T for variable x and I the vector of initial conditions needed to initiate the learning algorithm (both initial values of observables as well as shocks and Lagrange multipliers). The following sections describe the different elements entering (20). I initialize the Lagrange multipliers for the commitment problem on the forward-looking equations (1) and (2) at 0, implying that this hypothetical central bank does not have to honor any prior commitments at the start of the sample. The prior model probabilities p^c and p^d are each set to 0.5. Those initial values could be estimated, but, to economize on the number of estimated parameters, I calibrate them for now. I let the private sector use a sample of 10 years to form their model probabilities using (10). I have found that this sample size leads to a better overall fit than using a longer sample (or the entire sample for that matter) in terms of the likelihood function described in the next section. However, using a different sample length leads to results that are similar to those reported below. The data used in this paper is described in appendix A.

3.1 The Likelihood Function

Because the model is one of private-sector decision making, it does not have anything to say about how actual monetary policy is set and, instead, focuses on how the private sector thinks monetary policy will evolve. Accordingly, I use a likelihood function that conditions on the observed path of nominal interest rates:

$$p(y^T, \pi^T|\Theta, I, i^T) \quad (21)$$

This likelihood can be rewritten as follows:

$$p(y^T, \pi^T|\Theta, I, i^T) = p(y_1, \pi_1|\Theta, I, i_1) \prod_{t=2}^T p(y_t, \pi_t|\Theta, I, i^t, y^{t-1}, \pi^{t-1}) \quad (22)$$

Each of the densities in (22) can be evaluated by using the distributional assumptions on ε_t , equations (1)-(4) and the learning algorithm described earlier⁹. A more detailed description of the calculation of the likelihood can be found in appendix B.

This likelihood function will be used as an input for a Metropolis-Hastings algorithm, which will generate draws from the posterior distribution of the parameters. For a discussion of this algorithm see, for example, An & Schorfheide (2007).

⁹Note that the timing assumption used for the formation of expectations is very useful when writing down (22), as the expectations in (1) and (2) are functions of variables in the conditioning set of each of the densities in (22) at each point in time.

3.2 Priors

This section describes the priors that are used in combination with the likelihood described above to calculate the posterior distribution of the parameters. I will use two types of priors, one that I will call 'standard' in that it resembles priors used in previous studies on New Keynesian models, and one that puts more prior probability on regions of the parameter space where the learning models yield reasonable policy prescriptions:

$$\begin{aligned}
 \underbrace{p(\Theta|y^T, \pi^T, I, i^T)}_{\text{posterior}} &\propto \underbrace{\widehat{p}(\Theta)}_{\text{prior}} \underbrace{p(y^T, \pi^T|\Theta, I, i^T)}_{\text{likelihood}} & (23) \\
 = \underbrace{p_1(\Theta)}_{\text{"standard" prior}} &\underbrace{p_2(\Theta)}_{\text{prior on reasonable submodels}} \underbrace{p(y^T, \pi^T|\Theta, I, i^T)}_{\text{likelihood}}
 \end{aligned}$$

It turns out that when only the prior $p_1(\Theta)$ is used, the learning models of the agents will not fit particularly well, as that lack of fit is not directly punished by either the likelihood (which conditions on interest rates) or the prior. As long as the expectations in (1) and (2) lead to relatively small errors in these equations, the likelihood function will not be close to zero even if the interest rate prescriptions are very different from the observed interest rate. The fitted interest rates coming out of the two submodels are still highly correlated with the actual interest rate, but their variance is much higher, leading to highly improbable policy recommendations from each model.

I find it highly implausible that the private sector would use models that give unreasonable policy recommendations for over 40 years to learn about monetary policy. Thus, I want to put more prior mass on regions of the parameter space where the two submodels yield reasonable policy prescriptions. A shortcut to doing so is described below.

3.2.1 "Standard" Priors

I use a set of independent probability distributions to characterize "standard" prior beliefs about the parameters of the model. Priors for those parameters that are standard in New Keynesian models are in line with those used by Lubik & Schorfheide (2004) and Bianchi (2009) and can be found in appendix D, along with the priors for the loss functions of the hypothetical central banks. The discount factor β is calibrated and set equal to 0.99.

The priors for the loss function parameters of the two hypothetical central banks are set to capture the prior belief that the commitment central bank has a lower inflation target and a lower weight on output in the loss function. The variances on these priors are relatively large to ensure that the data have the final word on the values of these parameters. To make the loss function parameters more easily interpretable, the priors for the weights on the output gap are scaled by 1/16, as it seems more natural to think about a loss function where output deviations are compared to deviations of annualized inflation and interest rate from their respective targets, while the variables π_t and i_t in the model are defined as quarterly inflation and the quarterly nominal interest rate. This kind of rescaling is standard in the analysis of optimal policy in New Keynesian models.

3.2.2 Prior on Reasonable Policy Models

As mentioned before, estimating the model using the likelihood and "standard" priors described above leads to prescribed interest rates that, while correlated with the actual interest rates observed, are much too volatile. To correct this, I try to put more prior probability in regions of the parameter space where the two policy models fit well. Unfortunately, it is not easy to characterize that region of the parameter space. As a shortcut, the prior is augmented by the following probability distribution:

$$\prod_{t=1}^T (0.5l_t^c + 0.5l_t^d) \tag{24}$$

This is a weighted average of the likelihoods of the two policy models. It is important to remember that those are the likelihoods that the private sector uses to form model probabilities. Using this probability distribution as additional prior information ensures that regions of the parameter space that yield badly fitting policy models receive low prior probability.

Equation (24) is a data-dependent prior. While it is generally not desirable to have a prior depend on data, one should keep in mind that this data-dependent prior is just a shortcut to representing the prior belief that the private sector only uses a learning model with reasonable policy prescriptions.¹⁰ Furthermore, because the likelihood I use conditions on the observed path of interest rates, Bayes' law allows me to use a prior that also conditions on the path of interest rates. The dependence of (24) on i_t can thus be handled in a Bayesian framework. This prior also depends on the inflation rate and the output gap via the calculated values for z_t and g_t , though.

Another important issue with this construction of a prior is whether it too profoundly influences model probabilities. By looking at (24) one might have the impression that it will push the model probabilities towards .5, as it penalizes the likelihoods of both submodels symmetrically. While this is a priori a fair concern, it turns out that this concern is unwarranted; we will see below that while both submodels fit reasonably well there are still large swings in the estimated model probabilities.

One advantage of using (24) is that it penalizes a model with unreasonable policy prescriptions, even in periods when that model has a low model probability attached to it, and thus strengthens the identification of the loss function parameters. To see this, assume instead that I would use the following construction to penalize unreasonable learning models:

$$\prod_{t=1}^T (p_{t-1}^c l_t^c + p_{t-1}^d l_t^d) \tag{25}$$

While this might seem more appealing on theoretical grounds (after all, it is the perceived likelihood of the private sector), it only contains information

¹⁰Another interpretation of what I do here is that I take a penalized likelihood approach, with the penalty being data-dependent. For an introduction to penalized likelihood estimation see Green (1999).

about the loss function parameters of the hypothetical central bank in one of the submodels when the probability of that model is significantly different from 0. A prior of this sort would use a much smaller effective sample to estimate the loss function parameters. I found this not only to be a theoretical concern, but also to dramatically hinder the numerical estimation.

In future work, I plan to use the approach of Gallant & McCulloch (2009) instead of the shortcut presented here. Gallant & McCulloch (2009) present an approach that allows the econometrician to put priors not only directly on model parameters, but also on functions of model parameters (such as moments of the nominal interest rate). However, this approach is more time consuming than the shortcut I present here, as it requires the simulation of the model for each parameter draw.

3.3 Estimation Results

Table 3 contains the parameter values at the posterior mode, while table 2 in appendix E also displays the posterior mean and 95 % posterior density intervals.

In general, the mean and mode are very similar, with the only main difference between them being the weight on the output gap in the commitment loss function, which is almost five times larger at the posterior mean. Nonetheless, at both the posterior mean and the posterior mode, the same picture across the two sets of central bank preference parameters arises; the weight on the output gap, the weight on the interest rate deviations from the target and the inflation target are substantially larger for the hypothetical discretionary central bank.

