Implementable Fiscal Policy Rules*

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Abstract

We use a novel procedure to identify fiscal feedback rules for the US: We start by estimating a DSGE model and on that basis compute the Ramsey optimal responses to structural shocks. Then we let the policy maker choose from a general set of rules to match the dynamic behavior of a number of key variables like output, debt, and consumption, in the competitive equilibrium with their corresponding dynamic behavior in the Ramsey equilibrium. In the next step we estimate the model again but employ the contingency derived previously. The policy rules derived are general, not as complex as Ramsey and easily implementable.

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1 Introduction

What is fiscal policy and what are its effects? The latter part of the question cannot be answered without taking an explicit stand on the former. This paper sheds light on the former part, thus providing insights into the latter. While the literature agrees on how the private sector should be modeled, the fiscal policy sector is either modeled as a simple ad-hoc process or as welfare-optimizing (Ramsey). However, the first way to model fiscal policy probably assumes too little purposeful action by the policymaker, while the second implies an omnipotent and omniscient one. We develop a novel procedure to identify end estimate fiscal feedback rules for the US economy: First, we estimate a medium scale DSGE model using Bayesian estimation techniques. Second, we employ a Smyrnov-test to identify those policy coefficients significantly influencing the dynamics of the observable variables around the Ramsey steady state computed at the posterior mode. Third, we estimate the model again but employ the contingency derived previously. Thus we sidestep the pitfalls of both common approaches by on the one hand modeling fiscal policy behavior endogenously and not ad-hoc, while on the other hand, not assuming Ramsey optimal behavior.

We start by estimating a benchmark medium scale DSGE model as recently put forward by Schmitt-Grohé and Uribe (2007). We think of the DSGE model as containing two sets of behavioral equations: one describing the private sector and one describing the fiscal policy sector. The private sector is solely characterized by the solution to the households' and firms' problems and the corresponding structural model parameters.

We identify candidates for extensions of the stylized policy rules employed so far in the following way: Given the posterior distribution we compute the Ramsey solution at the posterior mode. For taxes on capital, private consumption, and labor, we specify very general policy rules consisting of policy coefficients linking the tax rates to a large set of key economic variables as output, government debt, private consumption, real wages, inflation, hours worked as well as the capital stock. We simulate artificial time series at the Ramsey solution. The general tax rules are estimated given the simulated time series. At the posterior mode we check whether the policy coefficients are identified and to which extend they influence the moments of the observable variables. The policy coefficients that influence significantly the dynamic behavior of the observable variables[1] around the Ramsey steady state constitute the extended policy rules to be estimated.

The relevant statistics are computed applying the local identification techniques as described in Iskrev (2010). More precisely, we compute the Jacobian of the moments of the observable variables with respect to the policy coefficients. Finally, we estimate the new contingencies by re-estimating the DSGE model.

The remaining paper is organized in the following way: section 2 summarizes the related literature. Section 3 describes the model. Afterwards we lay out the methodology how we determine the policy rules in section 4. In section 5 we present the results while section 6 concludes.

2 Related literature

After the study of Christiano, Eichenbaum, and Evans (2005), who have been among the first to extend a standard DSGE model with various features and frictions, DSGE models have been increasingly employed to estimate the dynamic effects of policy changes[2]. Fiscal policy in these models is, if at all present, modeled as an additional exogenous disturbance to the economy. However, as recently put forward by Curdia and Reis (2009), this way of describing the fiscal sector comes at the cost of misspecified models.

Building on the work of Baxter and King (1993), Galí, López-Salido, and Vallés (2007) and Leeper and Yang (2008) recent studies aimed at empirically characterizing the behavior of the fiscal policy sector. Forni, Monteforte, and Sessa (2009) characterize fiscal policy in a simple way by estimating feedback rules on debt following Bohn (1998). Similarly, Coenen and Straub (2005) estimate policy rule for lump sum taxes which responds to government

[1] As a starting point we consider the observable variables. However, it is straightforward to consider some unobserved variables, which are more relevant for welfare instead.

[2] See for instance Smets and Wouters (2003, 2007) for further examples.

debt and government expenditure. Jones (2002) assumes that that fiscal policy reacts on current and lagged output as well as hours worked. Leeper, Plante, and Traum (2009) include output as additional variable into the policy rules and consider potential correlations of the tax rates. The choice of fiscal policy coefficients are motivated by the following considerations: the first reason to include e.g. debt into the fiscal rules is to ensure the stability of the model. Output is chosen to capture the behavior fiscal stabilizers with respect to the business cycle.

Another strand of the literature investigates fiscal policy from an welfare maximizing perspective. While Schmitt-Grohé and Uribe (2005) estimate feedback parameters for monetary and fiscal policy rules which mimic the dynamic behavior of the welfare-optimizing Ramsey planner; Benigno and Woodford (2006b) derive optimal fiscal rules by deriving the correct feedback variables as well as corresponding parameter loadings by using their linear quadratic approach (Benigno and Woodford, 2006a).

3 Model

The DSGE model employed is a conventional New Keynesian model with a fiscal policy as augmented by Benigno and Woodford (2006b). The model incorporates nominal as well as real frictions as postulated by Christiano et al. (2005) and Smets and Wouters (2007). In particular, the economy faces real rigidities as internal habit formation, capital utilization, and investment adjustment costs. Additionally, there are two nominal rigidities for wages and prices, both following the adjustment process postulated by Calvo (1983).

Throughout the model description capital letters denote nominal and small letters real variables. An exception is investment, which is always expressed in real terms as I.

3.1 Households

In the economy exists a continuum of households indexed by $i \in (0,1)$. Each household i consumes $(c(i))$ and provides labor services (L). The preferences are characterized by the discount factor β, the inverse of the intertemporal substitution elasticity σ_c, the inverse of the labor supply elasticity with respect to wages σ_l. The parameter h measures the internal habit persistence regarding consumption. Utility takes the following functional form:

$$E_t \sum_{t=1}^{\infty} \beta^t \left[\frac{\left(c_t(i) - hC_{t-1}(i)\right)^{1-\sigma_c}}{1-\sigma_c} - \frac{L_t(i)^{1+\sigma_l}}{1+\sigma_l} \right] \tag{1}$$

Household i holds government bonds B yielding return R^b and invests $I(i)$ into capital k. Capital pays an interest rate R^k. Firms pay dividends d. The wage rate $W_t(i) \cdot L_t(i)$ is set after learning about whether it is allowed to optimize wages. The household i also receive $s_t(i)$ the net cash flow from state contingent securities. The existence of this security ensures that in equilibrium the households are homogenous with respect to consumption and asset holdings, but heterogenous with respect to wages and hours worked (e.g. Christiano et al., 2005). Finally, the household pays taxes τ_w and τ_k for labor income and capital income respectively. Finally, the budget constraint of the household is characterized by

$$\begin{aligned} c_t(i) + I_t(i) + b_t(i) = {} & (1-\tau_t^w) \frac{W_t(i)}{P_t} L_t(i) + \left(1-\tau_t^k\right) \frac{R_t^k u_t(i) k_{t-1}(i)}{P_t} L_t(i) \\ & + \frac{R_{t-1}^b b_{t-1}(i)}{\pi_t} + d_t(i) + s_t(i). \end{aligned} \tag{2}$$

Capital utilization can be varied equivalent to the assumption made by Smets and Wouters (2007). Cost of capacity utilization are given by $\phi_t(\cdot)$. As functional form we assume:

$$\phi_t(u) = \frac{(1-\bar{\tau}_k)\bar{R}^k}{\sigma_u} \left(\exp\left(\sigma_u (u_t - 1)\right) - 1\right) \tag{3}$$

Following Smets and Wouters (2007) we define a new parameter $\psi \in [0,1)$ such that $\sigma_u = \frac{\psi}{1-\psi}$. Following e.g. Smets and Wouters (2003) the law of motion for capital accumulation is given by

$$k_t\left(i\right) = \left(1-\delta\right)k_{t-1}\left(i\right) + \left[1 - s_t\left(\frac{\varepsilon_{i,t}I_t}{I_{t-1}}\right)\right]I_t\left(i\right), \tag{4}$$

where δ denotes the depreciation rate and $s\left(\cdot\right)$ an convex investment adjustment cost function. Additionally, ε_i is a investment specific efficiency shock to the adjustment costs and is supposed to follow the an autoregressive process:

$$\log \varepsilon_{i,t} = \rho_i \log \varepsilon_{i,t-1} + \epsilon_t^i, \tag{5}$$

with ϵ_t^i is assumed to be *i.i.d.* distributed. For the adjustment cost function we assume the following functional form:

$$s_t\left(\frac{\varepsilon_{i,t}I_t}{I_{t-1}}\right) = \frac{\nu}{2}\left(\frac{\varepsilon_{i,t}I_t}{I_{t-1}} - 1\right)^2 \tag{6}$$

3.2 Labor market

Following Erceg, Henderson, and Levin (2000), we model the wage setting analogously to staggered price setting introduced by Calvo (1983). Each household supplies a differentiated type of labor service, $l_t(i)$, which is aggregated into a homogenous labor good by a representative competitive firm. This firm uses the following technology:

$$l_t = \left[\int_0^1 l_t\left(i\right)^{\frac{\theta_w - 1}{\theta_w}}\right]^{\frac{\theta_w}{\theta_w - 1}},$$

where $\theta_w > 1$ is the elasticity of substitution. Finally, the demand for labor of type i is given by,

$$l_t(i) = \left[\frac{W_t\left(i\right)}{W_t}\right]^{-\theta_w} l_t^d, \tag{7}$$

where $W_t\left(i\right)$ is the nominal wage demanded by labor of type i and W_t is the wage index defined as

$$W_t = \left[\int_0^1 W_t\left(i\right)^{\theta_w - 1}\right]^{\frac{1}{\theta_w - 1}}.$$

Given the demand curve of labor, each household supplies as many labor services as demanded at this wage. The household has to set his wage. In each period the household can optimize his wage with probability $1 - \gamma_w$ and with probability γ_w he cannot. If the household can not optimize its wage, the wage rate in t is given by:

$$W_t(i) = \bar{\pi} W_{t-1}(i), \tag{8}$$

where $\bar{\pi}$ is the steady-state inflation rate of the economy, otherwise he would set the wage $W_t^\star$ The household optimizes its wage $W_t(i)$ by maximizing the following objective function:

$$E_t \left[\sum_{j=0}^{\infty} (\gamma_w \beta)^j \left[\lambda_{t+j} \bar{\pi}^j \frac{W_t(i)}{P_{t+j}} l_{t+j}(i) - U\left(l_{t+j}(i), c_{t+j}(i)\right) \right] \right] \tag{9}$$

The corresponding first order condition of the household is given by:

$$E_t \left[\sum_{j=0}^{\infty} (\gamma_w \beta)^j \left[\bar{\pi}^j \frac{W_t(i)}{P_{t+j}} l_{t+j}(i) - \frac{\theta_w - 1}{\theta_w} MRS_{t+j}\left(H_{t+j}(i), C_{t+j}(i)\right) \right] \right] = 0 \tag{10}$$

where $MRS = \frac{U_l}{U_c}$ is the marginal rate of intratemporal substitution between consumption and labor. Moreover, the nominal aggregate wage evolves according to

$$W_t = \left[\gamma_w \left(\bar{\pi} W_{t-1}\right)^{1-\theta_w} + (1 - \gamma_w)\left(W_t^\star\right)^{1-\theta_w} \right]^{\frac{1}{1-\theta_w}} \tag{11}$$

