Information, heterogeneity and market incompleteness

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Information in DGE models

An assumption of (most) DGE models is that

"...the state variables are observed, or are an invertible function of observables..." (Mehra and Prescott 1982)

This assumption is justified by

- identical agents

OR

- complete markets

If agents are heterogenous and markets are incomplete, agents must infer the states

- how can this be modelled?
- how does this affect DGE models?

This paper

The stochastic growth model with

- heterogeneity (both an aggregate and an idiosyncratic productivity shock)
- incomplete markets

We assume information is 'market consistent'

- households only gain information through their participation in markets

And we model

- the infinite regress of expectations that heterogeneity implies
- households as using a (modified) Kalman filter to infer the aggregate states

Contributions and relation to the literature

- a microfounded model of incomplete information
    - Bomfim (2001), Keen (2004), Collard and Dellas (2006), Lorenzoni (2006)
- the properties of the model change dramatically:
    - with 'market consistent' information the impact response of aggregate consumption to an aggregate productivity shock is **negative**
    - if we introduce some noisy public information, the path of aggregate consumption is **hump-shaped**
- incomplete markets matter
    - Levine and Zame (2002), Krueger and Lustig (2007)

This presentation

- the model
- information
- [analytical results]
- numerical results
  - market consistent information
- discussion
  - noisy public information
  - more financial assets

The model

- many households and many firms, divided into islands
- labour is subject to
  - an island-specific productivity shock
  - an aggregate productivity shock
- capital is homogenous and flows freely between islands
- markets are incomplete - the only tradeable asset is capital

Households

A typical household on island s maximises

$$E_t^s \sum_{i=0}^{\infty} \beta^i \left[\ln C_{t+i}^s + \theta \frac{\left(1 - H_{t+i}^s\right)^{1-\gamma}}{1-\gamma} \right]$$

subject to a resource constraint

$$R_t K_t^s + V_t^s H_t^s = C_t^s + I_t^s$$

and the evolution of the household's holdings of capital

$$K_{t+1}^s = (1-\delta) K_t^s + I_t^s$$

The rest of the economy

Firms

The production function of a typical firm on island s is

$$Y_t^s = (J_t^s)^{1-\alpha} (A_t Z_t^s H_t^s)^{\alpha}$$

Aggregates

Aggregates are defined explicitly, for example

$$C_t = \frac{1}{S} \sum_{s=1}^{S} C_t^s$$

Shocks

$$\begin{aligned} a_t &= \phi_a a_{t-1} + \varpi_t \\ z_t^s &= \phi_z z_{t-1}^s + \varpi_t^s, \text{ assume } \sum_{s=1}^{S} z_t^s = 0 \end{aligned}$$

The linearised model

Write the model as an Euler equation

$$E_t^s \Delta c_{t+1}^s = E_t^s r_{t+1}$$

and a law of motion for the economy that is symmetric across types:

$$W_{t+1}^s = F_W W_t^s + F_c c_t + F_s c_t^s + v_t^s$$

where the state vector $W_t^s = \left[\begin{array}{cc} \xi_t' & \chi_t^{s\prime} \end{array} \right]'$ comprises

- aggregate states $\xi_t = \left[\begin{array}{cc} k_t & a_t \end{array} \right]'$
- household states $\chi_t^s = \left[\begin{array}{cc} \kappa^s & z_t^s \end{array} \right]'$ $(\kappa_t^s = k_t^s - k_t)$.

Two benchmark cases

1. Complete markets
   - consumption is only a function of aggregate states $c_t^* = \eta_\xi^{*\prime} \xi_t$
   - full information $\Omega_t^* = \left[\xi_t, \{\chi_t^s\}_{s=1}^S, \Xi\right]$ is revealed
2. Incomplete markets and assumed complete information
   - the aggregate economy looks the same as with complete markets
   - incomplete markets do not matter for aggregates as long as there is complete information
   - $c_t^s | \Omega_t^* = \eta_W^{*\prime} W_t^s = \left[\begin{array}{cc} \eta_\xi^{*\prime} & \eta_\chi^{*\prime} \end{array} \right] \left[\begin{array}{c} \xi_t \\ \chi_t^s \end{array} \right]$