TABLE 3 HERE

To interpret the weight on the interest rate deviation from target and the inflation target, table 4 reports the estimated annualized inflation targets in percent at the posterior mode and the rescaled weight on the output gap, which I multiply by 16 to have a weight on the output gap that is comparable to weights on the inflation and interest targets, which are reported in annualized (instead of quarterly) units.

TABLE 4 HERE

The inflation target of the hypothetical commitment central bank is in the range of 2 %, a commonly assumed target value for the Federal Reserve today. The inflation target for the discretionary policymaker on the other hand seems very high at 5.48 %. Remember, however, that for reasons of identifiability I have abstracted from an average inflation bias, which could explain why this estimate is so high.

The rescaled values for the weights on the output gap show a big difference in the weight being placed on the output gap, with the discretionary policymaker caring over seven times more about the output gap than the hypothetical commitment central bank. Both values, though, are smaller than one, indicating that both hypothetical central banks care more about inflation than about the output gap.

Turning to the 95 % posterior density intervals, it is evident that all parameters are estimated tightly. This is due, at least in part, to the use of the second set of priors. Using these priors leads to a more peaked posterior, as regions of the parameter space where one of the two submodels does not fit well are given low posterior probability, even if the other model fits well and has much higher model probability attached to it in that region of the parameter space.

The estimates for the AR coefficients ρ_g and ρ_z are quite a bit lower than what is usually found in the literature (see, for example, Lubik & Schorfheide (2004)). It seems that the learning algorithm presented in this paper creates enough endogenous persistence to allow these parameters to be substantially lower than their prior mean (which is centered around usual estimates in the literature).

The estimate of the correlation between the innovations in (3) and (4), ρ_{gz} , is relatively high. Again, this is not surprising, as many non-linear models imply that the error terms in the linearized Euler equation (2) and New Keynesian Phillips curve 1 are correlated because an exogenous error term enters both of these equations. If shocks are redefined, as they are here, allowing for contemporaneous correlation seems important to do the underlying non-linear model justice.

Besides needing less exogenous persistence than other models in the literature, the estimate of κ is higher than what is found in most other studies (a notable exception being Lubik & Schorfheide (2004)¹¹). This high value implies a much higher probability of a firm being able to adjust its price each period in a Calvo framework. Note that the slope of the Phillips curve is $\kappa/(1 + \beta)$, not κ itself, so if one compares the slope estimates of New Keynesian Phillips curves across studies, the estimate found here, while being on the high end, is not unheard of.

4 Evolution of Beliefs

This section analyzes the private sector's beliefs by focusing mainly on the posterior mode estimates. Since the parameters are all tightly estimated, the error bands around most of the statistics of interest are narrow.

Figure 2 plots the sequence of estimated posterior model probabilities of the commitment model. The private sector quickly learned in the 1960s that the Federal Reserve was a discretionary policymaker. It took the Volcker disinflation at the beginning of the 1980s to slowly induce a change in these beliefs. While the policy actions in 1980 caused the posterior probability of the commitment model to increase substantially, the following 15 years are associated with large swings in the estimated model probabilities. There is an upward trend in p_t^c during that period, but fluctuations around that trend are volatile.

¹¹Like this paper Lubik & Schorfheide (2004) uses Hodrick-Prescott detrended output as an observable (even though the exact detrending method differs, see appendix A), while a lot of other studies use output growth and make actual output an unobservable state variable in a state-space representation of the linear(ized) model. This is a possible explanation for the similarity of some of the parameter estimates.

Also, it is worth noting that towards the end of the sample p_t^c decreases again.

FIGURE 2 HERE

In general, these model probabilities are estimated precisely. Figure 3 displays the 5th and 95th percentile of the estimated model probabilities of the commitment submodel, in addition to the estimated mean model probabilities. The mean probabilities are very similar to the model probabilities at the posterior mode estimates. Interestingly, there turns out to be substantial model uncertainty towards the end of the sample, making it hard to distinguish between the two submodels on the ground of the data used here for that period¹².

FIGURE 3 HERE

To gain further insight into how the private sector arrives at the posterior model probabilities plotted in figure 2, it is useful to analyze squared prediction errors coming from the two submodels, as they govern the Gaussian conditional likelihoods l_t^c and l_t^d .

FIGURE 4 HERE

Figure 4 plots the difference in squared forecast errors of the annualized interest rate:

$$(400i_t - 400i_t^d)^2 - (400i_t - 400i_t^c)^2 \quad (26)$$

A value of this statistic less than 0 implies a better fit of the discretion submodel. This statistic is very volatile and periods where one submodel is preferred by the date are often followed by a period where the other submodel is preferred. What determines the model probabilities is a 10 year average of this difference in squared prediction errors. This moving average is plotted in figure 5.

FIGURE 5 HERE

A positive value of the moving average implies evidence in favor of the commitment submodel. Because I set the prior model probabilities to 0.5, a positive value of this moving average implies that p_t^c is larger than 0.5. The evidence for or against one of the two submodels is strongest in the 1970s, where the magnitude of the moving average is much larger than what it is at any point after 1980.

The initial lack of credibility of the Volcker Federal Reserve has also been documented prominently by Goodfriend & King (2005). The approach there is different in that it is more narrative (even though an equilibrium model is used to illustrate the points made in that paper), and, in addition, uses long-term interest rates as a measure of inflation expectations, which the authors in turn interpret as a measure of credibility attributed to the Federal Reserve by the bond market. While it is beyond the scope of this paper, it would be very interesting to incorporate term structure information into the private sector's learning algorithm.

¹²For the other calculations in this section, posterior quantile bands do not convey substantial additional information beyond calculations carried out using the posterior mode estimates.

5 Interest Rate Prescriptions and Perceptions

This section analyzes the prescribed interest rates for the commitment and discretion submodels and the path for expected interest rates that those prescriptions and the calculated model probabilities imply.

At the end of each period, after inflation and the output gap for that period are realized, agents in this economy know the state variables for both submodels, X_t^c and X_t^d and they can determine the model probabilities taking into account that period's realization of exogenous shocks z_t and g_t . Firms and households can then calculate what they think interest rates should have been that period given the data¹³:

$$p_t^{c;c} + p_t^{d;d} \tag{27}$$

FIGURE 6 HERE

Figure 6 plots the historical path of the interest rate and the perceived path calculated using (27). While the perceived interest rate is more volatile than the actual Federal Funds rate, it does track medium- and low-frequency movements of the actual policy instrument reasonably well. Because the zero lower bound on nominal interest rates is not modeled here, perceived interest rates can go below zero and actually do so for a small number of periods. Taking into account the zero lower bound would require using techniques that would make the estimation of this model infeasible¹⁴.

While figure 6 displays the private sector's view about monetary policy once model uncertainty is taken into account, it is also useful to ask how different the policy prescriptions coming out of each model are. After all, this difference plays a major role in determining model probabilities.¹⁵

Figure 7 plots the difference between the policy prescriptions of the models. We can see that for the beginning and the end of the sample, the discretionary central bank would have set higher interest rates on average than its committed counterpart. At the end of the 1970s and the beginning of the 1980s, the commitment central bank would have set interest rates higher to combat inflation. An interesting question is why the discretionary central bank would have set higher interest rates for most of the sample. To do so, it is instructive to look at the two policy rules $f^c(X_t^c)$ and $f^d(X_t^d)$:¹⁶

$$i_t^c = 0.0103 - 0.56z_t + 0.41g_t + 0.38\pi_{t-1} + \text{terms depending on } \lambda_{NKPC,t-1} \text{ and } \lambda_{IS,t-1} \tag{28}$$

$$i_t^d = 0.0136 - 1.51z_t + 0.92g_t + 0.56\pi_{t-1} \tag{29}$$

¹³I use the private sector's information set at the end of each period so that I do not have to take a stand on how the agents forecast the Lagrange multipliers that are state variables for the commitment problem.