Finally, we define real wage inflation π^w as:

$$\pi_t^w = \frac{w_t}{w_{t-1}} \pi_t \tag{12}$$

and using our definitions for the labor demand (eq.7) to re-write the wage setting problem in recursive form as follows:

$$K_t^w = \left(l_t^d\right)^{1+\sigma_l} + \beta\gamma_w \left(\frac{\bar{\pi}}{\pi_{t+1}^w}\right)^{-\theta_w(1+\sigma_l)} K_{t+1}^w \tag{13}$$

$$F_t^w = \frac{(\theta_w - 1)}{\theta_w} \left(1-\tau_t^w\right) l_t^d \chi_t + \beta\gamma_w \left(\frac{\pi_{t+1}}{\pi_{t+1}^w}\right)^{-\theta_w} \left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{1-\theta_w} F_{t+1}^w \tag{14}$$

$$\frac{Kw}{Fw} = \frac{1}{\psi_l} \left(w_t^*\right)^{1+\theta_w\sigma_l} w_t \tag{15}$$

where w_t is the real wage and $w_t^* = \frac{W_t^\star}{W_t}$ and follows the following law of motion:

$$1 = \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w}\right)^{1-\theta_w} + (1-\gamma_w)\left(w_t^*\right)^{1-\theta_w} \tag{16}$$

3.3 Firms

The production sector consists of intermediate and final goods producing firms. The final good, Y_t, is produced under the constant-return-to-scale production function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta_p - 1}{\theta_p}}\right]^{\frac{\theta_p}{\theta_p - 1}},$$

where $Y_t(i)$ is the intermediate good and let $P_t(i)$ be its nominal price, such that the corresponding price index, P_t is given by:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_p}\right]^{\frac{1}{1-\theta_p}}.$$

It is assumed that households and the government demand the final good. The demand curve of the final good is given by:

$$Y_t(i) = \left[\frac{P_t}{P_t(i)}\right]^{\theta_p} Y_t^d \tag{17}$$

The intermediate goods are produced by an existing continuum of monopolistically competitive firms $j \in [0,1]$ using the production function:

$$y_t(j) = (u_t(j)\, k_{t-1}(j))^{\alpha} (l_t(j)\, \varepsilon_{z,t})^{1-\alpha}, \tag{18}$$

where α denote the output elasticity with respect to capital. $\varepsilon_{z,t}$ denotes a labor augmenting productivity shock and is assumed to follow a process given by:

$$\log \varepsilon_{z,t} = \rho_z \log \varepsilon_{z,t-1} + \epsilon_t^z \tag{19}$$

Firms minimize costs according to:

$$\max_{u_t \cdot k_{t-1}, l_t} \left[\left[\frac{P_t(i)}{P_t} \right]^{-\theta_p} (u_t k_{t-1}(j))^{\alpha} (l_t(j)\, \varepsilon_{z,t})^{1-\alpha} - W_t l_t(j) - R_t^k k_{t-1}(j) \right], \tag{20}$$

The first order conditions combined with the production function and its derivative yield expressions for relative prices, the capital market equilibrium and marginal costs:

$$\frac{R_t^k}{W_t} = \frac{\alpha}{1-\alpha} \frac{l_t^P(j)}{u_t k_{t-1}(j)} \tag{21}$$

and the finally necessary capital market equilibrium in real terms:

$$u_t k_{t-1} = \left(\frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k} \right)^{1-\alpha} y_t \varepsilon_{z,t}^{\alpha-1} \tag{22}$$

$$mc_t = \zeta w_t^{1-\alpha} (r_t^k)^{\alpha} \tag{23}$$

with

$$\zeta = \frac{\varepsilon_{z,t}^{\alpha-1}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}$$

Firms nominal profits are given by:

$$D_t = \left[\frac{P_t\,(i)}{P_t}\right]^{1-\theta_p} Y_t - MC_t Y_t \tag{24}$$

As postulated by Calvo (1983) we assume that the prices are staggered. This means that the monopolistic firm can adjust her prices, $P_t^{\star}$, with probability $1-\gamma_p$, independently from other and independently of the subsequent price setting. Thus, a fraction of $1-\gamma_p$ monopolistic firms adjust their prices in period t, while the rest of monopolistic firms γ_p cannot adjust their prices and set $P_t\,(i) = \bar{\pi} P_{t-1}$. These assumption ca be written as aggregate price index in form of:

$$P_t = \left[\gamma_p \left(\bar{\pi} P_{t-1}\right)^{1-\theta_p} + \left(1-\gamma_p\right)\left(P_t^{\star}\right)^{1-\theta_p}\right]^{\frac{1}{1-\theta_p}} \tag{25}$$

Firms who can reset their price choose $P_t\,(i)$ to maximize the expected sum of discounted future profits:

$$\max_{P_t(i)} \quad E_t \sum_{j=0}^{\infty} \gamma_p^j m_{t+j} \left[\bar{\pi}^j P_t\,(i)\, Y_{t+j}\,(i) - MC_{t+j} Y_{t+i}\,(i)\right] \; . \tag{26}$$

MC_t denotes the nominal marginal cost and m_t is the real stochastic discount factor given as $m_{t+j} = \beta^j \frac{\chi_{t+j} P_t}{\chi_t P_{t+j}}$ with χ_t marginal utility with respect to consumption. The first-order condition of this maximization problem implies that prices in period t are set according to:

$$\frac{P_t\,(i)}{P_t} = \frac{\theta_p}{\theta_p - 1} \frac{E_t\left[\sum_{j=0}^{\infty} \theta_p^j m_{t+j} mc_{t+j} Y_{t+j}\,(i)\, \frac{P_{t+j}}{P_t}\right]}{E_t\left[\sum_{j=0}^{\infty} \theta_p^j m_{t+j} \bar{\pi}^j Y_{t+j}\,(i)\right]}, \tag{27}$$

where the mc_{t+j} refers to real marginal costs.

Similar to the labor market we can re-write the price setting problem by using de demand equation above and defining $p_t^* = \frac{P_t^\star}{P_t}$ in the following way:

$$F_t^p = y_t^d \chi_t + \gamma_p \beta \left(\frac{\bar{\pi}}{\pi_{t+1}} \right)^{1-\theta_p} F_{t+1}^p \tag{28}$$

$$K_t^p = \frac{\theta_p}{\theta_p - 1} y_t^d \chi_t mc_t + \gamma_p \beta \left(\frac{\bar{\pi}}{\pi_{t+1}} \right)^{-\theta_p} K_{t+1}^p \tag{29}$$

$$\frac{K_t^p}{F_t^p} = p_t^* \tag{30}$$

$$1 = \gamma_p \left(\frac{\bar{\pi}}{\pi_t} \right)^{1-\theta_p} + (1-\gamma_p) (p_t^*)^{1-\theta_p} \tag{31}$$

3.4 Policy sector

3.4.1 Monetary authority

The monetary authority sets nominal interest rates according a Taylor rule. In particular, the interest rate responds to its lagged value, current inflation and lagged output:

$$\log R_t = \rho_R \log R_{t-1} + (1-\rho_R) \left(\bar{R} + \rho_\pi (\log \pi_t - \log \bar{\pi}) + \rho_y (\log y_{t-1} - \log \bar{y}) \right) + \epsilon_t^m \tag{32}$$

where ϵ_t^m denotes an *iid* error term.

3.4.2 Fiscal authority

The fiscal authority receives tax income t and issues bonds b to finance government consumption expenditure c^g. The government budget constraint therefore reads:

$$\left[\frac{b_t \pi_{t+1}}{R_t} - b_{t-1} \right] = c_t^g - t_t \tag{33}$$

where government tax revenues are equal to:

$$t_t = \tau_t^w w_t l_t + \tau_t^k \left[r_t^k u_t k_{t-1} + d_t \right] \tag{34}$$

Government consumption expenditures evolve according to an exogenous process:

$$\log c_t^g = \rho_{cg} \log c_{t-1}^g + (1 - \rho_{cg}) \log \bar{c}^g + \epsilon_t^{cg} \tag{35}$$

where ϵ_t^{cg} represent an *iid* error term[3].

In the focus of the analysis are policy rules for capital- and labor- taxes. We first estimate the model to obtain a description of the household and firm behavior. To close the model we assume standard feedback rules as in e.g. Forni et al. (2009) and denote them as:

$$\log \tau_t^w = (1 - \rho_w) \left(\log \bar{\tau}^w - \eta_w \log \bar{b} \right) + \rho_w \log \tau_{t-1}^w + (1 - \rho_w) \eta_w \log b_{t-1} + \epsilon_{t,\tau^w} \tag{36}$$

$$\log \tau_t^k = (1 - \rho_k) \left(\log \bar{\tau}^k - \eta_k \log \bar{b} \right) + \rho_k \log \tau_{t-1}^k + (1 - \rho_k) \eta_k \log b_{t-1} + \epsilon_{t,\tau^k} \tag{37}$$

where ϵ_{t,τ^w} and ϵ_{t,τ^k} denote *iid* error terms.

After we have obtained the description of the household and firms behavior we specify different policy rules for (36) and (36) according to the procedure laid out in section 4.

3.5 Aggregation, market clearing and equilibrium

Because the price stickiness implies relative price dispersion across varieties, the common resource constraint doesn't (e.g. Schmitt-Grohé and Uribe, 2006). For this reason we derive the following law of motion to capture price dispersions

$$p_t^+ = (1 - \gamma_p) (p_t^*)^{-\theta_p} + \gamma_p \left(\frac{\bar{\pi}}{\pi_t} \right)^{-\theta_p} p_{t-1}^+ \tag{38}$$

Finally, the resource constraint is given by

$$p_t^+ y_t = y_t^d \tag{39}$$

$$p_t^+ (u_t k_{t-1})^\alpha \left(l_t^d \varepsilon_{z,t} \right)^{1-\alpha} = c_t + I_t + c_t^g + \psi_t (u_t) k_{t-1} \tag{40}$$

[3]As a starting point, we derive more elaborate policy rules for the tax rates on capital and labor only.

Similarly, because of wage stickiness there exists wage dispersion across labor inputs. For this circumstance we define:

$$w_t^+ = (1 - \gamma_w)(w_t^*)^{-\theta_w} + \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w} \right)^{-\theta_w} w_{t-1}^+ \tag{41}$$

which measures the wage dispersion. The labor market is cleared given $l_t = w_t^+ \cdot l_t^d$. Moreover, the wage dispersion across labor input lead to a dispersion in utility across households. To measure the degree of dispersion we define:

$$\tilde{w}_t^+ = (1 - \gamma_w)(w_t^*)^{-\theta_w(1+\sigma_l)} + \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w} \right)^{-\theta_w(1+\sigma_l)} \tilde{w}_{t-1}^+ \tag{42}$$

Finally, the aggregated utility across households can be written as:

$$U_t = \frac{(c_t - hc_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \psi_l \frac{\tilde{w}_t^+ \left(\frac{l_t}{w_t^+} \right)^{1+\sigma_l}}{1+\sigma_l} \tag{43}$$

4 Determination of fiscal policy rules

This section sets out how we determine and estimate implementable fiscal policy rules.

4.1 Choice of the benchmark model

The choice of the benchmark model has to fulfil two main characteristics: it should provide a good description of the private sector and should include (at least) some fiscal policy instruments. The model laid out in detail in section 3 is well suited as a benchmark model. As it in the succession of Christiano et al. (2005) and Smets and Wouters (2007) it is designed to capture the behavior well. It also exhibits a rich specification of the government sector including feedback rules for tax rates, government consumption expenditure as well as public labor services. In order to obtain estimates of the private sector on which basis we are going to derive the fiscal policy rules, we estimate the model with simple fiscal feedback rules first.