Market-consistent information

- households only obtain information from the markets they participate in so the information set of household s at time t is

$$\Omega_t^s = \left[\{r_i\}_{i=0}^{t}, \{v_i^s\}_{i=0}^{t}, \{k_i^s\}_{i=0}^{t}, \Xi \right]$$

- define a measurement vector $i_t^s = \begin{bmatrix} r_t & v_t^s & k_t^s \end{bmatrix}'$
- then the information set evolves according to

$$\Omega_{t+1}^s = \Omega_t^s \cup i_{t+1}^s$$

- and we can write the measurement vector as

$$i_t^s = H_w W_t^s + H_c c_t$$

The hierarchy of expectations

The consumption function of a household of type s is

$$c_t^s = \eta' E_t^s X_t^s$$

where the (infinite dimension) state vector is

$$X_t^s = \left[\begin{array}{ccccc} W_t^s & W_t^{(1)} & W_t^{(2)} & W_t^{(3)} & \dots \end{array} \right]'$$

The first-order average expectation $W_t^{(1)}$ is an average over all households' expectations of their idiosyncratic state vector

$$W_t^{(1)} = \frac{1}{S} \sum_{s=1}^{S} E_t^s W_t^s$$

and higher-order expectations are given by

$$W_t^{(k)} = \frac{1}{S} \sum_{s=1}^{S} E_t^s W_t^{(k-1)}; k > 1$$

...following Townsend (1983), Woodford (2002), Nimark (2007)

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- then aggregate consumption will be $c_t = \eta_2' \begin{bmatrix} W_t^{(1)} \\ W_t^{(2)} \end{bmatrix}$
- hence the household state vector should again be augmented to include $W_t^{(2)}$, and so forth.

The Kalman filter

Households use their information set Ω_t^s to form estimates of the infinite dimension state vector X_t^s. The standard Kalman filter must be modified in two ways

1. The states depend on the household's choice variable so we need the endogenous Kalman filter (Pearlman et al, 1986; Baxter,Graham and Wright, 2007)
2. The states depend on other households' decisions

Assumption 1: *It is common knowledge that all households' expectations are rational (model consistent).*

The complete system

Each household faces a signal extraction problem of the form

$$\begin{aligned} X_{t+1}^s &= Lc_t^s + MX_t^s + Nv_{t+1}^s \\ i_t^s &= H' X_t^s \end{aligned}$$

Optimal consumption will satisfy the Euler equation

$$E_t^s \Delta c_{t+1}^s = E_t^s r_{t+1}$$

The method of undetermined coefficients:

$$c_t^s = \eta' E_t^s X_t^s$$

where the state estimates are updated according to innovations in the observed variables

$$E_t^s X_t^s - E_{t-1}^s X_{t-1}^s = \beta \left(i_t^s - E_{t-1}^s i_t^s \right)$$

Analytical results - very briefly

- the equilibrium is a fixed point of an iterative system of matrix equations
- if the variance of the idiosyncratic shocks is non-zero the economy can never replicate the full information economy
- a positive aggregate productivity shock always leads to households lowering their estimate of aggregate capital
- consumption is only certainty-equivalent $c_t^s = \eta_W^{*\prime} E_t^s W_t^s$ in two limiting cases
    1. Perfect homogeneity
    2. Perfect heterogeneity

Aside: the invertibility of information sets

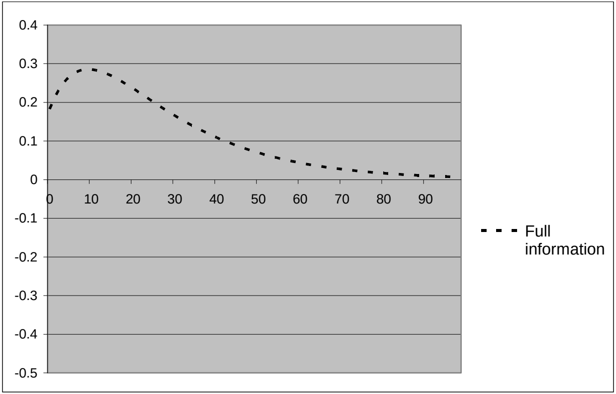
- Baxter, Graham and Wright (2007) "Invertible and non invertible information sets in dynamic general equilibrium"
- some information sets are invertible i.e. full information can be recovered from time t information
- some are asymptotically invertible i.e. full information can be recovered with lots of information
- but in general information sets will not be invertible