¹⁴For a treatment of optimal policy under discretion and commitment that explicitly takes into account the zero lower bound on nominal interest rates see Adam & Billi (2006), Adam & Billi (2007) and Eggertsson & Woodford (2003).

¹⁵It is the relative squared distance to the actual observed interest rate, though, that ultimately determines model probabilities. This can be seen in figures 4 and 5.

¹⁶The negative sign on z_t in both policy rules is a result of that shock entering with a minus sign in equation (1), which is a standard way of writing this equation (see, for example, Lubik & Schorfheide (2004)).

Part of the answer is the higher inflation target of the discretionary central bank. It is worth noting, however, that this higher inflation target does *not* translate one-to-one to a higher constant in the policy rule for that central bank. Instead, while the policy rule for that central bank does imply a steady state value for the nominal interest rate equal to $\pi^d + r^*$, part of that steady state interest rate comes from the coefficient on lagged inflation. In fact, the difference in the constants of the two policy rules in annualized percentage terms is only 1.32%. The discretionary central bank reacts more strongly to both shocks and the lagged inflation rate than the commitment central bank, but it obviously lacks the terms depending on the Lagrange multipliers $\lambda_{NKPC,t-1}$ and $\lambda_{IS,t-1}$, coming from the nature of the commitment solution. This stronger contemporaneous response is a standard feature of discretionary policymaking, as described in, for example, Woodford (2003). Because the terms depending on Lagrange multipliers are hard to interpret for the commitment central bank, I plot the contributions of those terms in figure 8. To facilitate comparison, figure 8 also plots the contributions of the other state variables, and figure 9 plots the same decomposition for the opportunistic central bank.

To come back to the question posed above, the combination of the stronger contemporaneous and the higher inflation target lead to the higher interest rate prescriptions of the opportunistic central bank for most of the sample. Inspecting figure 8, we see that the contribution of the Lagrange multipliers drives the higher prescribed interest rate for the commitment model during the run-up of inflation in the 1970s and 1980s.

6 Private Sector Expectations

Because one-step-ahead forecasts of inflation and the output gap are important factors in determining date t values for those variables, as is evident from the New Keynesian Phillips curve (1) and the representative household's consumption Euler equation (2), it is important to ask what kind of expectations agents hold when they use the learning algorithm described in this paper. Figure 10 plots $E_t\pi_{t+1}$ and $E_t y_{t+1}$ versus the actual outcomes those expectations try to predict.

FIGURE 10 HERE

The private sector's expectations track the actual inflation rate very well and do reasonably well for the output gap. The learning algorithm presented in this paper thus endows the private sector with very reasonable expectations of future economic outcomes.

A related question is how well the expectations calculated using the learning algorithm presented here track survey measures of inflation expectations. Figure 11 contrasts the median one-year-ahead inflation expectation of the University of Michigan's Survey of Consumers ¹⁷ with one-year-ahead infla-

¹⁷Inflation expectations are surveyed every month. I use a three month average to make the survey comparable to the output of the model, which uses quarterly variables. The source for

tion expectations calculated using the learning algorithm described earlier. Note that the inflation expectations plotted in figure 11 are not directly comparable to those plotted in figure 10 because the former are one-year-ahead expectations, while the latter are one-quarter-ahead expectations. Also, the dates on the x-axis in figure 11 refer to the dates at which expectations are formed, in contrast to the dates on the x-axis of figure 10, which refers to the date inflation is actually realized.

FIGURE 11 HERE

The model consistent expectations track the lower frequency movements of survey expectations well. The decrease in expected inflation at the beginning of the 1980s is evident in both series. Because there are a number of issues commonly raised when it comes to survey measures of inflation, I will not compare the two series in greater detail. It is, however, worth remembering that the learning algorithm presented here endows the agents with expectations broadly similar to those measured in surveys.

7 The Perceived Inflation Target

As the private sector updates model probabilities, it also changes its perceptions of the long-run level of inflation as given by equation (19). The model presented here can thus be reinterpreted as a theory of perceived changes in the inflation target. It is important to emphasize that this model of private-sector behavior has nothing to say about whether or not the actual inflation target of the central bank (which is not modeled here) changed. Instead, by focusing on the private sector's perceptions, this model complements studies such as Erceg & Levin (2003), Ireland (2005) and Liu, Waggoner & Zha (2007) that explicitly model changes in the inflation target of the central bank in their models.

Figure 12 plots the perceived steady state value of inflation. The perceived steady state moves to the inflation target of the discretionary central bank at the beginning of the sample, even though inflation is still relatively low. The reason for this is that the private sector learns from interest rates and not from inflation directly and as a consequence p_i^c can move to 0, even though inflation is still far away from the inflation target of the discretionary central bank. Comparing figure 12 to the top panel of figure 10, it is evident that most of the low-frequency movement of inflation expectations is governed by changes in the perceived steady state of inflation, but that there are considerable medium- and high-frequency dynamics in inflation expectations beyond those dynamics in perceived steady states.

FIGURE 12 HERE

the survey data is the FRED database of the Federal Reserve Bank of St. Louis.

8 Gains from Transparency and Commitment

To address the issue of gains from transparency, I will calculate several counterfactual scenarios. In what follows, I will mainly focus on the gains from transparency of a commitment central bank. The results show that there are no gains from transparency for a discretionary central bank.¹⁸

The counterfactual scenarios differ from the actually estimated model along two dimensions: the interest rate set in the economy and the beliefs of the private sector.¹⁹ The first counterfactual asks what inflation and the output gap would have been if the Federal Reserve had actually set the interest at its historical levels, but the private sector believed that the Federal Reserve was acting under commitment ($p_t^c = 1 \forall t$). As we will see below, this change in beliefs will have substantial effects on inflation and the output gap. It is thus reasonable to question the assumption that the Federal Reserve would have set the Federal Funds rate to the historically observed levels. To remedy this shortcoming and to analyze a situation where beliefs about policy are correct, I turn to the second counterfactual. This counterfactual asks what output and inflation would have been if the Federal Reserve had not only convinced the public that it was a central bank acting under commitment, but had also actually followed that policy ($i_t = i_t^c$). I then ask how outcomes differ if certainty about the Federal Reserve's conduct of monetary policy is removed from the private sector. The third counterfactual sets $p_t^c = 0.5 \forall t$ while retaining the assumption that interest rates are actually set according to the commitment policy rule. The fourth and final counterfactual endows the private sector with the belief that Federal Reserve is acting under discretion and assumes that this belief is indeed correct (so that $i_t = i_t^d$). Comparing outcomes under the second and third counterfactuals gives an estimate of the effects of transparency on inflation and the output gap for a committed central bank. Table 5 shows the mean and variance of inflation and the output gap multiplied by 100 for the data and the four counterfactuals (denoted CF1, CF2, CF3 and CF4 in that table).

TABLE 5 HERE

A few patterns emerge when comparing these outcomes across counterfactuals:

1. Comparing the data to counterfactuals 1 and 2, we see that endowing the

¹⁸The results also suggest that a discretionary central bank could improve economic outcomes by convincing the public that it is a central bank acting under commitment. This type of scenario is analyzed by, among others, Barro (1986), King, Lu & Pastén (2008), Levine, McAdam & Pearlman (2008) and Sleet & Yeltekin (2007). A closely related question is that of optimal policy when the central bank has an informational advantage over the private sector. For a recent treatment of this question, see Mertens (2008), who solves for the Markov perfect equilibrium of an economy similar to the one presented in this paper when the private sector does not directly observe the central bank's time varying output gap target.

¹⁹Counterfactuals are calculated using a subsample of 50000 draws from the original Metropolis-Hastings sample. The reported results are averages across draws. Because most parameters are tightly estimated, these averages adequately characterize the posterior distribution of outcomes under the counterfactual scenarios.