Denote the vector of deep parameters as ϑ^D, the vector of policy parameters[4] as ϑ^P and the combination of both as $\vartheta = [\ \vartheta^D \vartheta^P\ ]$. Given the estimation results of the model economy in which the policy rules are specified according to (36) and (37). This yields a posterior distribution for every deep parameter: $p(\vartheta^D|Y)$.

4.2 Ramsey optimal equilibrium

Given the structural estimates derived in section 4.1 we compute the Ramsey optimal equilibrium in the following way. The dynamic economy model described above contains N endogenous variables. The private sector equilibrium conditions are represented by $N-2$ conditions [5]. Instead to close the economy by characterizing fiscal policy rules as before, we assume the benevolent government implements the Ramsey optimal equilibrium.

Following Schmitt-Grohé and Uribe (2006) we assume that the government has operate for infinite number of periods and honors its commitments made in the past. As mentioned by Woodford (2003) this kind of policy under commitment is optimal from a timeless perspective.

In particular, the Ramsey equilibrium for the model proposed in the present paper can be defined as a set of the stationary variables c_t, l_t, I_t, k_t, b_t, p_t^+, w_t^+, $\tilde{w}_t^+$, p_t^*, w_t^*, y_t, u_t, w_t, π_t, π_t^w, t_t, K_t^w, F_t^w, K_t^p, F_t^p, r_t^k, R_t, mc_t, d_t, χ_t, $\varepsilon_{i,t}$, $\varepsilon_{z,t}$, and c_t^g for $t \geq 0$ that maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t - hc_{t-1}, l_t\right), \tag{44}$$

subject to the the $N-2$ competitive equilibrium conditions for $t \geq -\infty$, given the exogenous stochastic processes $\{\epsilon_t^z, \epsilon_t^i, \epsilon_t^{cg}, \epsilon_t^m\}$, values of the N endogenous variables dated $t < 0$, and values of the $N-2$ Lagrangian multipliers associated with the private sector equilibrium

[4] As mentioned before, we consider as policy parameters only parameters, which are contained in policy rules of interest. In our example those are the feedback rules for tax rates. All other parameters in policy rules are considered as deep parameters of the economy.

[5] Because we are interested in fiscal policy tax rules, we also consider the monetary policy rule as private sector equilibrium condition.

constraints dated $t < 0$.

The non-stochastic steady state of the maximization problem above is assumed to be the long-run state of the Ramsey equilibrium in the economy without uncertainty. We compute this Ramsey steady state by varying the steady state values of $\bar{\tau}_k$ and $\bar{\tau}_w$, until the system defined above is solved. Because we are just interested in tax rates, we fix the long-run Ramsey equilibrium of inflation to the one of the competitive equilibrium $\bar{\pi} = 0.5\%$. The remaining parameter are those at the posterior mean.

4.3 Local identification and sensitivity analysis of the policy coefficients

We define the extended policy rule as exhibiting those policy coefficients that are first identifiable and second influence significantly the dynamics of the observable variables at the Ramsey equilibrium. To that end, we first simulate artificial time series from at the Ramsey solution given the posterior distribution of the benchmark model. We postulate a very general policy rule for the tax rates and estimate the policy coefficients employing the simulated time series. The identification and sensitivity analysis is then conducted at the so derived posterior distribution.

To check whether the policy coefficients are identified we employ the methodology laid out by Iskrev (2010), i.e. we compute the Jacobian of the moments of the observable variables with respect to the policy coefficients. In the case the Jacobian has full rank the policy coefficients are identified. The Jacobian is the product of one Jacobian containing the derivatives of the recursive laws of motion of the DSGE model with respect to the policy coefficients and one Jacobian containing the derivatives of the moments of the observable variables with respect to the recursive laws of motions. The Jacobian matrices are calculated locally, i.e. given one vector of policy coefficients. In the case that we would like to consider the whole distribution estimated at the artificial time series, we have to conduct this analysis for across the posterior distribution.

After we have checked the identification of the policy coefficients and discarded those coefficients not identified, we compute the sensitivity of the moments of the observable variables with respect to the remaining policy coefficients. We rank them according to their importance and thereby determine the new, extended policy rule for the extended model.

4.4 The extended model

Given the results derived in section 4.3, we define the new policy rules . Those rules are substituted into the DSGE model instead of (37) and (36).

Depending on the size of the DSGE model the parameter of the new policy rules ϑ^P are estimated either jointly with the parameters of the private sector ϑ^D or using a Gibbs sampling algorithm[6]. In this application, we choose to estimate the vector of parameters jointly. If the estimates of the structural parameters describing the private sector differ from the initial estimates, we repeat the procedure as described in the former subsections.

5 Results

5.1 Data

We employ quarterly US data from 1953:1 to 2005:3. Since we have six shocks we choose six observable variables. We use two time series associated with fiscal policy: the tax rates on capital and wages. As additional observable variables we choose private consumption, GDP, inflation, and private investment. Except for the tax rates, the data was obtained from NIPA or FRED2. A detailed description of the source can be found in appendix A. The tax rates were computed as in Jones (2002). Whenever necessary, the data was transformed into real terms and per capita. Finally, since the employed model does not exhibit an endogenous trend we take de-trend the data using an one-sided HP filter with $\lambda = 1600$ to avoid the

[6]In the case of the Gibbs sampling algorithm the joint posterior distribution of ϑ^P and ϑ^D, $p(\vartheta^P, \vartheta^D|Y)$, is evaluated by estimating the conditional distributions $p(\vartheta^P|Y, \vartheta^D)$ and $p(\vartheta^D|Y, \vartheta^P)$.

endpoint problem which occurs by using the standard two-sided HP filter. The ones-sided HP filter is implemented for each time series by using a initialization window of 40 quarters. Finally, we use 171 observation corresponding to the data from 1963:1 to 2005:3 during the estimation.

5.2 Prior Choice and calibrated parameters

We calibrate $\beta = 0.9926$ in order to have a quarterly real interest rate of 1.25%. The elasticity of capital in the production is set $\alpha = 0.3$. Together with a depreciation rate of capital $\delta = 0.025$ this implies a investment to output ratio of 10.85% after taxes. The elasticity of substitution between goods is chosen to yield a steady state price mark up of 30%. Additionally, we choose the elasticity of substitution among labor inputs to receive a wage mark up of 50%. Both mark ups are identically to Smets and Wouters (2007). The steady state values for the the government expenditures to GDP ratio $\bar{c}^g/\bar{y}$ is set to 28%, which implies an private consumption to output ratio $\bar{c}/\bar{y}$ of approximately 60%. Capital taxes $\bar{\tau}^k = 0.3898$ and labor taxes $\bar{\tau}^w = 0.2006$ are calculated according to our time series. Similar to Christiano, Motto, and Rostagno (2009) we set steady state annual inflation $\bar{\pi} = 1.0202$.

Description	Symbol	Value
discount factor	β	0.9926
capital share	α	0.3
depreciation rate	δ	0.025
price markup	$\theta_p/(\theta_p - 1)$	1.3
wage markup	$\theta_w/(\theta_w - 1)$	1.5
annualized inflation	$\bar{\pi}$	1.0202
fraction of government consumption to output	$\bar{c}^g/\bar{y}$	0.28
total supply steady state capital tax rate	$\bar{\tau}_k$	0.3898
steady state labor tax rate	$\bar{\tau}_w$	0.2006

Table 1: Parameter calibration.

The remaining parameters are estimated. In general we follow the standard literature in the formulation of the prior distribution (e.g. Smets and Wouters, 2007; Christiano et al., 2009). An overview of the employed prior distributions is given in Table 2. For the inverse intertemporal elasticity of substitution and the inverse Frisch elasticity we formulate a gamma distribution with a standard deviation of 0.5 and a mean of 2 and 3 respectively. Those values are slightly higher as in Smets and Wouters (2007), but in line with Forni et al. (2009). The habit parameter is assumed to be beta distributed with mean 0.5 and a standard deviation of 0.15, which is more diffuse than often used in the literature.

In line with the literature, investment adjustment costs are assumed to be gamma distributed with mean 4 and standard distribution 1.25. For capital utilization costs (σ_u) we follow Smets and Wouters (2007) and define that $1/\sigma_u = \psi/(1-\psi)$ to normalize the the capital utilization costs between 0 and 1. The corresponding prior is beta distributed with mean 0.5 and a standard deviation of 0.15. Similarly, the prior of the Calvo probabilities with respect to wages and prices are assumed to be Beta distributed with mean 0.5 and a standard deviation of 0.15. This suggests an average duration of price and wage contracts of two quarters.

Since, as starting point, we employ the same fiscal policy rules as Forni et al. (2009). The AR(1) coefficients of the policy rules are assumed to be beta distributed with mean 0.7 and a standard deviation of 0.2 and the coefficients on government debt are gamma distributed with mean 0.4 and a standard deviation of 0.2.

For the coefficients of the monetary policy rule we follow Christiano et al. (2009) choosing a beta distribution with mean 0.8 and standard deviation 0.1 for the interest rate smoothing coefficient, a gamma distribution with mean 1.7 and standard deviation 0.1 for the policy coefficient on inflation, and a normal distribution with mean 0.125 and standard deviation 0.05 for the policy coefficient on output. The prior distributions for AR(1) coefficients of the shock processes are chosen to be beta distributions with mean 0.85 and standard deviation 0.1. The standard deviations of the structural shocks are assumed to be inverse-gamma distributed with mean 0.01 and 4 degrees of freedom.

Parameter	Symbol	Domain	Density	Para(1)	Para(2)
inv. intertemp. subst. elasticity	σ_c	$\mathbb{R}^+$	Gamma	1.75	0.5
inverse Frisch elasticity	σ_l	$\mathbb{R}^+$	Gamma	2.0	0.5
habit persistence	h	$[0,1)$	Beta	0.5	0.15
capital utilization cost	ψ	$\mathbb{R}^+$	Gamma	0.5	0.15
Calvo parameter prices	γ_p	$[0,1)$	Beta	0.5	0.15
Calvo parameter wages	γ_w	$[0,1)$	Beta	0.5	0.15
investment adjustment cost	ν	$\mathbb{R}^+$	Gamma	4	1.25
interest rate AR coefficient	ρ_R	$[0,1)$	Beta	0.8	0.1
inflation coefficient	ρ_π	$\mathbb{R}^+$	Gamma	1.7	0.1
output coefficient	ρ_y	$\mathbb{R}$	Normal	0.125	0.05
labor tax AR coefficient	ρ_w	$[0,1)$	Beta	0.7	0.2
labor tax debt coefficient	η_w	$\mathbb{R}^+$	Gamma	0.4	0.2
capital tax AR coefficient	ρ_k	$[0,1)$	Beta	0.7	0.2
capital tax debt coefficient	η_k	$\mathbb{R}^+$	Gamma	0.4	0.2
adjustment costs AR coefficient	ρ_i	$[0,1)$	Beta	0.85	0.1
technology AR coefficient	ρ_z	$[0,1)$	Beta	0.85	0.1
public consumption AR coefficient	ρ_{cg}	$[0,1)$	Beta	0.85	0.1
s.d. adjustment costs shock	ϵ_i	$\mathbb{R}^+$	InvGam	0.01	4.0
s.d. technology shock	ϵ_z	$\mathbb{R}^+$	InvGam	0.01	4.0
s.d. wage tax shock	ϵ_{τ^w}	$\mathbb{R}^+$	InvGam	0.01	4.0
s.d. capital tax shock	ϵ_{τ^k}	$\mathbb{R}^+$	InvGam	0.01	4.0
s.d. public consumption shock	ϵ_{cg}	$\mathbb{R}^+$	InvGam	0.01	4.0
s.d. monetary policy shock	ϵ_m	$\mathbb{R}^+$	InvGam	0.01	4.0

Table 2: Prior distribution for model parameters. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distribution.

