Risk premiums and Macroeconomic Dynamics in a Heterogeneous Agent Model

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1 Introduction

Financial asset prices contain potentially interesting information about growth and inflation expectations of private agents. However, extracting this information and integrating financial asset prices in a DSGE model is complicated because of the presence of time varying risk premiums. The standard DSGE model with endogenous capital and labor has problems to generate sufficiently large premiums and realistic real statistics at the same time. Various solutions have been suggested in the literature to overcome this problem in a standard representative agent model: Boldrin et al. (1999) suggest frictions in the labor allocation between sectors, Uhlig (2007) proposes real wage rigidity as a possible solution. In this paper, we follow Guvenen (2005) and Danthine & Donaldson (2002-2007), and concentrate on the impact of heterogeneous capital market participation across agents. This setup implies a number of interesting features that can facilitate the joint explanation of real and financial statistics. First, in such setup, it is no longer aggregate consumption that drives the pricing kernel of asset prices. There is a well documented literature that suggest that the consumption of wealthy agents, that hold the majority of the capital stock, is more volatile than aggregate consumption. Second, in a context of heterogeneous agents, the valuation of the capital stock is not only determined by aggregate risk, but potentially also by distribution risk. The volatile and highly procyclical nature of the profit stream, can potentially contribute significantly to the explanation of the equity risk premium and can help to differentiate between stock and bond risk premiums. The risk sharing between heterogeneous agents does not only affect the pricing of the claims on future profits but offers also the natural context to explain the observed acyclical behavior of real wages and the countercyclical behavior of the wage share. Third, an explanation of the risk premium based on the heterogeneous capital market participation across agents has important empirical implications for the financial behavior

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of the different agents for instance in terms of wealth accumulation and the resulting wealth distribution. Therefore, this approach has the advantage that the underlying assumptions can be validated more easily compared to alternative explanations which are often based on non-observable features of the utility functions (another popular solution to the equity premium in the context of a representative agent model).

We start from a general setup in which we consider three types of agents (see Chien et al. (2007) for a similar general setup). A first group of agents are the standard active portfolio investors that allocate their wealth between stock and bonds. These agents act as the marginal investors that clear the bond and stock market and their stochastic discount factor will determine the pricing of the corresponding risk. Based on empirical evidence, we will assume that the active portfolio managers are characterized by a lower risk aversion than the other agents in the economy. A second group of agents participate in the capital market by buying a portfolio fund with fixed weights of bonds and equity. The crucial assumption is that these agents have no separate pricing equation for equity and their consumption behavior is therefore not contributing directly to the pricing of the equity stock. The precautionary savings argument will determine the wealth accumulation of these agents which turns out to depend on the composition of their portfolio and the corresponding expected return. Finally, a third group of agents, the workers, does not participate in the capital market at all and basically consumes immediately its income from labor. In order to smooth their marginal utility, these agents are completely dependent on the labor contract which provides the only opportunity for them to share their income risk with the other agents in the economy and more specifically with the active shareholders as owners of the firms. We will consider two types of optimal contracts. In a context of continued labor-firm relations, the optimal labor contract guarantees a fixed relation between the marginal utility of the workers and the marginal shareholder of the firms. In a context of one period contracts between workers and firms, these contracts will guarantee a fixed relation between the one-period ahead expected marginal utility of the workers and that of the marginal shareholder. More risk averse type 2 and type 3 agents will shift some of the aggregate risk towards the active shareholders, either via savings or via the wage contract, and in exchange these will require a higher return. The model will endogenously determine the wealth distribution in the stochastic economy and this result can be helpful to calibrate the proportion of the three agents in the economy and their risk aversion.

This general setup allow us to review specific cases that have been considered previously in the literature. If we only consider type 1 agents in the economy, we are back in the representative agent model. This allows us to review the implications of the various model assumptions in a more standard setup. More specifically we discuss the important implications of alternative specifications of the utility function. We show that it makes a major difference whether the utility function is assumed to be separable or non-separable between consumption and labor. In order to clarify these implications we will consider three different utility specifications which are standard in the DSGE modelling work: the separable power utility function, the King Plosser Rebelo (KPR) utility function and Greenwood Hercovitz Huffman (GHH) utility function as examples of a non-separable utility function. Furthermore, we will consider the impact of habits, investment adjustment costs, and real wage rigidity in this context. The combination of the type 1 and type 2 agents results in a setup that is similar to the one considered in Guvenen, in particular if we assume that the type two agents hold a pure bond portfolio. The combination of type 1 and type 3 agents is similar to the setup assumed in Danthine and Donaldson. Our setup is more general than the one considered in these papers as the labor supply decision is endogenous in our model. We illustrate how their results are robust under endogenous labor and for different functional forms of the utility function.