- private sector with the belief that the Federal Reserve is a central bank acting under commitment ($p_t^c = 1$) leads to inflation that is both lower on average and less volatile than what we have observed in the data.
2. This belief alone can lead to a substantially lower and more volatile output gap compared to the data, if the policy the Federal Reserve actually follows does not agree with the beliefs of the private sector. This can be seen by comparing the data to counterfactual 1.
 3. Comparing counterfactuals 1 and 2, we see that if the Federal Reserve had actually followed the commitment policy and convinced the public of that, it could have achieved both low and stable inflation without a deterioration in the output gap, both in average terms and in terms of volatility.
 4. Removing the private sector’s uncertainty is crucial in achieving lower and less volatile inflation even when the Federal Reserve follows the commitment policy rule, as can be seen by comparing counterfactuals 2 and 3.
 5. A discretionary Federal Reserve, in combination with correct beliefs of the private sector about monetary policy, would have led to very high and volatile inflation. In particular, the average inflation level would have been substantially higher after 1980.

A comparison of counterfactuals 2 and 3 highlights the *gains from transparency* for a committed central bank, while looking at counterfactual 1 shows that convincing the public of the central bank’s policy rule even without necessarily following through would have lead to lower and less volatile inflation, albeit at a cost in terms of output. Figures 13 to 16 plot the counterfactual inflation series, while figure 17 plots the difference between the actual and counterfactual output gap series.

The counterfactual calculations so far use the entire sample. Next, I turn to an analysis of the run-up in US inflation and the eventual disinflation, the period from 1970 to 1984. Looking at figures 13 to 15, we see that the first three counterfactuals lead to substantially lower inflation in that period. Following Bianchi (2009), I calculate *counterfactual sacrifice ratios*, which are defined as follows:

$$\sum_{t=1970:Q1}^{1984:Q1} (y_t - y_t^{CF}) / \sum_{t=1970:Q1}^{1984:Q1} (\pi_t - \pi_t^{CF}) \quad (30)$$

where variables with a CF superscript denote counterfactual outcomes. These counterfactual sacrifice ratios can be interpreted as the cost of lowering inflation in terms of the output gap. Table 6 shows the counterfactual sacrifice ratios for the first three counterfactuals ²⁰.

TABLE 6 HERE

²⁰As there is no disinflation in the fourth counterfactual, the counterfactual sacrifice ratio would have been harder to interpret for that counterfactual.

While the sacrifice ratio is highest in the first counterfactual, owing to the large deterioration in output gaps, there is also a substantial difference between the second and the third counterfactual. This highlights again the *gains from transparency*.

9 The Effect of Policy Actions on Beliefs

Having estimated the private sector’s beliefs about the Federal Reserve, it seems natural to ask how much the Federal Reserve could have done to change those beliefs at any point in time. Certainly a significantly different path for the nominal interest rate could have changed beliefs substantially. Rather, I focus here on *one-time changes* in the nominal interest rate i_t . In particular, I ask the following: How would a one-time change in the nominal interest rate, ranging from a 5% decrease in annualized terms to a 5% increase in annualized terms, have changed beliefs at time t ? I conduct this experiment for all sample periods. The calculation presented below takes into account the zero lower bound on the nominal interest rate, so if a counterfactual decrease calls for a negative interest rate, I set the counterfactual interest rate to zero. The results described below, particularly the asymmetry of policy effects, remain qualitatively unchanged if I disregard the zero lower bound.

FIGURE 18 HERE

Figure 18 plots the difference between the mean probabilities of the commitment model implied by the changed interest rates (one series for each change in 1% increments) and the mean probability of the commitment model from the draws of the Metropolis-Hastings algorithm. It is important to keep in mind that these series represent the results of a series of experiments, namely a series of one-time changes in i_t . Note that the effects of policy on beliefs are highly asymmetric and vary greatly across time. The possible losses in credibility (i.e., decreases in p_t^c) are larger in absolute value than the possible gains. Thus, it is easier to lose credibility than to gain it.

In general, the magnitude of the change in beliefs can be driven by different variables: the interest rate prescriptions from both submodels, the relative difference between these prescriptions and the counterfactual interest rate and prior beliefs. To see this, it is instructive to look at $\frac{\partial p_t^c}{\partial i_t}$:

$$\frac{\partial p_t^c}{\partial i_t} = \frac{p_{t-1}^c}{p_{t-1}^c l_t^c + p_{t-1}^d l_t^d} \left(\partial l_t^c / \partial i_t - l_t^c \frac{p_{t-1}^c (\partial l_t^c / \partial i_t) + p_{t-1}^d (\partial l_t^d / \partial i_t)}{p_{t-1}^c l_t^c + p_{t-1}^d l_t^d} \right) \quad (31)$$

This derivative is close to zero either when beliefs are close to dogmatic ($p_{t-1}^c \approx 1$ or $p_{t-1}^c \approx 0$) or when the interest rate prescriptions of both models are very similar, leading to very similar likelihoods for the two submodels ($l_t^d \approx l_t^c$).

Each of these drives the implied changes in beliefs at different points in the sample. In the 1970s, a one-time change in i_t of the magnitudes considered here would have had hardly any effect on beliefs. This can be explained by very firmly held beliefs of the private sector during that period ($p_t^c \approx 0$), which makes

changes in those beliefs very hard to come by. In the latter part of the sample, both forces described earlier are at play when the changes in beliefs are close to zero. It seems very important for any central bank to consider which of the two forces leads to unchanged beliefs; it might be very hard to change strongly held beliefs, but policy prescriptions of the two submodels could change quite a bit from period to period, as seen in previous sections.

10 Conclusion

This paper presents a theory of private-sector decision making with a particular focus on belief formation about monetary policy²¹. The theory confirms anecdotal evidence that the Federal Reserve was seen as a policymaker acting under discretion in the 1970s and that it took the drastic policy measures of the Volcker Federal Reserve to change those beliefs. However, the policy actions of the 1980s did not leave firms and households certain that the Federal Reserve acted under commitment.

The gains from transparency for a central bank acting under commitment would have been large for the sample considered here. Such a transparent central bank could have avoided the large increase in inflation in the 1970s.

Finally, this paper shows how much influence the Federal Reserve had on beliefs at different points in the last 40 years and clarifies why that influence varied substantially across time.

The estimation algorithm used here highlights the fact that the estimation of learning models is possible without taking a stand on the true data-generating process of the aspects of the economy about which economic agents are uncertain.

While the specific model presented here does not discuss financial frictions, the methodology presented in this paper could be used to study issues such as the recent financial crisis. In particular, by modifying the equations governing private-sector behavior (by introducing a housing sector, for example), as well as the models the private sector uses to form expectations about the Federal Reserve, the methodology presented here could deliver useful insights into the interaction between the private sector and monetary policy during a financial crisis.

²¹The learning algorithm presented here endows the private sector with a large amount of information and only leaves firms and households uncertain about a particular feature of the economy. In contrast, a considerable amount of previous work on learning in macroeconomics, such as Milani (2007), endows the private sector with considerably less information. Another approach that endows economic agents with more information about the structure of the economy is presented in Preston (2005).

A Data

In this paper I use a quarterly sample starting in the first quarter of 1960 and ending in the third quarter of 2004. The following data series are used:

- quarterly PCE inflation
- the quarterly average Federal Funds rate.
- deviations of log per capita real output from a trend calculated using the Hodrick-Prescott Filter (Hodrick & Prescott (1997)) in real time. I calculate the trend using the Hodrick-Prescott filter for every period in the sample (using only data up until that time period), then calculate the deviations of the most recent observation from the most recent value for the trend and build up a sample for the output gap that way. I do so since I use conditional one-step-ahead forecast densities to calculate the likelihood, which could be problematic if the trend is calculated only once using the entire sample.

The raw data for the Federal Funds rate and per capita output are the same that are used in Smets & Wouters (2007) and more information about the data can be found in that paper. The source for the PCE price index used for calculating PCE inflation is the Bureau of Economic Analysis. Per capita real output and the PCE price index are seasonally adjusted.