5.3 Estimation results benchmark model

The following results are based on a Bayesian estimation of the benchmark model, where we first estimated the posterior mode of the distribution. Afterwards we employed a Random-walk-Metropolis-Hastings algorithm to approximate the distribution around the posterior mode. We run two chains, each with 300,000 parameter vectors draws. The first 270,000 have been discarded.[7]

Illustrations of the estimation results, i.e. prior vs. posterior distribution plots, can be found in Figures 1 and 2. The plots indicate that the posterior distributions of all structural parameters are well approximated around. Furthermore, most of the parameters are identified as substantially different from their prior distribution. The table 3 shows detailed posterior statistics, e.g. posterior mean and the HPD interval of 10% and 90%.

The estimates of the parameters associated with the preferences of the households are well in line with the literature. The estimate of the inverse elasticity of the intertemporal substitution ($\sigma_c = 1.65$) and of the inverse Frisch elasticity ($\sigma_l = 2.08$) are similar to those obtained by Smets and Wouters (2007) ($\sigma_c = 1.39$) and ($\sigma_l = 1.92$). The posterior mean of the habit parameter ($h = 0.38$) is lower than the estimate found by Smets and Wouters (2007) (0.71) but higher than the estimate by Levin, Onatski, Williams, and Williams (2005) (0.29).

The estimates of the monetary policy rule are close to other studies in the literature: the interest rate smoothing coefficient $\rho_r = 0.85$, the inflation coefficient $\rho_\pi = 1.71$ and the coefficient on output $\rho_y = 0.12$ are among others found by Smets and Wouters (2007).

In the present paper the wage stickiness and the price stickiness are estimated at $\gamma_w = 0.65$ and $\gamma_p = 0.92$ respectively. Both estimates imply a duration of wage and price contracts of five and six quarters respectively. In contrast to Smets and Wouters (2007) we find not that wages stickiness is higher than price stickiness. However, these results are in line with more recent studies by Sahuc and Smets (2008) and Traum and Yang (2009), who get similar

[7]Convergence statistics and further diagnostics can be obtained upon request.

results. The AR(1) coefficients of the shock processes are well identified as are the standard deviation of the shock processes.

Summarizing the subsection we find our estimation results are well identified and similar to other studies and therefore represent a well good description of the private sector of the economy and a good starting point for the subsequent identification of implementable fiscal policy rules.

Parameter	Symbol	Post. mean	HPDinf	HPDsup
inv. intertemp. subst. elasticity	σ_c	1.6499	1.1100	2.1551
inverse Frisch elasticity	σ_l	2.0852	1.2614	2.8954
habit persistence	h	0.3860	0.2644	0.4968
capital utilization cost	ψ	0.6157	0.4533	0.7956
price stickiness	γ_p	0.9177	0.8966	0.9388
wage stickiness	γ_w	0.6531	0.5003	0.8047
investment adjustment cost	ν	4.4156	2.5551	6.1555
interest rate AR coefficient	ρ_R	0.8540	0.8101	0.8966
inflation coefficient	ρ_π	1.7161	1.5515	1.8761
output coefficient	ρ_y	0.1166	0.0715	0.1610
labor tax AR coefficient	ρ_w	0.7782	0.6939	0.8617
labor tax debt coefficient	η_w	0.1658	0.0435	0.2839
capital tax AR coefficient	ρ_k	0.8385	0.7693	0.9084
capital tax debt coefficient	η_k	0.2041	0.0519	0.3617
adjustment costs AR coefficient	ρ_i	0.3100	0.2018	0.4144
technology AR coefficient	ρ_z	0.9379	0.9014	0.9750
public consumption AR coefficient	ρ_{cg}	0.7695	0.7093	0.8302
s.d. adjustment costs shock	ϵ_i	0.0559	0.0494	0.0620
s.d. technology shock	ϵ_z	0.0109	0.0070	0.0145
s.d. wage tax shock	ϵ_{τ^w}	0.0270	0.0245	0.0294
s.d. capital tax shock	ϵ_{τ^k}	0.0227	0.0206	0.0246
s.d. public consumption shock	ϵ_{cg}	0.0143	0.0130	0.0156
s.d. monetary policy shock	ϵ_m	0.0029	0.0019	0.0039

Table 3: Posterior distribution of benchmark model parameters.

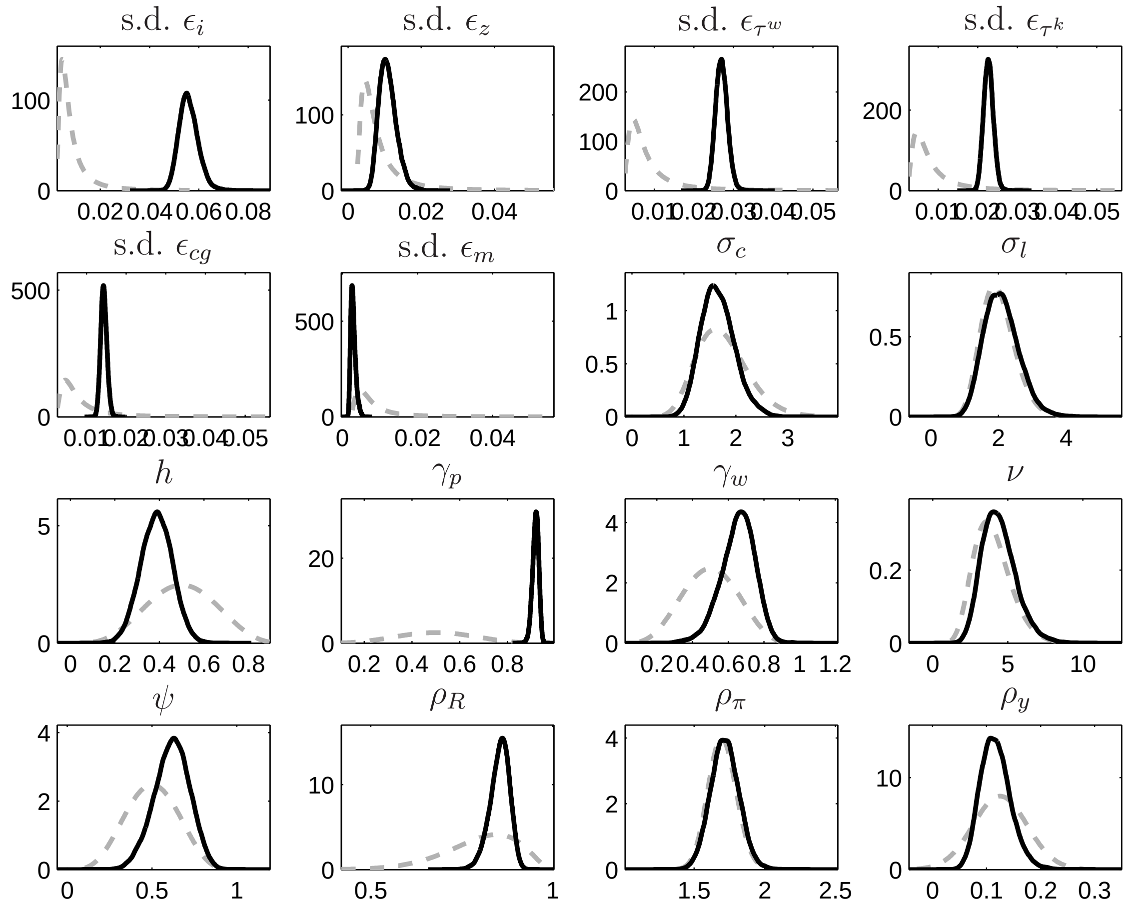

Figure 1: Prior (grey dashed) and posterior (black solid) distribution for the benchmark model.

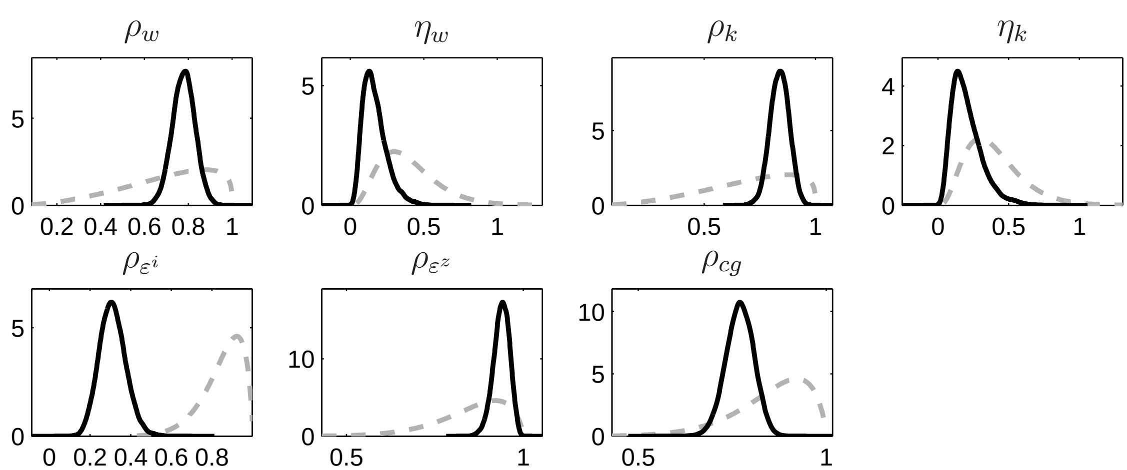

Figure 2: Prior (grey dashed) and posterior (black solid) distribution for the benchmark model.

5.4 Ramsey equilibrium

We find the Ramsey optimal steady state for $\bar{\tau}_k = -0.2498$ and $\bar{\tau}_w = -0.6605$ respectively. Given the model price and wage dispersions are the most important frictions. For welfare maximization it becomes apparent that subsidizing capital reduces the price markup, which is the inverse of the marginal cost, and leads to a more efficient steady state. Similarly, the subsidy of labor reduces the friction on the labor market. To ensure that the government budget constraint is fulfilled, these subsidies are financed by lump-sum taxation.

5.5 Identified policy coefficients

Given the Ramsey steady state we simulate time series for consumption, investment, output, and the risk-free nominal interest rate. We choose these variables, because the economy is affected by investment specific shock, technology shocks, government consumption shocks, and monetary policy shocks. The chosen time series are good indicators of the dynamic economic behavior as well as good proxies of indicators a fiscal authority could be interested in.

Afterwards, we define general tax policies to close the competitive equilibrium. In particular, we therefore specify the new policy rules in the following way:

$$\tau_t^w = f\left(y_t, b_{t-1}, k_{t-1}, c_t, l_t, I_t, \pi_t\right) \tag{45}$$

$$\tau_t^k = f\left(y_t, b_{t-1}, k_{t-1}, c_t, l_t, I_t, \pi_t\right) \tag{46}$$

We estimate the corresponding feedback parameters of these rule. In particular, we define uninformative prior distributions for these feedback parameters, a normal distribution with mean zero and a standard deviation of ten. The deep model parameters of the private sector ϑ^D as well as parameters of the exogenous shocks $\{\epsilon_t^z, \epsilon_t^i, \epsilon_t^{cg}, \epsilon_t^m\}$ are fixed at the posterior mean or calibrated as before. Finally, we run two Random-walk-Metropolis-Hastings chains, each with 300,000 parameter vectors draws. The first 270,000 have been discarded.

Illustrations of the estimation results, i.e. prior vs. posterior distribution plots, can be found in Figures 3. The plots indicate that the posterior distributions of all policy feedback parameters are well approximated and their posterior distribution is substantially different from their prior distribution. The table 4 shows detailed posterior statistics, e.g. posterior mean and the HPD interval of 10% and 90%.