Calibration

- standard RBC values for most parameters $\delta = 0.025$, $\alpha = 0.667$, $\beta = 0.99$, $H = 0.33$
- intertemporal elasticity of labour supply 0.2
- persistence of aggregate productivity shock 0.9, innovation standard deviation 0.7% per quarter
- the idiosyncratic shock
    - we use Guvenen's (2005, 2007) estimates of labour income processes
    - this gives a persistence equal to that of aggregate productivity, but a much higher innovation variance of around 5% per quarter

The path of consumption

Response of aggregate consumption to a positive aggregate technology shock



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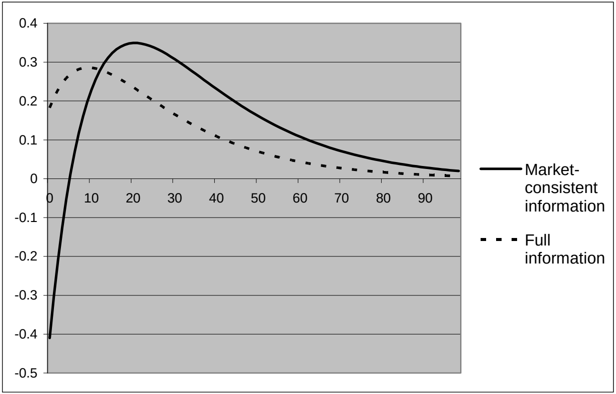


Intuition

- the aggregate productivity shock is observed by households as innovations in factor returns
- these innovations are used to update the state estimates
- we show that consumption can be divided into a certainty equivalent part and a part dependent on the hierarchy

$$c_t^s = \eta_W^{*\prime} E_t^s W_t^s + \sum_{k=1}^{\infty} \mu_k E_t^s \left[a_t - a_t^{(k)} \right]$$

- write the certainty-equivalent part as

$$c_1^s = \begin{bmatrix} \eta_k & \eta_a & \eta_\kappa & \eta_z \end{bmatrix} \begin{bmatrix} E_1^s k_1 \\ E_1^s a_1 \\ E_1^s \kappa_1^s \\ E_1^s z_1^s \end{bmatrix}$$

Intuition - how do the state estimates change on impact?

1. Aggregate capital
   - the estimate of aggregate capital must fall on impact (analytical result)
2. Aggregate productivity
   - a positive innovation in returns could have been caused by
     - a positive innovation in productivity today
     - past over-estimates of productivity implied by over-estimates of capital $r_t = \lambda_3 (a_t - k_t) = \lambda_3 (E_t^s a_t - E_t^s k_t)$
3. Idiosyncratic productivity
   - an increase in the wage always causes households to increase their estimates of idiosyncratic productivity.

Intuition - why a negative response of consumption?

$$c_1^s = \begin{bmatrix} \eta_k & \eta_a & \eta_\kappa & \eta_z \end{bmatrix} \begin{bmatrix} E_1^s k_1 \\ E_1^s a_1 \\ E_1^s \kappa_1^s \\ E_1^s z_1^s \end{bmatrix}$$

- the idiosyncratic states have only a small effect on household consumption
- the impact effect on the estimate of productivity is close to zero because of the two offsetting effects
- so the negative change in the estimate of aggregate capital dominates

What about the hierarchy of expectations?

$$c_t^s = \eta_W^{*\prime} E_t^s W_t^s + \sum_{k=1}^{\infty} \mu_k E_t^s \left[a_t - a_t^{(k)} \right]$$

- the calibrated case is very close to the limiting case of perfect heterogeneity
- so the impact of the term dependent on the hierarchy is small
- BUT the hierarchy means that households make better forecasts of the states