B Recursive Computation of the Likelihood

The components of (22) can be recursively computed as follows:

1. Using prior model probabilities p^c and p^d , initial state variables for both submodels, X_0^c and X_0^d and initial observations π_0 , y_0 and i_0 as well as initial values for z_0 and g_0 (set to 0 in the actual estimation) and a parameter vector Θ , do the following:
 - calculate the optimal policies under discretion and commitment (5) and (6)
 - solve for the policy prescriptions of both models at time 0 using the policy rules (5) and (6) and initial state vectors for both models X_0^c and X_0^d
 - use those policy prescriptions and i_0 to calculate the likelihoods of the submodels at time 0
 - use the likelihoods, p^c and p^d , and equation (10) to calculate p_0^c and p_0^d
 - use X_0^c and X_0^d to calculate $E_0^c(\pi_2)$, $E_0^c(y_2)$, $E_0^d(\pi_2)$ and $E_0^d(y_2)$ which can be computed using the solution to the two optimal policy problems
 - use the calculated expectations and time 1 data to back out z_1 and g_1 using (1) and (2)

- given observations for z and g dated 0 and 1 back out ε_1 and use that to update the multipliers using (14)
 - calculate the time 1 contribution to the likelihood of the entire model using ε_1 and its distributional assumption
2. for all $t = 2, \dots, T$ do the following:
- solve for the policy prescriptions of both models at time $t - 1$ using the policy rules (5) and (6) and state vectors for both models X_{t-1}^c and X_{t-1}^d
 - use those policy prescriptions and i_{t-1} to calculate the likelihoods of the submodels at time $t - 1$
 - use the likelihoods, p^c and p^d , and equation (10) to calculate p_{t-1}^c and p_{t-1}^d
 - use X_{t-1}^c and X_{t-1}^d to calculate $E_{t-1}^c(\pi_{t+1})$, $E_{t-1}^c(y_{t+1})$, $E_{t-1}^d(\pi_{t+1})$ and $E_{t-1}^d(y_{t+1})$ which can be computed using the solution to the two optimal policy problems
 - use the calculated expectations and time t data to back out z_t and g_t using (1) and (2)
 - given observations for z and g dated $t - 1$ and t back out ε_t and use that to update the multipliers using (14)
 - calculate the time t contribution to the likelihood of the entire model using ε_t and its distributional assumption

C Numerical Implementation of the Metropolis-Hastings Algorithm

I use a standard Random Walk Metropolis-Hastings algorithm to generate draws from the posterior distribution of the parameters. I generate $N = 1100000$ draws, where I discard the first 100000 to allow for effects of the initial condition to wear off.

To generate a starting value Θ_0 I start off different optimizers²² at 1000 randomly chosen points in the parameter space to get a preliminary estimate of the posterior mode²³. I also calculated the negative of the inverse Hessian at that point. Usually a scaled version of that matrix is used as the innovation matrix in the random walk proposal, but I found for my purposes that a weighted average (with weight .5) of that matrix and the prior covariance matrix worked better numerically. Note that nothing in the theory of the Metropolis-Hastings algorithm requires the innovation matrix to be a scaled version of the negative inverse Hessian.

A detailed description of the Metropolis-Hastings algorithm can be found in, for example, An & Schorfheide (2007). The posterior distributions of all counterfactuals and statistics such as p_t^c are calculated using 50000 draws, which are drawn uniformly from the entire set of 1000000 draws.

²²I used the Knitro suite of optimizers.

²³I maximized the posterior kernel since the normalizing constant is notoriously hard to estimate.

D "Standard" Prior

Table 1: Description of "standard" priors. For parameters with restricted range (relative to how the usual range for the relevant distribution) the moments refer to the prior distribution *without* the restriction.

| Variable | Prior Distribution | Range | Prior Mean | Prior Standard Deviation |
|---------------|--------------------|---------------------|------------|--------------------------|
| κ | Gamma | $[0, \infty]$ | 0.3 | 0.1 |
| σ | Gamma | $[0, \infty]$ | 2 | 0.2 |
| ρ_g | Inverse Gamma | $[0, \infty]$ | 0.8 | 0.1 |
| ρ_z | Inverse Gamma | $[0, \infty]$ | 0.8 | 0.1 |
| σ_ν | Inverse Gamma | $[0, \infty]$ | 0.01 | 0.005 |
| σ_z | Inverse Gamma | $[0, \infty]$ | 0.01 | 0.005 |
| σ_g | Inverse Gamma | $[0, \infty]$ | 0.004 | 0.003 |
| ρ_{gz} | Normal | $[-1,1]$ | 0 | 0.3 |
| π^c | Normal | $[-\infty, \infty]$ | 0.005 | 0.0025 |
| π^d | Normal | $[-\infty, \infty]$ | 0.015 | 0.0025 |
| λ^c | Normal | $[0, \infty]$ | 0.05 | 0.03125 |
| λ^d | Normal | $[0, \infty]$ | 0.13 | 0.03125 |
| λ_i^c | Normal | $[0, \infty]$ | 0.1 | 0.1 |
| λ_i^d | Normal | $[0, \infty]$ | 0.1 | 0.1 |
| r | Normal | $[-\infty, \infty]$ | 0.005 | 0.0025 |

E Posterior Distribution

Table 2: Statistics of Posterior Distribution

| Variable | Posterior Mode | Posterior Mean | 5th Percentile | 95th Percentile |
|---------------|----------------|----------------|----------------|-----------------|
| κ | 0.6994 | 0.7304 | 0.6818 | 0.7581 |
| σ | 1.6055 | 1.6026 | 1.4855 | 1.6906 |
| ρ_g | 0.4021 | 0.4105 | 0.3851 | 0.4359 |
| ρ_z | 0.5727 | 0.5926 | 0.5651 | 0.6211 |
| σ_ν | 0.0061 | 0.0063 | 0.0057 | 0.0071 |
| σ_z | 0.0078 | 0.0081 | 0.0075 | 0.0086 |
| σ_g | 0.0157 | 0.0155 | 0.0148 | 0.0161 |
| ρ_{gz} | 0.6074 | 0.6326 | 0.5656 | 0.6815 |
| π^c | 0.0044 | 0.0048 | 0.0044 | 0.0052 |
| π^d | 0.0137 | 0.0128 | 0.0119 | 0.0137 |
| λ^c | 0.0041 | 0.0189 | 0.0014 | 0.0423 |
| λ^d | 0.0309 | 0.0330 | 0.0180 | 0.0451 |
| λ_i^c | 0.1128 | 0.1099 | 0.0997 | 0.1199 |
| λ_i^d | 0.3149 | 0.3123 | 0.2608 | 0.3689 |
| r | 0.0076 | 0.0059 | 0.0043 | 0.0081 |

F Tables for Main Body of the Paper

Table 3: Posterior

| Variable | Posterior Mode |
|---------------|----------------|
| κ | 0.6994 |
| σ | 1.6055 |
| ρ_g | 0.4021 |
| ρ_z | 0.5727 |
| σ_ν | 0.0061 |
| σ_z | 0.0078 |
| σ_g | 0.0157 |
| ρ_{gz} | 0.6074 |
| π^c | 0.0044 |
| π^d | 0.0137 |
| λ^c | 0.0041 |
| λ^d | 0.0309 |
| λ_i^c | 0.1128 |
| λ_i^d | 0.3149 |
| r | 0.0076 |

Table 4: Rescaled Posterior Mode Estimates

| Annualized Inflation Targets in Percent | |
|---|--------|
| commitment | 1.76 |
| discretion | 5.48 |
| Rescaled Weights on Output Gap in Loss Function | |
| commitment | 0.0656 |
| discretion | 0.4944 |

Table 5: Counterfactual outcomes

| | data | CF1 | CF2 | CF3 | CF4 |
|--------------------------|------|-------|------|------|------|
| mean, inflation | 3.72 | 1.13 | 1.91 | 3.40 | 5.17 |
| variance, inflation | 6.45 | 1.57 | 1.38 | 2.28 | 5.14 |
| mean, 100*output gap | 0.01 | -0.27 | 0.02 | 0.05 | 0.06 |
| variance, 100*output gap | 2.64 | 2.86 | 2.40 | 2.35 | 2.34 |

Table 6: Counterfactual sacrifice ratios

| | CF1 | CF2 | CF3 |
|---|------|------|------|
| counterfactual sacrifice ratio, 1970-1984 | 0.42 | 0.17 | 0.27 |