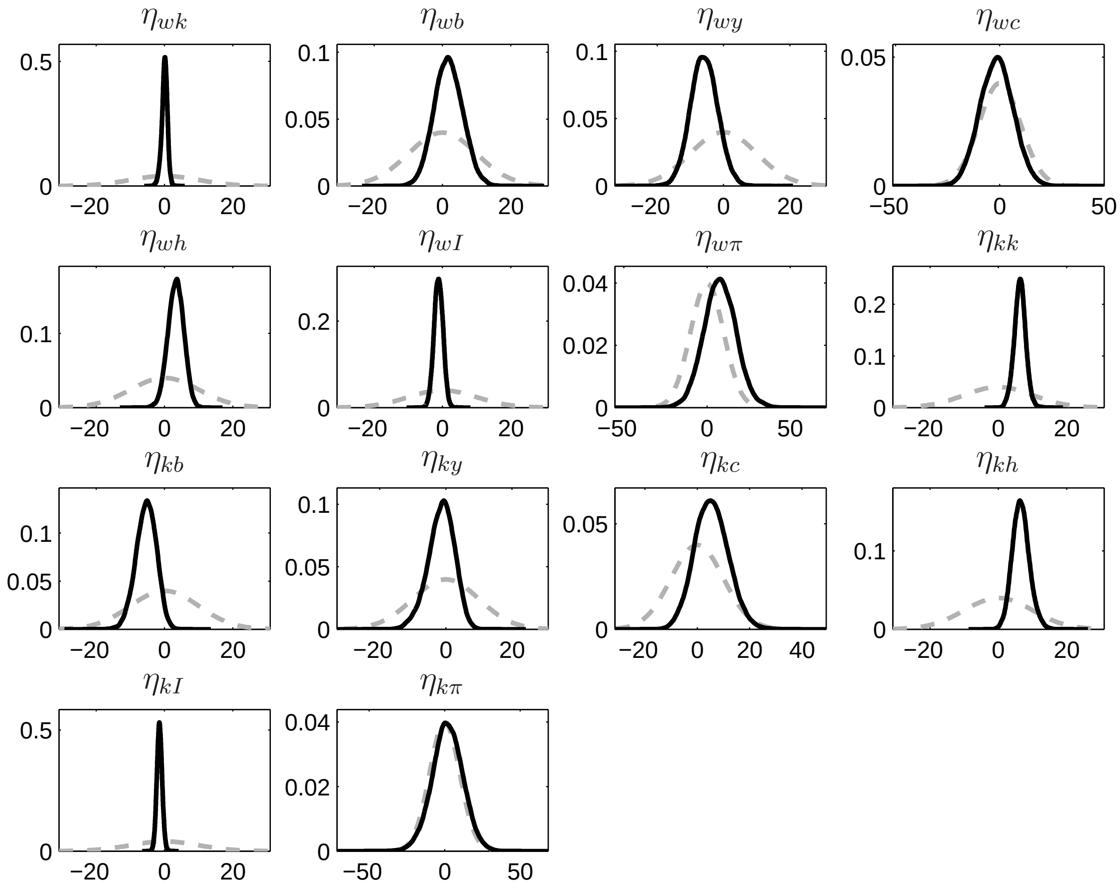

Figure 3: Prior (grey dashed) and posterior (black solid) distribution for the benchmark model.

The figures 4, 5, 6, and 7 display the impulse response function for the optimized rules at the posterior mean and the Ramsey equilibrium and illustrate how good the optimized policy mimic the optimal economic behavior. Especially the dynamics of the observed variables $\{c_t, y_t, I_t, R_t^b\}$, but also the short run behavior of hours worked and capital and the inflation and real wage dynamics after productivity shocks are well described. We therefore employ these optimized rules as a starting point to identify the variables which drive the dynamic

Parameter	Symbol	Post. mean	HPDinf	HPDsup
labor tax capital coefficient	η_{wk}	-0.0099	-1.4176	1.3654
labor tax debt coefficient	η_{wb}	1.6995	-4.9420	8.5536
labor tax output coefficient	η_{wy}	-5.9018	-12.4741	0.7477
labor tax consumption coefficient	η_{wc}	-1.5982	-14.8283	11.3848
labor tax hours worked coefficient	η_{wh}	3.2743	-0.5035	6.9919
labor tax investment coefficient	η_{wI}	-1.1759	-3.3623	0.9152
labor tax inflation coefficient	$\eta_{w\pi}$	7.6320	-8.2278	23.3325
capital tax capital coefficient	η_{kk}	6.2947	3.4846	8.9941
capital tax debt coefficient	η_{kb}	-5.4289	-10.1284	-0.4745
capital tax output coefficient	η_{ky}	-1.4779	-7.7885	5.0950
capital tax consumption coefficient	η_{kc}	5.3656	-5.1622	15.7195
capital tax hours worked coefficient	η_{kh}	6.5662	2.4493	10.5702
capital tax investment coefficient	η_{kI}	-1.4776	-2.7003	-0.1792
capital tax inflation coefficient	$\eta_{k\pi}$	1.7029	-14.9051	18.3034

Table 4: Posterior distribution of optimized feedback coefficients.

behavior of the economy more than the others.

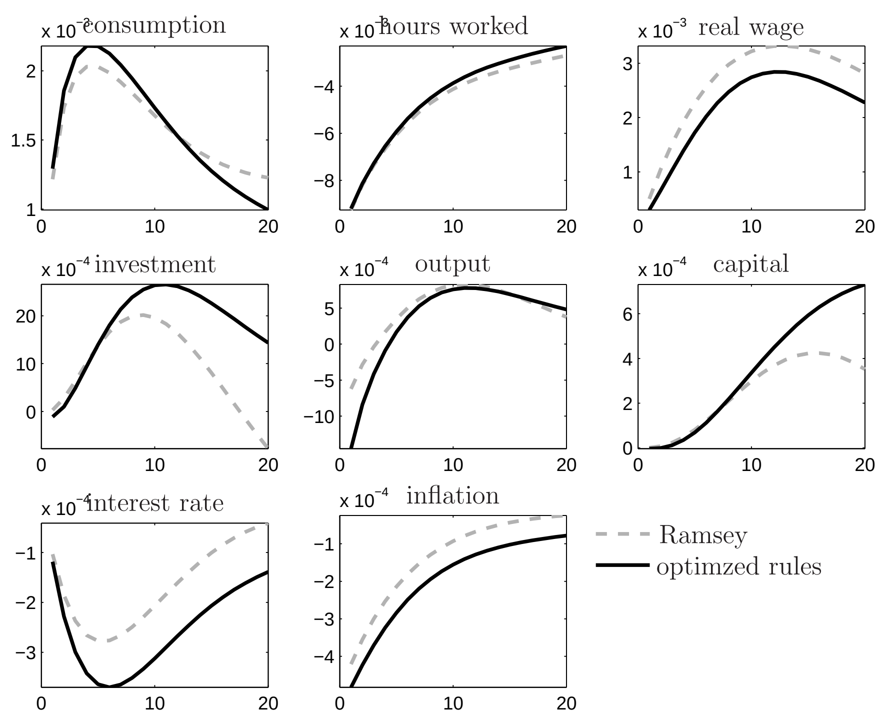

Figure 4: Impulse responses for the competitive (solid) and Ramsey (dashed) equilibrium. Technology shock.

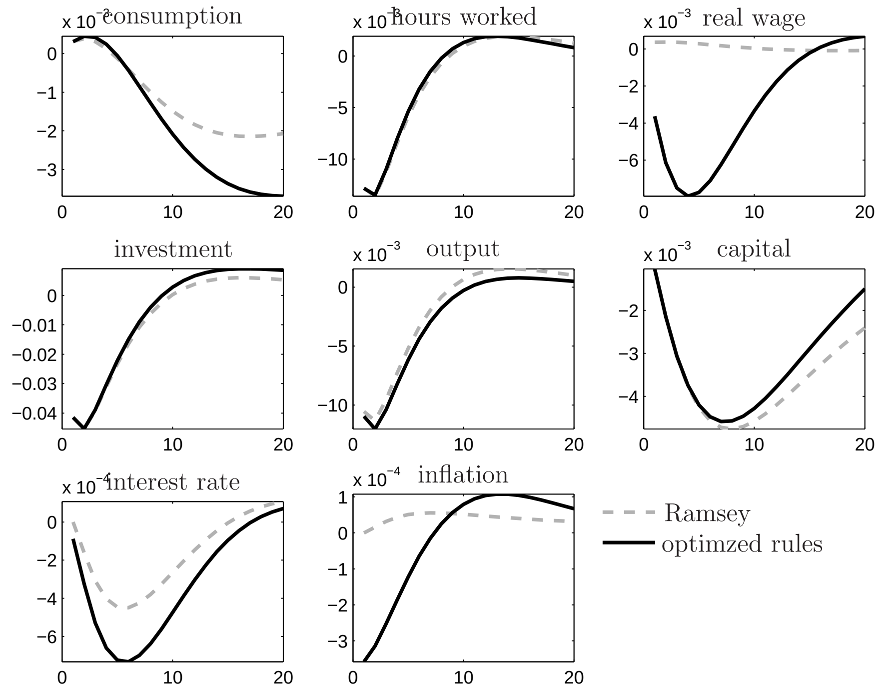

Figure 5: Impulse responses for the competitive (solid) and Ramsey (dashed) equilibrium. Investment adjustment cost shock.

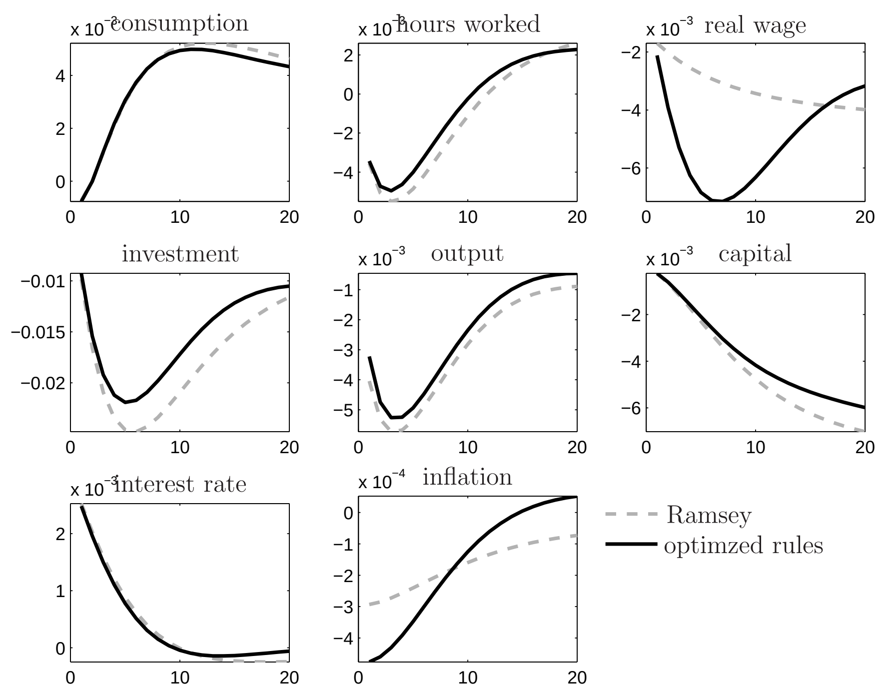

Figure 6: Impulse responses for the competitive (solid) and Ramsey (dashed) equilibrium. Monetary policy shock.