Finally, we consider the general model where the three types of agents are simultaneously present. We show that this model driven by a combination of aggregate productivity and distribution risk is able to generate significant risk premiums. Under GHH preferences, the model also produces quite realistic real statistics for aggregate volatilities and correlations. In particular, the optimal labor contract motivated by risk sharing considerations, explains the observed rigidity and low volatility in the real wages and the countercyclical wage share. Combined with stochastic distribution risk, that takes up possible shifts in the bargaining power between workers and firms, these features deliver a high volatility in profits, returns to equity and price dividend ratios. In doing so, the model is able to produce differentiated risk premiums for equity and bonds, although the results are not yet optimal in that respect and other features, like financial leverage are needed to improve on that dimension. Finally we will also consider how the risk premium and the other properties of the model are changed if we allow for sticky prices, an active role for monetary policy and inflation risk in the model.

We consider this paper as a first necessary step towards the construction of a DSGE model that allows a combined analysis of financial asset price and real fluctuations in the economy. Up to now, our analysis was based on the first and second order approximation of the model, which allows for an analysis of the average risk premium. A next step, before we can take the model to the actual data, is to analyze the time varying nature of these risk premiums in a third order approximation of the model.

Part I
The model

2 Households

There are three different types of households in our model economy: Active shareholders, passive shareholders and workers. The types of households differ from one another in the way they insure against pervasive risks in the economy. The different groups of households have unequal access to financial markets. All agents maximize expected utility, which is a positive function of consumption and negatively depends on the amount of labor supplied.

2.1 Type 1 agents: Active shareholders

The first type of agents are active shareholders. These are households which are freely able to invest both in stocks and bonds. They choose the amount of working hours ($N_{1,t}$)

they supply at the prevailing spot market wage rate W_t^s. The decision problem for these shareholders is thus:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U_1\left(C_{1,t}, N_{1,t}\right)$$

subject to the requirement:

$$C_{1,t} + \frac{P_t^B}{P_t} B_{1,t+1} + \frac{P_t^S}{P_t} S_{1,t+1} \leqslant \frac{B_{1,t}}{P_t} + S_{1,t} \frac{\left(P_t^S + D_t\right)}{P_t} + \frac{W_t^s}{P_t} N_{1,t} + \frac{\Gamma_t}{P_t}$$

In words, the active shareholders' budget constraint states that their expenditures on consumption ($C_{1,t}$), bonds ($B_{1,t+1}$) and stocks ($S_{1,t+1}$), cannot exceed total income. The aggregate price level is denoted by P_t. Bonds are sold at a price P_t^B, while shares trade at price P_t^S. In addition to labor income ($W_t^s N_{1,t}$), active shareholders obtain funds from previous bond holdings ($B_{1,t}$), from selling stocks ($S_{1,t} P_t^S$) and from receiving dividends of firms ($S_{1,t} D_t$) and the financial intermediary (Γ_t, see below). This maximization problem results in the following FOC.

First Order Conditions

($\partial C_{1,t}$)

$$\frac{\partial U_1\left(C_{1,t}, N_{1,t}\right)}{\partial C_{1,t}} - \lambda_{1,t} = 0$$

($\partial N_{1,t}$)

$$\frac{\partial U_1\left(C_{1,t}, N_{1,t}\right)}{\partial N_{1,t}} + \lambda_{1,t} \frac{W_t^s}{P_t} = 0$$

($\partial B_{1,t+1}$)

$$\beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{1}{P_t^B} \frac{P_t}{P_{t+1}} \right\} = \beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{1}{P_t^B \pi_{t+1}} \right\} = \beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} R_{t+1}^f \frac{1}{\pi_{t+1}} \right\} = 1$$

($\partial S_{1,t+1}$)

$$\beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{\left(P_{t+1}^S + D_{t+1}\right)}{P_t^S} \frac{1}{\pi_{t+1}} \right\} = \beta E_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} R_{t+1}^S \frac{1}{\pi_{t+1}} \right\} = 1$$

($\partial \lambda_{1,t}$)

$$C_{1,t} + \frac{P_t^B}{P_t} B_{1,t+1} + \frac{P_t^S}{P_t} S_{1,t+1} = \frac{B_{1,t}}{P_t} + S_{1,t} \frac{\left(P_t^S + D_t\right)}{P_t} + \frac{W_t^s}{P_t} N_{1,t} + \frac{\Gamma_t}{P_t}$$