G Figures for Main Body of the Paper

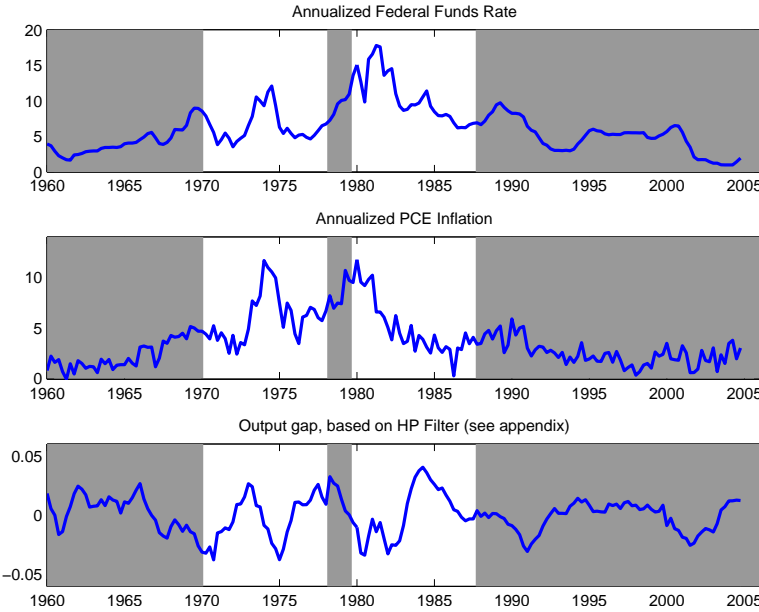


Figure 1: Data used in this paper

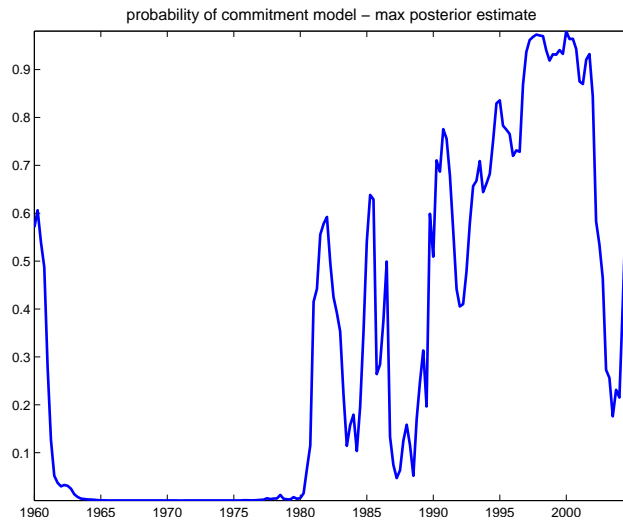


Figure 2: p_t^c at the posterior mode estimates

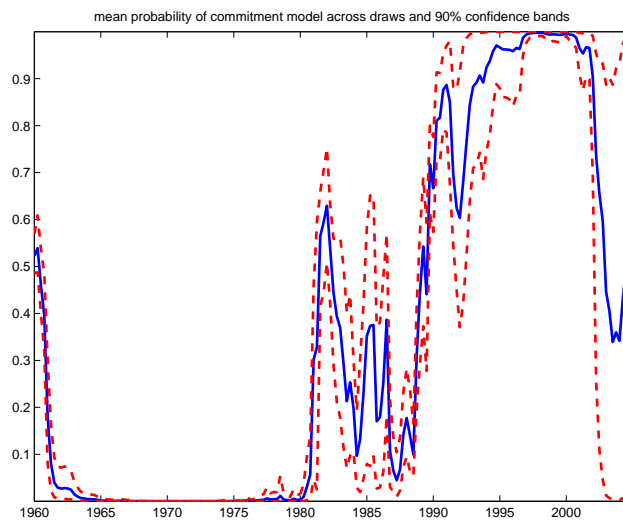


Figure 3: Posterior distribution of p_t^c

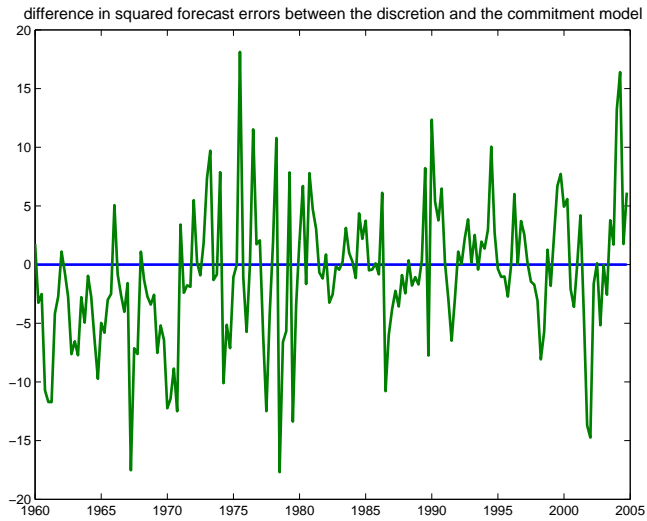


Figure 4: Difference in squared prediction errors across submodels

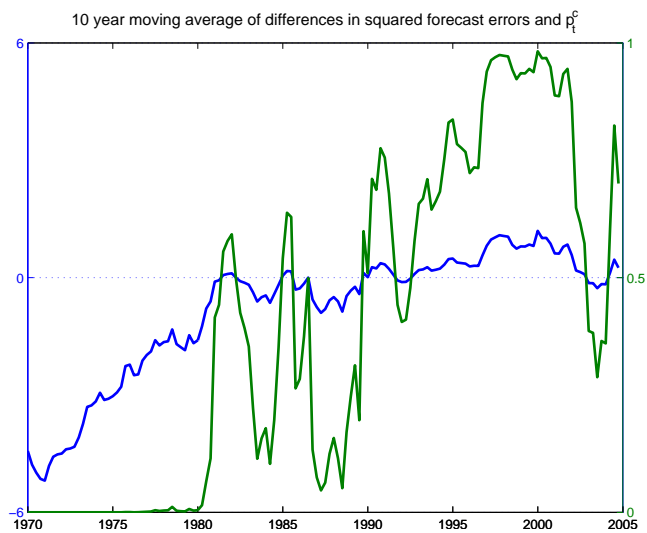


Figure 5: Moving average of difference in squared forecast errors vs p_t^c

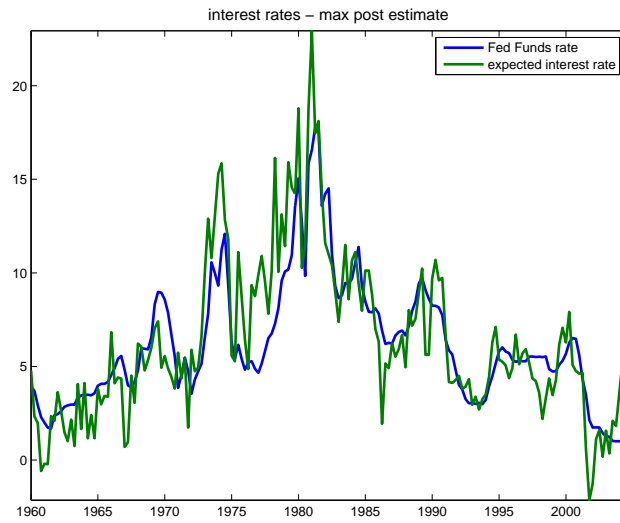