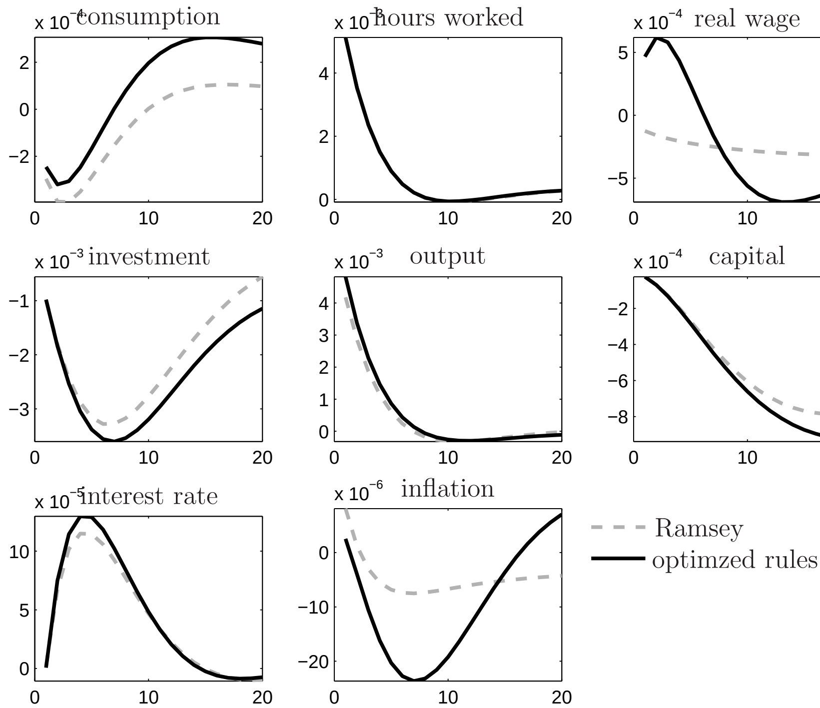

Figure 7: Impulse responses for the competitive (solid) and Ramsey (dashed) equilibrium. Government expenditures shock.

We continue to determine the feedback parameters which are identifiable with respect to the first and second moments of the observable variables using the methodology of Iskrev (2010). Table 5 shows the sensitivity of the moments of each observable variable to each one of the 14 policy feedback parameters.[8] Our findings show that all parameters are identified, i.e. each feedback parameter effects the moments of the observable variables uniquely. We cannot discard any feedback so far.

However, the table illustrates that small changes of the feedback parameters have different strong effects on the first and second moments. Given these result we identify the feedback variables most important to mimic the optimal behavior of the economy with respect to the chosen observable variables. We find the for both tax rules the reaction on public debt and inflation is important. The identification of inflation is just on a first view surprising, because it is well known that minimizing inflation is welfare enhancing. Additionally, the

[8]Following Iskrev (2010) the sensitivity is computed as the Euclidian norm of the vector of elasticities of the mean, variance and first order auto-covariance to each feedback parameter.

reaction of wage taxes on output is important but not the reaction of capital taxes on output. In contrast, capital taxes should react on changes of capital. Especially, the moments of investment are very sensitive with respect to changes in the policy parameters.

Parameter	Symbol	c_t	y_t	R_t	I_t
labor tax capital coefficient	η_{wk}	0.002	0.001	0.001	0.016
labor tax debt coefficient	η_{wb}	3.303	20.131	2.375	501.806
labor tax output coefficient	η_{wy}	9.505	2.908	2.195	745.950
labor tax consumption coefficient	η_{wc}	0.149	0.052	0.079	4.723
labor tax hours worked coefficient	η_{wh}	0.214	0.018	0.079	0.444
labor tax investment coefficient	η_{wI}	0.019	0.151	0.048	0.369
labor tax inflation coefficient	$\eta_{w\pi}$	1.809	0.215	0.392	39.932
capital tax capital coefficient	η_{kk}	0.947	0.299	0.247	5.212
capital tax debt coefficient	η_{kb}	6.625	7.882	2.095	733.088
capital tax output coefficient	η_{ky}	0.026	0.014	0.006	2.852
capital tax consumption coefficient	η_{kc}	0.084	0.126	0.033	2.067
capital tax hours worked coefficient	η_{kh}	0.429	0.127	0.123	0.608
capital tax investment coefficient	η_{kI}	0.004	0.463	0.024	1.358
capital tax inflation coefficient	$\eta_{k\pi}$	3.037	2.252	0.529	109.447

Table 5: Sensitivity of the moments of each observable variable with respect to the fiscal feedback coefficients.

We also calculate the sensitivities of the impulse response function for the seven variables plotted in figures 4-7 for each shock separately. Table 6 shows the sensitivity of the impulse response functions for the first five periods with respect to the fiscal feedback coefficients. The overall results for this identification procedure is similar to the one discussed before. However, the results nicely illustrates the different importance of the feedback variables for different tax rules bus also for different shocks hitting the economy.

The results indicate that, if we assume a fiscal authority which sets tax rates in reaction to changes of some key macroeconomic variables, we have to think about different feedback variables for each fiscal rule. This result is in line with Benigno and Woodford (2006b) who also suggests different rules for simple wage and capital tax rules.

Parameter	Symbol	ϵ^m	ϵ^{cg}	ϵ^z	ϵ^i
labor tax capital coefficient	η_{wk}	0.008	0.002	0.009	0.008
labor tax debt coefficient	η_{wb}	156.053	74.722	257.857	157.182
labor tax output coefficient	η_{wy}	21.596	12.907	46.104	1.565
labor tax consumption coefficient	η_{wc}	0.498	0.807	1.788	0.211
labor tax hours worked coefficient	η_{wh}	1.103	0.190	2.72	1.152
labor tax investment coefficient	η_{wI}	0.192	0.632	1.911	0.694
labor tax inflation coefficient	$\eta_{w\pi}$	10.544	3.806	12.551	5.501
capital tax capital coefficient	η_{kk}	3.146	1.224	3.890	3.251
capital tax debt coefficient	η_{kb}	236.468	121.074	406.766	243.252
capital tax output coefficient	η_{ky}	0.065	0.004	0.013	0.029
capital tax consumption coefficient	η_{kc}	0.373	0.2610	0.404	0.092
capital tax hours worked coefficient	η_{kh}	1.745	0.189	1.038	1.407
capital tax investment coefficient	η_{kI}	0.113	0.113	0.346	0.295
capital tax inflation coefficient	$\eta_{k\pi}$	14.531	3.765	1.373	6.327

Table 6: Sensitivity of the impulse response functions for the first five periods with respect to the fiscal feedback coefficients.

On the grounds of this sensitivity analysis we specify the extended contingencies as:

$$\hat{\tau}_t^w = \rho_w \hat{\tau}_{t-1}^w + (1-\rho_w)\left(\eta_{wb}\hat{b}_{t-1} + \eta_{wy}\hat{y}_t + \eta_{w\pi}\hat{\pi}_t\right) + \epsilon_{t,\tau^w} \tag{47}$$

$$\hat{\tau}_t^k = \rho_k \hat{\tau}_{t-1}^k + (1-\rho_k)\left(\eta_{kb}\hat{b}_{t-1} + \eta_{kk}\hat{k}_{t-1} + \eta_{k\pi}\hat{\pi}_t\right) + \epsilon_{t,\tau^w} \tag{48}$$

where all variables are written logarithmic deviations from steady state and ϵ_{t,τ^w} and ϵ_{t,τ^k} denote *iid* error terms.

5.6 Estimation of the model including the new contingencies

The extended model is estimated given the data, calibration, and prior distribution laid out ind the subsections 5.1 and 5.2. The difference to the benchmark model is that we replace the fiscal rules 36 and 37 by 47 and 48. For all included policy coefficients we specify a prior which is normally distributed with mean 0 and standard deviation 10.

The model is estimated by running two Random-walk-Metropolis-Hastings chains, each with 300,000 parameter vectors draws. The first 270,000 have been discarded. An overview of the posterior estimates is given in Tables 7. Prior and posterior distributions are illustrated in figure 8 and 9.

Parameter	Symbol	Post. mean	HPDinf	HPDsup
inv. intertemp. subst. elasticity	σ_c	1.7495	1.1917	2.3076
inverse Frisch elasticity	σ_l	2.0300	1.2330	2.8142
habit persistence	h	0.3806	0.2672	0.4957
capital utilization cost	ψ	0.6662	0.5077	0.8194
price stickiness	γ_p	0.9153	0.8918	0.9400
wage stickiness	γ_w	0.7055	0.5480	0.8630
investment adjustment cost	ν	4.3103	2.4817	6.1143
interest rate AR coefficient	ρ_R	0.8465	0.8034	0.8913
inflation coefficient	ρ_π	1.7046	1.5428	1.8686
output coefficient	ρ_y	0.1223	0.0663	0.1744
labor tax AR coefficient	ρ_w	0.7627	0.6743	0.8562
labor tax debt coefficient	η_{wb}	0.0686	-0.0343	0.1705
labor tax output coefficient	η_{wy}	1.0241	0.2033	1.8139
labor tax inflation coefficient	$\eta_{w\pi}$	3.4970	-2.0876	9.3867
capital tax AR coefficient	ρ_k	0.8606	0.7887	0.9351
capital tax capital coefficient	η_{kk}	1.7947	-0.7012	4.1036
capital tax debt coefficient	η_{kb}	0.1844	-0.0038	0.3847
capital tax inflation coefficient	$\eta_{k\pi}$	-6.5813	-15.9807	2.9396
adjustment costs AR coefficient	ρ_i	0.2990	0.1913	0.4002
technology AR coefficient	ρ_z	0.9311	0.8920	0.9692
public consumption AR coefficient	ρ_{cg}	0.7719	0.7109	0.8354
s.d. adjustment costs shock	ϵ_i	0.0566	0.0500	0.0627
s.d. technology shock	ϵ_z	0.0119	0.0076	0.0163
s.d. wage tax shock	ϵ_{τ^w}	0.0265	0.0241	0.0288
s.d. capital tax shock	ϵ_{τ^k}	0.0228	0.0208	0.0249
s.d. public consumption shock	ϵ_{cg}	0.0144	0.0131	0.0157
s.d. monetary policy shock	ϵ_m	0.0031	0.0020	0.0043

Table 7: Posterior distribution of the extended model parameters.