In case the active shareholders can also invest in a long horizon bond, their budget constraint becomes

$$\begin{aligned} & C_{1,t} + \frac{P_t^B}{P_t} B_{1,t+1} + \frac{P_t^{B,long}}{P_t} B_{1,t+1}^{long} + \frac{P_t^S}{P_t} S_{1,t+1} \\ = \quad & \frac{B_{1,t}}{P_t} + B_{1,t}^{long} \frac{\left(P_t^{B,long} + Coupon\right)}{P_t} + S_{1,t} \frac{\left(P_t^S + D_t\right)}{P_t} + \frac{W_t^s}{P_t} N_{1,t} + \frac{\Gamma_t}{P_t} \end{aligned}$$

with the first order condition associated to B^{long}_{t+1}:
$\left(\partial B^{long}_{1,t+1}\right)$

$$\beta E_t\left\{\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\frac{\left(P^{B,long}_{t+1}+Coupon\right)}{P^B_t}\frac{1}{\pi_{t+1}}\right\}=\beta E_t\left\{\frac{\lambda_{1,t+1}}{\lambda_{1,t}}R^{f,long}_{t+1}\frac{1}{\pi_{t+1}}\right\}=1$$

The FOCs for the shareholder are standard. In particular, they mimic the well known conditions for consumption, labor and asset holdings in a standard representative agent model. At the margin, the agent should be indifferent between changes in asset holdings, consumption and labor allocations.

2.2 Type 2 Agents: Passive shareholders

Passive shareholders differ in one crucial manner from type 1 agents. These agents do not actively manage their stock portfolio. Rather, they hold funds with a fixed portion (α^B) invested in bonds and the remainder ($1-\alpha^B$) in stocks. A particular version of the model assumes they do not hold stocks at all ($\alpha^B=1$), effectively making them pure bondholders. While passive shareholders may have a different momentary utility function, relative to active shareholders (different in the degree of risk aversion in particular), they are otherwise very similar. In particular, the type 2 agents also work at the spot wage and thus maximize:

$$\max E_0\sum_{t=0}^{\infty}\beta_2^t U_2\left(C_{2,t},N_{2,t}\right)$$

subject to:

$$\begin{aligned}C_{2,t}+\frac{\left[P^B_tB_{2,t+1}+P^S_tS_{2,t+1}\right]}{P_t}\frac{1}{\phi(B_{2,t+1})}&\leqslant B_{2,t}\frac{1}{P_t}+S_{2,t}\frac{\left(P^S_t+D_t\right)}{P_t}+\frac{W^s_t}{P_t}N_{2,t}\\ P^B_tB_{2,t+1}&=\alpha^B\left[P^B_tB_{2,t+1}+P^S_tS_{2,t+1}\right]\end{aligned}$$

Passive shareholders engage in asset accumulation via a financial intermediary. In doing so, they are subject to a portfolio cost $\phi(B_{2,t+1})$. We introduce such a cost for bond holdings so that the return on bonds will depend on the macro bond supply[1]. The more passive shareholders save, the lower the return. The more debt they take, the higher the cost. This cost is taken as given from the point of view of an individual passive shareholder. This mechanism is the same as in Benigno (2007) who uses it in a two-country model. The introduction of such an intermediation margin is necessary to avoid infinite bond holdings or borrowing. This assumption is similar to the discrete constraints on bond positions in Guvenen (2005) that one cannot use when applying perturbation methods to solve the model, as we do below. Profits made by the financial intermediary Γ_t are rebated to the active shareholders.

[1] Since these agents' stock holdings are a constant fraction or multiple of their bond position, their budget constraint remains the same irrespective of whether the portfolio cost is paid on only the bonds, or on both stocks and bonds.

First Order Conditions

$(\partial C_{2,t})$

$$\frac{\partial U_2\left(C_{2,t}, N_{2,t}\right)}{\partial C_{2,t}} - \lambda_{2,t} = 0$$

$(\partial N_{2,t})$

$$\frac{\partial U_2\left(C_{2,t}, N_{2,t}\right)}{\partial N_{2,t}} + \lambda_{2,t}\frac{W_t^s}{P_t} = 0$$

$(\partial B_{2,t+1})$

$$\beta E_t\left\{\frac{\lambda_{2,t+1}}{\lambda_{2,t}}\phi(B_{2,t+1})\left[\alpha^B\frac{1}{P_t^B} + (1-\alpha^B)\frac{P_{t+1}^S + D_{t+1}}{P_t^s}\right]\frac{1}{\pi_{t+1}}\right\} = 1$$

$(\partial \lambda_{2,t})$

$$C_{2,t} + \frac{1}{\alpha^B}\frac{P_t^B}{P_t}B_{2,t+1}\frac{1}{\phi(B_{t+1})} = B_{2,t}\frac{1}{P_t} + S_{2,t}\frac{\left(P_t^S + D_t\right)}{P_t} + \frac{W_t^s}{P_t}N_{2,t}$$

2.3 Type 3 Agents: Workers

The third type of agents also derive utility from consumption and labor, with felicity function $U_3(.)$.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U_3\left(C_{3,t}, N_{3,t}\right)$$