Figure 6: Perceived and actual values for i_t

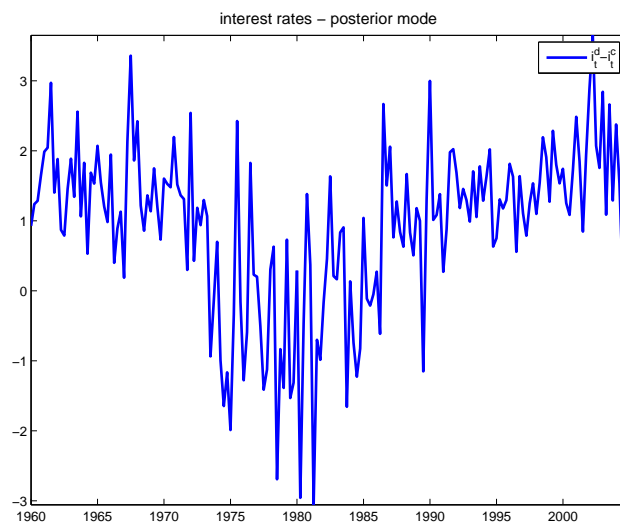


Figure 7: Difference in interest rate prescriptions

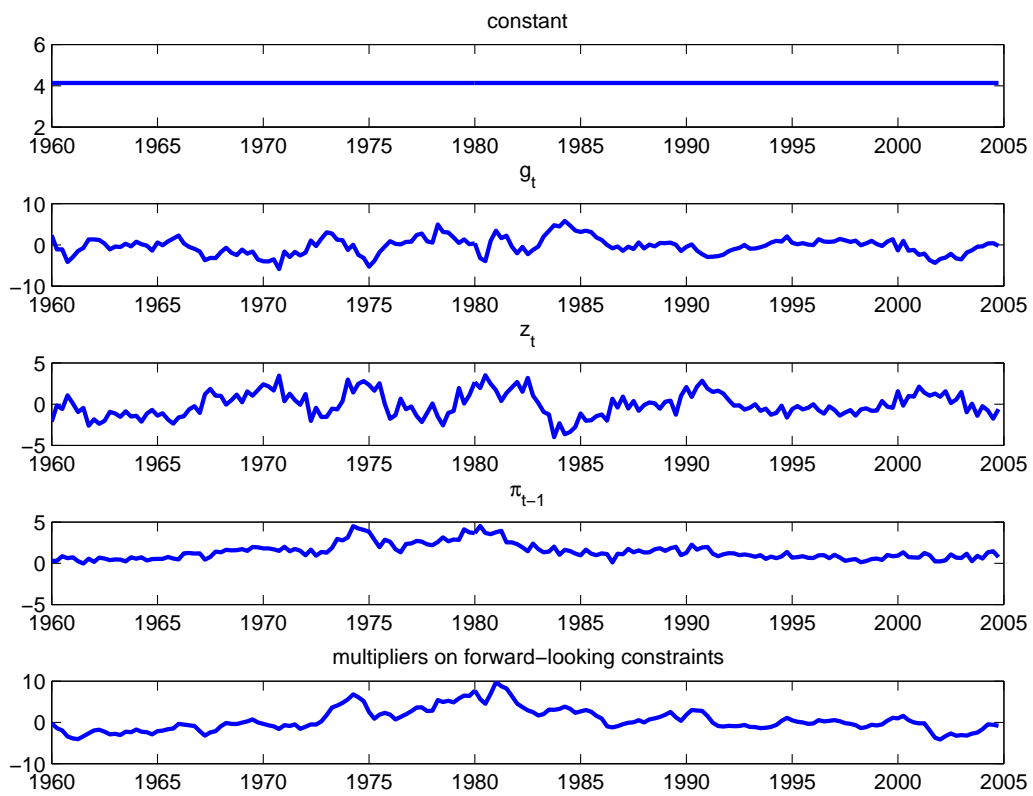


Figure 8: Contributions of different state variables to i_t^c

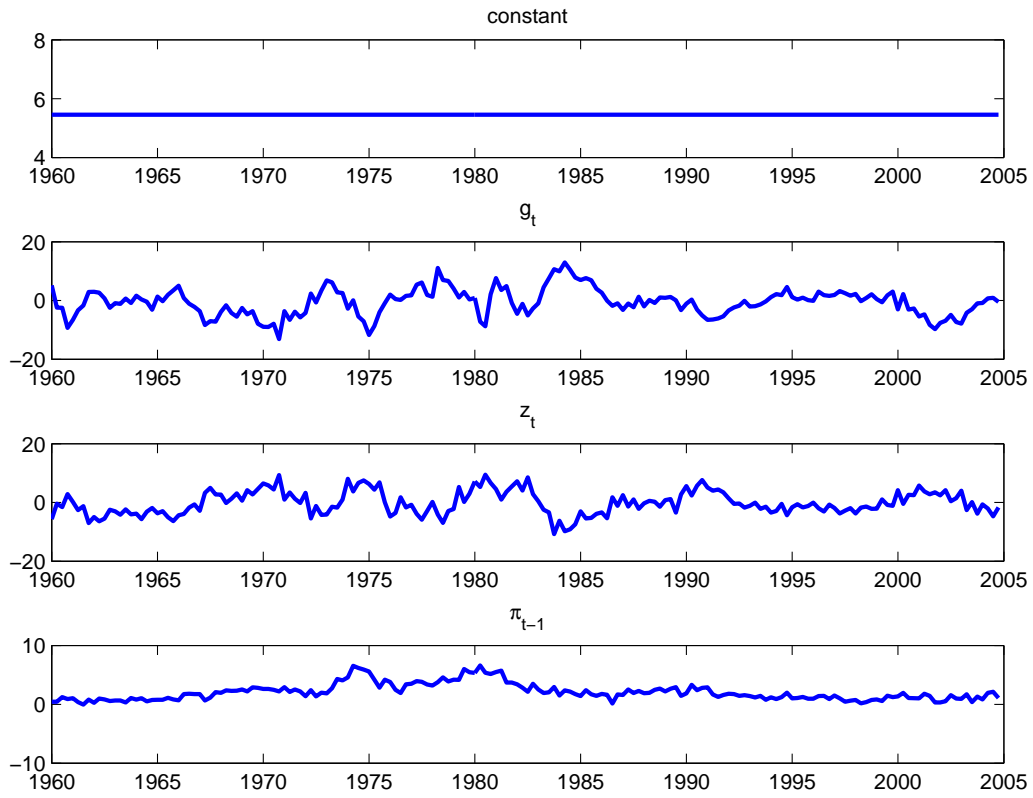


Figure 9: Contributions of different state variables to i_t^d

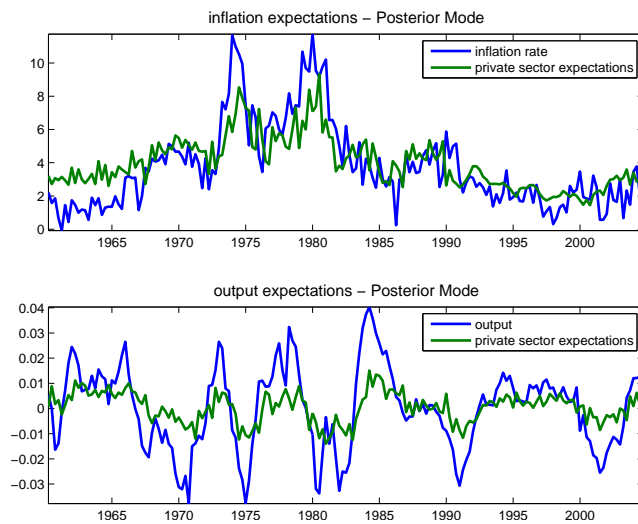


Figure 10: Private sector expectations

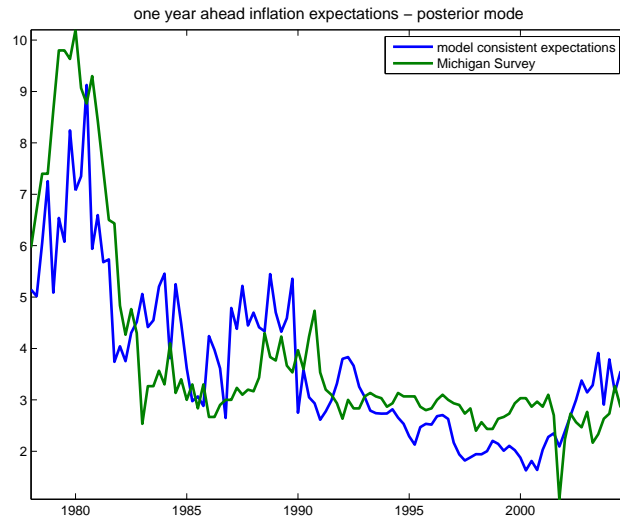


Figure 11: Private sector expectations from the model and a survey measure of inflation expectations