Comparing the estimation of the extended model with the benchmark model we find that: the deep parameters and the parameters governing the shock process of the DSGE

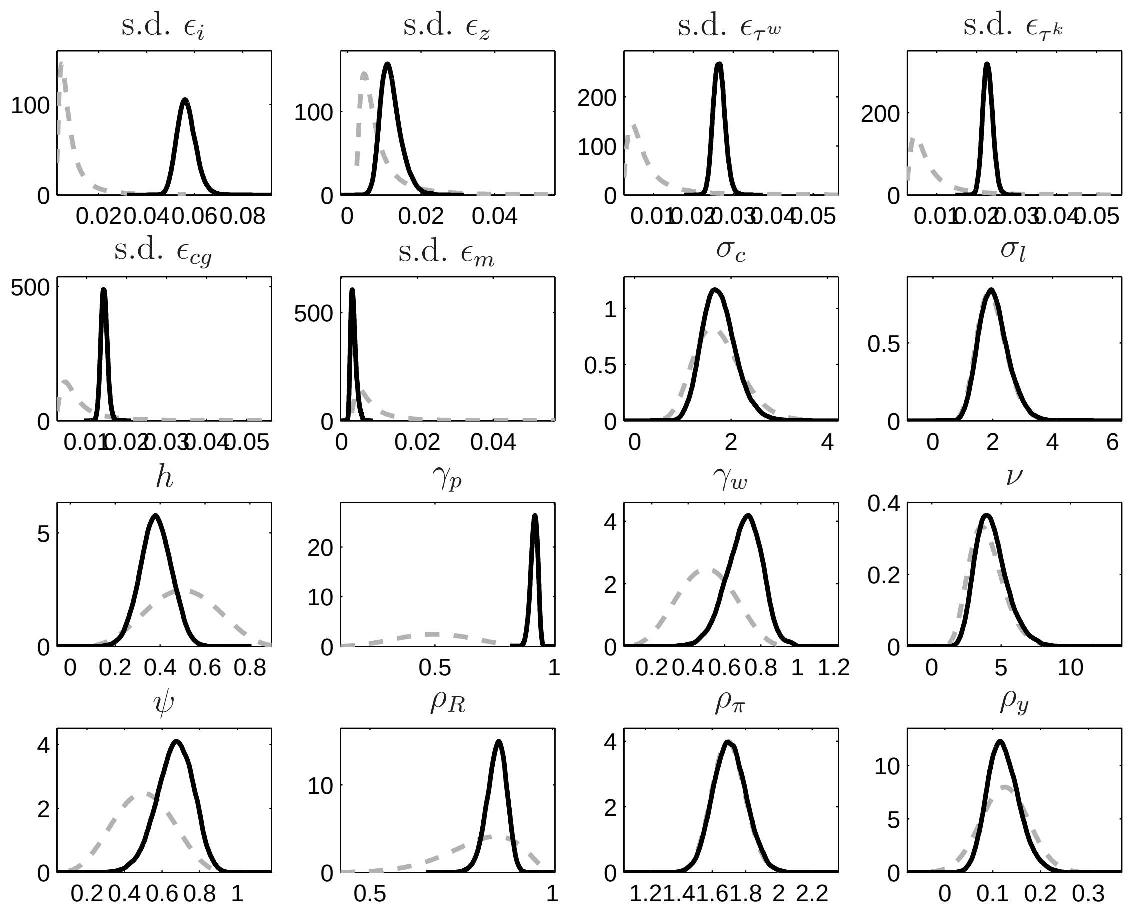

Figure 8: Prior (grey dashed) and posterior (black solid) distribution for the extended model.

model are just slightly different.[9]

The posterior distribution of the fiscal policy feedback parameters, are different from the prior distribution. While the autoregressive parameters as well as the feedback parameters are similar to the estimation results of the benchmark model, we can also identify the parameters on output and capital respectively quite well. However, the wide posterior distribution of of the feedback parameters on inflation suggest a weak identification in the data.

[9] To check robustness of our results with respect to the slightly different parameter estimates, we conduct the procedure again, but with the new estimates of the deep parameters and the shock process. The results are robust to the small changes of those parameters.

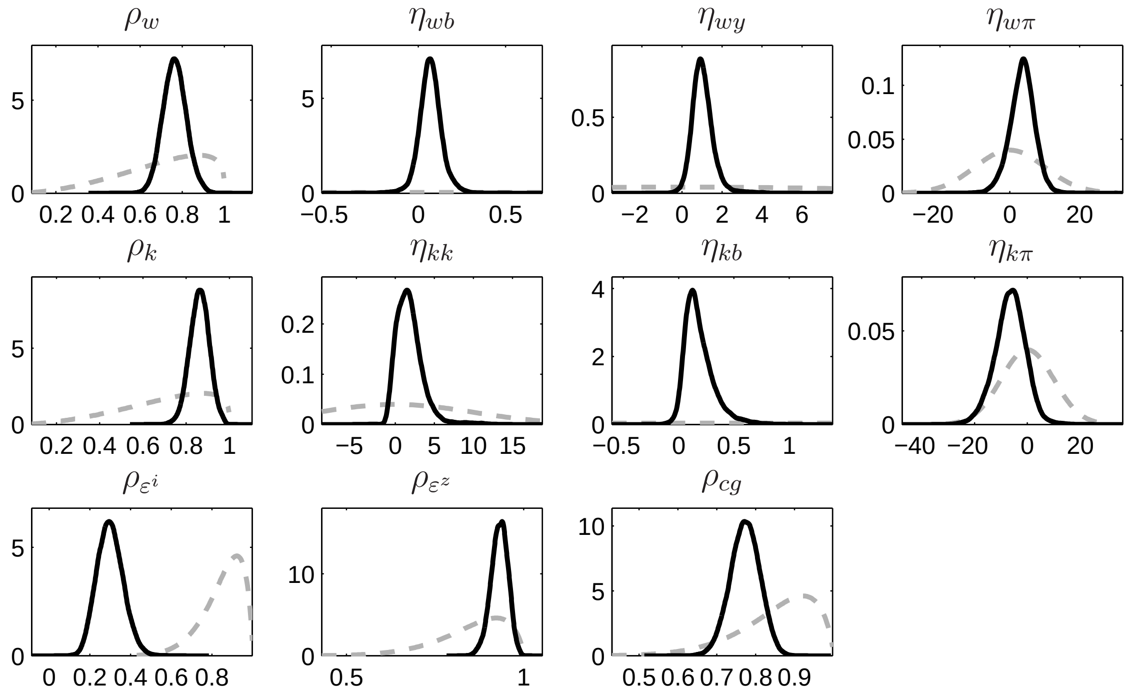

Figure 9: Prior (grey dashed) and posterior (black solid) distribution for the extended model.

6 Conclusion

In this paper we have set out how to determine implementable fiscal feedback rules in an estimated DSGE model. Key ingredient in the analysis is that we allow for endogenously determined policy rules, which are not determined as optimal policy rules.

We have started by estimating a standard DSGE model. Given the posterior distribution of the structural parameters we have continued in the following way: The policymaker is given the opportunity to choose from a large set of rules. The policymaker is restricted in the sense that she is not allowed to implement the Ramsey equilibrium. Instead she chooses the a policy rule for the competitive equilibrium that implies for a limited number of key variables similar dynamic behavior as in the Ramsey equilibrium. Afterwards we estimate the policy rules by repeating the procedure.

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A Data description

The frequency of all data used is quarterly[10].

Real GDP: This series is *BEA NIPA table 1.1.6 line 1.*

Nominal GDP: This series is: *BEA NIPA table 1.1.5 line 1.*

Implicit GDP Deflator: The implicit GDP deflator is calculated as the ratio of **N**ominal GDP to **R**eal GDP.

Private Consumption: This series is obtained from *BEA NIPA table 1.1.5 line 5+6.* We consider consumption of non-durable goods and services.

Private Investment: This series is *BEA NIPA table 1.1.6 line 7.* We add durable good consumption.

Civilian noninstitutional population: This series is taken from: *http://research.stlouisfed.org/fred2/series/CNP16OV?cid=104.*

Tax rates: Capital, labor and consumption tax rates are calculated following Jones (2002). Explanations are provided on his website: *http://www.albany.edu/ jbjones/fispol/Fispol.htm.*

B Model solution

B.1 Competitive equilibrium conditions

The following set of equations are the necessary competitive equilibrium conditions to resolves the model as described in 3. All variables are denoted in real terms, a line over a variable indicates its steady state value:

[10]Note that nominal data is always chained 2000.

Welfare & Utility:

$$U_t = \frac{(c_t - hc_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \psi_l \frac{\tilde{w}_t^+ \left(\frac{l_t}{w_t^+}\right)^{1+\sigma_l}}{1+\sigma_l} \tag{49}$$

$$\mathcal{W}_t = U_t + \beta \mathcal{W}_{t+1} \tag{50}$$

Household:

$$\chi_t = (c_t - hc_{t-1})^{-\sigma_c} - h\beta \left(c_{t+1} - hc_t\right)^{-\sigma_c} \tag{51}$$

$$\frac{1}{R_t} = \beta E_t \left[\frac{\chi_{t+1}}{\chi_t \pi_{t+1}}\right] \tag{52}$$

$$q_t = \frac{1 - \beta E_t \left[\frac{\chi_{t+1}}{\chi_t} q_{t+1} s'_{t+1} \varepsilon_{i,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2\right]}{1 - s_t - s'_t \frac{\varepsilon_{i,t} I_t}{I_{t-1}}} \tag{53}$$

$$s_t = \frac{\nu}{2}\left(\frac{\varepsilon_{i,t} I_t}{I_{t-1}} - 1\right)^2 \tag{54}$$

$$s'_t = \nu \left(\frac{\varepsilon_{i,t} I_t}{I_{t-1}} - 1\right) \tag{55}$$

$$q_t = \beta E_t \left[\frac{\chi_{t+1}}{\chi_t}\left(\psi'\left(u_{t+1}\right) u_{t+1} - \psi\left(u_{t+1}\right) + q_{t+1}\left(1-\delta\right)\right)\right] \tag{56}$$

$$\psi_t\left(u_t\right) = \frac{\bar{r}^k\left(1-\bar{\tau}^k\right)}{\sigma_u}\left(\exp\left(\sigma_u\left(u_t - 1\right)\right) - 1\right) \tag{57}$$

$$\psi'\left(u_t\right) = \bar{r}^k\left(1-\bar{\tau}^k\right)\exp\left(\sigma_u\left(u_t-1\right)\right) \tag{58}$$

$$\psi'\left(u_t\right) = r_t^k\left(1-\bar{\tau}_t^k\right) \tag{59}$$

$$k_t = \left(1-\delta\right) k_{t-1} + \left(1-s_t\right) I_t \tag{60}$$

Staggered Price & Wages:

$$p_t^+ = \left(1-\gamma_p\right)\left(p_t^*\right)^{-\theta_p} + \gamma_p \left(\frac{\bar{\pi}}{\pi_t}\right)^{-\theta_p} p_{t-1}^+ \tag{61}$$

$$F_t^p = y_t \chi_t + \gamma_p \beta \left(\frac{\bar{\pi}}{\pi_{t+1}} \right)^{1-\theta_p} F_{t+1}^p \tag{62}$$

$$K_t^p = \frac{\theta_p}{\theta_p - 1} y_t \chi_t mc_t + \gamma_p \beta \left(\frac{\bar{\pi}}{\pi_{t+1}} \right)^{-\theta_p} K_{t+1}^p \tag{63}$$

$$\frac{K_t^p}{F_t^p} = p_t^* \tag{64}$$

$$1 = \gamma_p \left(\frac{\bar{\pi}}{\pi_t} \right)^{1-\theta_p} + (1 - \gamma_p) \left(p_t^* \right)^{1-\theta_p} \tag{65}$$

$$K_t^w = \left(\frac{l_t}{w_t^+} \right)^{1+\sigma_l} + \beta \gamma_w \left(\frac{\bar{\pi}}{\pi_{t+1}^w} \right)^{-\theta_w (1+\sigma_l)} K_{t+1}^w \tag{66}$$

$$F_t^w = \frac{(\theta_w - 1)}{\theta_w} \left(1 - \tau_t^w \right) \frac{l_t}{w_t^+} \chi_t + \beta \gamma_w \left(\frac{\pi_{t+1}}{\pi_{t+1}^w} \right)^{-\theta_w} \left(\frac{\bar{\pi}}{\pi_{t+1}} \right)^{1-\theta_w} F_{t+1}^w \tag{67}$$

$$\frac{Kw}{Fw} = \frac{1}{\psi_l} \left(w_t^* \right)^{1+\theta_w \sigma_l} w_t \tag{68}$$

$$\pi_t^w = \frac{w_t}{w_{t-1}} \pi_t \tag{69}$$

$$1 = \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w} \right)^{1-\theta_w} + (1 - \gamma_w) \left(w_t^* \right)^{1-\theta_p} \tag{70}$$

$$w_t^+ = (1 - \gamma_w) \left(w_t^* \right)^{-\theta_w} + \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w} \right)^{-\theta_w} w_{t-1}^+ \tag{71}$$

$$\tilde{w}_t^+ = (1 - \gamma_w) \left(w_t^* \right)^{-\theta_w (1+\sigma_l)} + \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w} \right)^{-\theta_w (1+\sigma_l)} \tilde{w}_{t-1}^+ \tag{72}$$