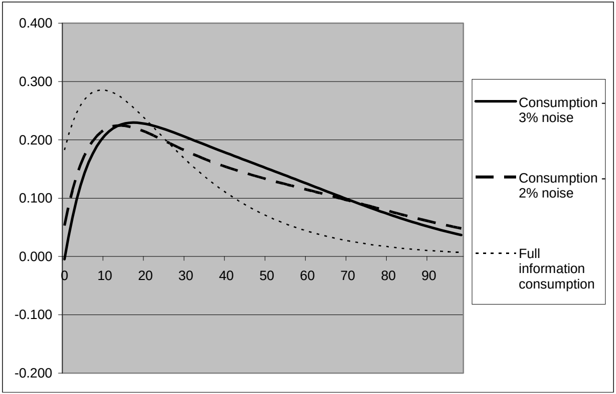
Sensitivities to the properties of idiosyncratic shock

	σ_z / σ_a					
ϕ_z	∞	10	5	2	1	0
0.95	−0.352	−0.345	−0.338	−0.273	−0.113	0.183
0.9	−0.440	−0.425	**−0.410**	−0.276	0.022	0.183
0.85	−0.474	−0.448	−0.424	−0.211	0.058	0.183
0.7	−0.510	−0.438	−0.376	−0.009	0.126	0.183
0.5	−0.526	−0.365	−0.245	0.0763	0.160	0.183

Sensitivity to public information

- Watson (2007) asks "How Accurate are Real-Time Estimates of Trends and Gaps?"
  - answer: "not very"
- Orphanides (2001) compares "real time" estimates of the output gap with "final" estimates
- finds a standard deviation of around 2%
- what if we feed such a noisy signal of output into our model?

Noisy public information



Adding a risk free bond

- if we define the return on a risk free bond by

$$E_t^s \Delta c_{t+1}^s = r_{ft}$$

- observing this return is equivalent to observing a linear combination of the hierarchy

$$r_{ft} = R' M T_1 T_2 X_t^s$$

- however numerically this hardly changes our results.
- why? It only gives information on aggregates...

Adding a stock market

- the stock market can be similarly priced using the Euler equation, giving

$$p_t = J \sum_{i=0}^{\infty} M^i (1-\delta)^i X_t^{(i+1)}$$

- we conjecture that this won't change our results much either
- but what about dividends? In levels:

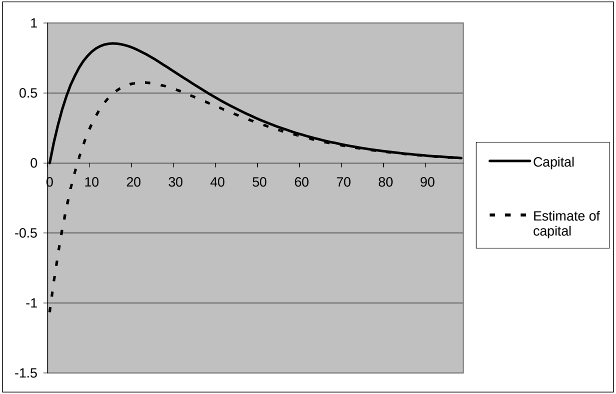
$$D_t = (1-\alpha) Y_t$$

Conclusion

- links between complete markets, full information and the representative agent
- does market incompleteness matter?
- do we overstate the informational problem...
  - more securities
  - statistical offices
- ...or understate it?
  - only a single source of aggregate uncertainty
  - only a single source of idiosyncratic uncertainty
  - strong assumptions of symmetry

How close are estimates of capital to the truth?

- the standard deviation of the estimate of capital around the true value of capital is 2% per quarter



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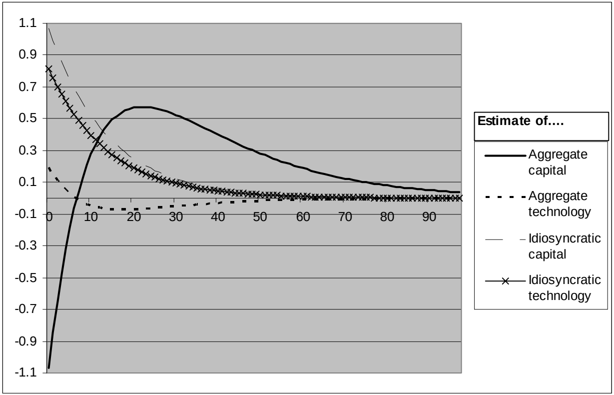

Innovations to returns