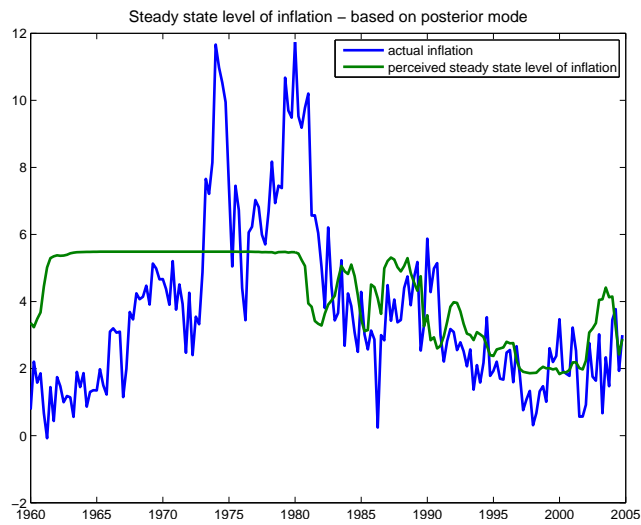


Figure 12: Perceived steady state of inflation

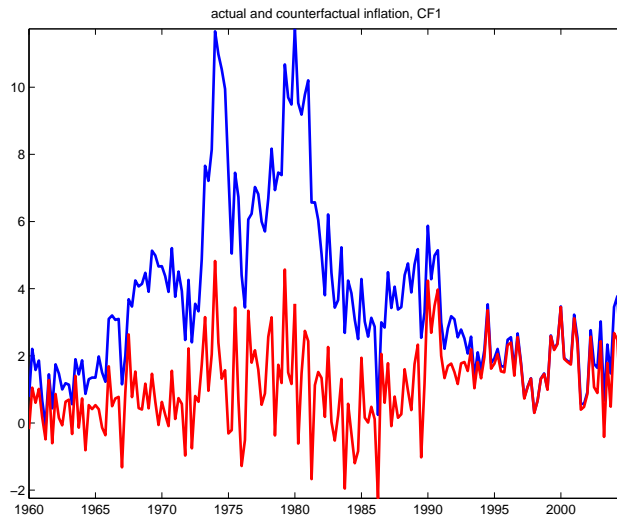


Figure 13: First counterfactual

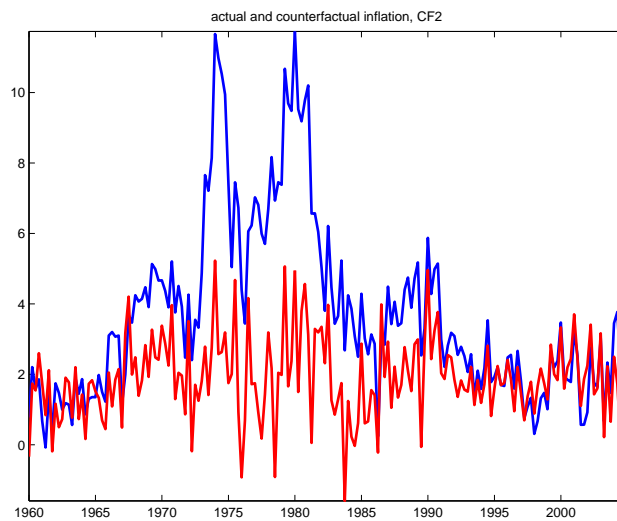


Figure 14: Second counterfactual

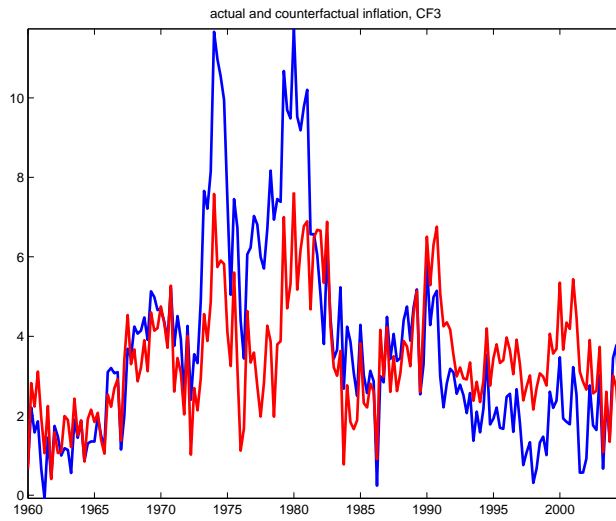


Figure 15: Third counterfactual

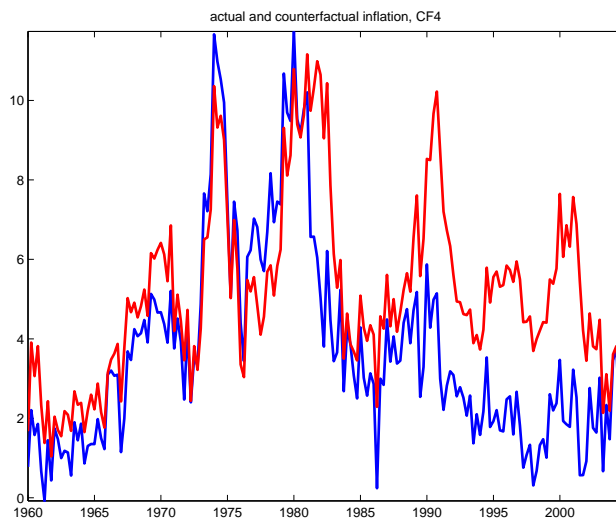


Figure 16: Fourth counterfactual

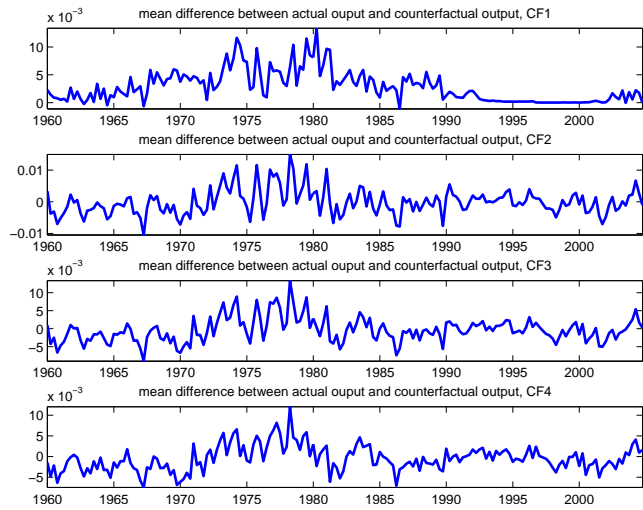


Figure 17: Counterfactual output gaps

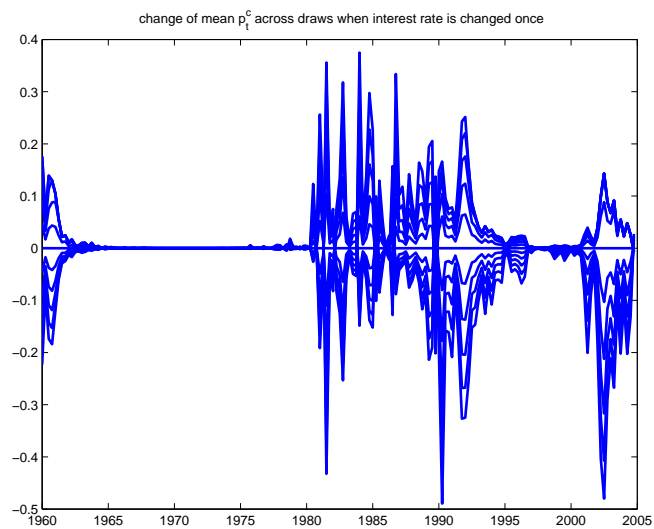


Figure 18: Difference between the mean counterfactual probability of the commitment model and the mean probability calculated from data

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