Firm:

$$mc_t = \frac{\varepsilon_{z,t}^{\alpha-1}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} w_t^{1-\alpha} (r_t^k)^\alpha \tag{73}$$

$$d_t = y_t^d - mc_t y_t^d \tag{74}$$

$$u_t k_{t-1} = \left(\frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k} \right)^{1-\alpha} y_t \varepsilon_{z,t}^{\alpha-1} \tag{75}$$

Supply & Demand:

$$y_t = c_t + I_t + c_t^g + \psi(u_t) k_{t-1} \tag{76}$$

$$p_t^+ y_t = (u_t k_{t-1})^\alpha \left(\frac{l_t}{w_t^+} \varepsilon_{z,t} \right)^{1-\alpha} \tag{77}$$

Government:

$$\left[\frac{b_t \pi_{t+1}}{R_t} - b_{t-1} \right] = c_t^g - tx_t \tag{78}$$

$$tx_t = \tau_t^w w_t l_t + \tau_t^k \left[r_t^k u_t k_{t-1} + d_t \right] \tag{79}$$

Policy Rules:

$$\log\left(\frac{R_t}{\bar{R}}\right) = \rho_R \log\left(\frac{R_{t-1}}{\bar{R}}\right) + (1-\rho_R)\left(\rho_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \rho_y \log\left(\frac{y_{t-1}}{\bar{y}}\right)\right) + \log \varepsilon_t^m \tag{80}$$

$$\log\left(\frac{\tau_t^w}{\bar{\tau}^w}\right) = \rho_{\tau^w} \log\left(\frac{\tau_{t-1}^w}{\bar{\tau}^w}\right) + (1-\rho_{\tau^w})\,\eta_{\tau^w} \log\left(\frac{b_{t-1}}{\bar{b}}\right) + \log \varepsilon_t^{\tau^w} \tag{81}$$

$$\log\left(\frac{\tau_t^k}{\bar{\tau}^k}\right) = \rho_{\tau^k} \log\left(\frac{\tau_{t-1}^k}{\bar{\tau}^k}\right) + (1-\rho_{\tau^k})\,\eta_{\tau^k} \log\left(\frac{b_{t-1}}{\bar{b}}\right) + \log \varepsilon_t^{\tau^k} \tag{82}$$

Exogenous Variables:

$$\log\left(\frac{c_t^g}{\bar{c}^g}\right) = \rho_{cg} \log\left(\frac{c_{t-1}^g}{\bar{c}^g}\right) + \log \varepsilon_t^{cg} \tag{83}$$

$$\log \varepsilon_{z,t} = \rho_z \log \varepsilon_{z,t-1} + + \epsilon_t^z \tag{84}$$

$$\log \varepsilon_{i,t} = \rho_i \log \varepsilon_{i,t-1} + + \epsilon_t^i \tag{85}$$

B.2 Steady-State

To solve for the steady state we take the following as given: $\bar{\tau}^k$, $\bar{\tau}^w$, $\bar{c}^g/\bar{y}$,$\bar{\varepsilon}_i = 1$, and $\bar{\varepsilon}_z = 1$. Moreover it is easy to figure out that:

$$\bar{u} = 1 \; \bar{s} = \bar{s}' = 0, \tag{86}$$

that the Tobin's q condition is satisfied for:

$$\bar{q} = 1, \tag{87}$$

and the capital adjustment cost equations can be solved for:

$$\bar{\psi} = 0 \; \bar{\psi}' = \bar{r}^k \left(1 - \bar{\tau}^k\right) \tag{88}$$

The steady state inflation rate follows the long-run growth ρ:

$$\bar{\pi} = 1 + \rho \ , \tag{89}$$

moreover, in the steady state inflation of wages and prices are identical:

$$\bar{\pi}^w = \bar{\pi} \ . \tag{90}$$

Given these results we can solve the euler equation and receive:

$$\bar{R} = \frac{\pi}{\beta} \tag{91}$$

The marginal costs are equal the price markup:

$$\overline{mc} = \frac{\theta_p - 1}{\theta_p} \tag{92}$$

From the households FOC w.r.t. capital we get:

$$\bar{r}^k = \frac{1 - \beta\,(1 - \delta)}{\beta\,(1 - \bar{\tau}^k)} \tag{93}$$

and real wages can be solved as follows:

$$\bar{w} = (1-\alpha)\left(\bar{r}^k\right)^{\frac{-\alpha}{(1-\alpha)}}\left(\alpha^\alpha \overline{mc}\right)^{\frac{1}{1-\alpha}} \tag{94}$$

$$\frac{\bar{k}}{\bar{y}} = \left(\frac{\alpha}{1-\alpha}\frac{\bar{w}}{\bar{r}^k}\right)^{1-\alpha} \tag{95}$$

$$\frac{\bar{c}}{\bar{y}} = 1 - \frac{\bar{c}^g}{\bar{y}} - \delta\frac{\bar{k}}{\bar{y}} \tag{96}$$

$$\bar{l} = \left(\bar{w}^{1-\sigma_c}\frac{\theta_w - 1}{\theta_w}\overline{mc}^{\sigma_c}(1-\alpha)^{\sigma_c}(1-h)^{-\sigma_c}(1-\bar{\tau}^w)\frac{(1-\beta h)}{\psi_l}\frac{\bar{c}}{\bar{y}}^{-\sigma_c}\right)^{\frac{1}{\sigma_c+\sigma_l}} \tag{97}$$

$$\bar{k} = \left(\frac{\bar{k}}{\bar{y}}\right)^{\frac{1}{1-\alpha}}\bar{l} \tag{98}$$

$$\bar{y} = \frac{\bar{y}}{\bar{k}}\bar{k} \tag{99}$$

$$\bar{I} = \delta\bar{k}; \tag{100}$$

$$\bar{d} = \frac{1}{\theta_p}\bar{y} \tag{101}$$

$$\bar{c} = \frac{\bar{c}}{\bar{y}}\bar{y} \tag{102}$$

$$\bar{c}^g = \frac{\bar{c}^g}{\bar{y}}\bar{y} \tag{103}$$

$$\bar{tx} = \bar{\tau}^w \bar{w}\bar{l} + \bar{\tau}^k \left(\bar{r}^k \bar{k} + \bar{d}\right) \tag{104}$$

$$\bar{b} = \frac{\left(-\bar{tx} + \bar{c}^g\right)}{\beta - 1} \tag{105}$$

C Log-Linearization

Household:

$$(1 - \beta h)\,\hat{\chi}_t = \frac{-\sigma_c}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right) + \frac{h\beta\sigma_c}{1-h}\left(\hat{c}_{t+1} - h\hat{c}_t\right) \tag{106}$$

$$0 = \hat{\chi}_{t+1} - \hat{\chi}_t - \hat{\pi}_{t+1} + \hat{R}_t \tag{107}$$

$$\hat{k}_t = (1-\delta)\,\hat{k}_{t-1} + \delta\hat{I}_t \tag{108}$$

$$\hat{I}_t = \frac{\hat{I}_{t-1}}{(1+\beta)} + \frac{\beta\hat{I}_{t+1}}{(1+\beta)} + \frac{\hat{q}_t}{\nu\,(1+\beta)} + \frac{\beta\hat{\varepsilon}_{i,t+1}}{(1+\beta)} - \frac{\hat{\varepsilon}_{i,t}}{(1+\beta)} \tag{109}$$

$$\hat{\chi}_t + \hat{q}_t = \hat{\chi}_{t+1} + \beta\left[(1-\delta)\,\hat{q}_{t+1} + \bar{r}^k\left(1-\bar{\tau}^k\right)\hat{r}^k_{t+1} - \bar{r}^k\bar{\tau}^k\hat{\tau}^k_{t+1}\right] \tag{110}$$

$$\sigma_u\hat{u}_t = \hat{r}^k - \frac{\bar{\tau}^k}{1-\bar{\tau}^k}\hat{\tau}^k_t \tag{111}$$

Staggered Prices & Wages:

$$\hat{\pi}^w_t = \beta\hat{\pi}^w_{t+1} + \frac{(1-\gamma_w)\,(1-\beta\gamma_w)}{\gamma_w\,(1+\theta_w\sigma_l)}\left(\sigma_l\hat{l}_t - \hat{\chi}_t - \hat{w}_t + \frac{\bar{\tau}^w}{(1-\bar{\tau}^w)}\hat{\tau}^w\right) \tag{112}$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{(1-\gamma_p)\,(1-\beta\gamma_p)}{\gamma_p}\widehat{mc}_t \tag{113}$$

$$\hat{\pi}^w_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{114}$$

Firm:

$$\widehat{mc}_t = (\alpha - 1)\,\hat{\varepsilon}_{z,t} + (1-\alpha)\,\hat{w}_t + \alpha\hat{r}^k_t \tag{115}$$

$$\hat{u}_t + \hat{k}_{t-1} = \hat{y}_t + (1-\alpha)\left(\hat{w}_t - \hat{r}_t^k - \hat{\varepsilon}_{z,t}\right) \tag{116}$$

$$\hat{d}_t = \hat{y}_t + (1-\theta_p)\,\widehat{mc}_t; \tag{117}$$

Supply & Demand:

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\left(\hat{l}_t + \hat{\varepsilon}_{z,t}\right) + \alpha\hat{u}_t \tag{118}$$

$$\bar{y}\hat{y}_t = \bar{c}\hat{c}_t + \bar{I}\hat{I}_t + \bar{c}^g\hat{c}_t^g + \bar{r}^k\left(1-\bar{\tau}^k\right)\bar{k}u_t \tag{119}$$

Government:

$$\beta\bar{b}\left(\hat{b}_t + \hat{\pi}_{t+1} - \hat{R}_t\right) = \bar{b}\hat{b}_{t-1} + \bar{c}^g\hat{c}_t^g - \bar{tx}\hat{tx}_t \tag{120}$$

$$\bar{tx}\hat{tx}_t = \bar{\tau}^w\bar{w}\bar{l}\left(\hat{\tau}_t + \hat{w}_t + \hat{l}_t\right) + \bar{\tau}^k\bar{r}^k\bar{k}\left(\hat{\tau}_t^k + \hat{r}_t^k + \hat{u}_t + \hat{k}_{t-1}\right) + \bar{\tau}^k\bar{d}\left(\hat{d}_t + \hat{\tau}_t^k\right) \tag{121}$$

Policy Rules:

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left(\rho_\pi\hat{\pi}_t + \rho_y\hat{y}_{t-1}\right) + \hat{\epsilon}_t^m \tag{122}$$

$$\hat{\tau}_t^w = \rho_{\tau^w}\hat{\tau}_{t-1}^w + (1-\rho_{\tau^w})\,\eta_{\tau^w}\hat{b}_{t-1} + \hat{\epsilon}_t^{\tau^w} \tag{123}$$

$$\hat{\tau}_t^k = \rho_{\tau^k}\hat{\tau}_{t-1}^k + (1-\rho_{\tau^k})\,\eta_{\tau^k}\hat{b}_{t-1} + \hat{\epsilon}_t^{\tau^k} \tag{124}$$

Exogenous Variables:

$$\hat{c}_t^g = \rho_{cg}\hat{c}_{t-1}^g + \hat{\epsilon}_t^{cg} \tag{125}$$

$$\hat{\varepsilon}_{z,t} = \rho_z\hat{\varepsilon}_{z,t-1} + \hat{\epsilon}_t^z \tag{126}$$

$$\hat{\varepsilon}_{i,t} = \rho_i\hat{\varepsilon}_{i,t-1} + \hat{\epsilon}_t^i \tag{127}$$