The main difference from the other types of agents is that workers do not participate in financial markets at all and cannot accumulate wealth.[2]. As a result, these agents consume their entire labor income each period:

$$C_{3,t} = W_t^c N_{3,t}$$

This does not, however, mean that these agents bear all the risk. Rather to the contrary, similar to Danthine and Donaldson (2002), we assume that workers engage in permanent relations with the owners of the firms, the active shareholders. The workers earn the contract wage (W_t^c) and the amount of labor they supply is determined in the contract that allows them to share risk and smooth consumption. The contract solves:

$$\max E_t\left\{v_t U_1\left(C_{1,t}, N_{1,t}\right) + (1-v_t)U_3\left(C_{3,t}, N_{3,t}\right)\right\}$$

[2] In fact, this feature follows endogenously from the fact that these agents are able to sign optimal risk sharing contracts with the firm owners. Given these contracts, they will be indifferent to participate in the capital market or not. In order to get a well defined solution, we need to make some additional assumption on their financial behaviour. But the results of the model, will probably be unaffected if we would take some alternative assumption on this. In that sense, the model might not be in conflict with the empirically observed financial wealth that is hold by these agents as well.

subject to:

$$
\begin{aligned}
C_{1,t} &= F(K_t, N_t) - W_t^c N_{3,t} - W_t^s N_{2,t} - W_t^s N_{1,t} - I_t \\
C_{3,t} &= W_t^c N_{3,t}
\end{aligned}
$$

The FOC are:
(∂W^c)

$$
\begin{aligned}
\frac{\partial U_1 \left(C_{1,t}, N_{1,t}\right)}{\partial C_{1,t}} &= \frac{(1-v_t)}{v_t} \frac{\partial U_3 \left(C_{3,t}, N_{3,t}\right)}{\partial C_{3,t}} \\
U_{1,t}^C &= ds_t U_{3,t}^C \\
where\ ds_t &= \frac{(1-v_t)}{v_t}
\end{aligned}
$$

$(\partial N_{3,t})$

$$
\begin{aligned}
v_t \frac{\partial U_1 \left(C_{1,t}, N_{1,t}\right)}{\partial C_{1,t}} [\frac{\partial F(K_t, N_t)}{\partial N_{1,t}} - W_t^c] + (1-v_t) \left\{ \frac{\partial U_3 \left(C_{3,t}, N_{3,t}\right)}{\partial C_{3,t}} W_t^c + \frac{\partial U_3 \left(C_{3,t}, N_{3,t}\right)}{\partial N_{3,t}} \right\} &= 0 \\
v_t U_{1,t}^C [F_t^N - W_t^c] + (1-v_t) \left\{ U_{3,t}^C W_t^c + U_{3,t}^N \right\} &= 0
\end{aligned}
$$

Combining both first order conditions shows that their labor supply guarantees that their marginal rate of substitution between labor and consumption is equal to their marginal productivity. The contract wage has only distributive effects, but does not create any allocative distortion. With a fixed value of v, the contract provides optimal insurance against aggregate risk and reproduces the same outcome as the exchange of contingent securities (constant relative marginal utilities), at given wealth distribution. The steady state level of v is chosen such that the income distribution resulting from the contract is similar to the outcome under spot labor markets. ν can be considered as constant or as time varying, driven by exogenous shocks to the bargaining power. We will refer to these shocks as distribution risk where:

$$
\log(v_t) = (1-\rho_v)\log(\overline{v}) + \rho_v \log(v_{t-1}) + \varepsilon_t^v
$$

As an alternative setup, we consider one period contracts between workers and firms. These contracts equalize the expected marginal utility of both parties to their relative bargaining power. The implications for the labor supply will be similar (See Boldrin and Horvath (1995), and Appendix 4 [TBA] for more details).

3 Firms

Firms maximize the present value of the nominal dividend streams using the active shareholders' stochastic discount factor.[3]

$$
\max E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{P_t}{P_{t+j}} D_{t+j} \right]
$$

[3] In the current version of the model, when type 2 agents are present, they are pure bondholders ($\alpha^B = 1$).

where current period dividends (D_t) are the fraction of sales $P_t(i) Z_t K_t^{\theta} (N_t)^{(1-\theta)}$ that remains after paying operational costs. The firms incur three different types of costs. First, it pays the wage bill, $W_t N_t = W_t^s N_{1,t} + W_t^s N_{2,t} + W_t^c N_{3,t}$. While active and passive shareholders earn the spot market wage, workers are paid the contractual wage.

Second, firms can change their goods' prices, but doing so comes at a cost, as in Rotemberg (1982). Specifically, the cost of changing prices is $\frac{\chi}{2}\left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} - 1\right)^2$. The relative demand curves that firms are faced with are $Y_{t+j}(i) = \left(\frac{P_{t+j}(i)}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j}$, subject to the requirement that firms willingly supply all demand, or $Z_{t+j} K_{t+j}^{\theta} (N_{t+j})^{(1-\theta)} \geqslant Y_{t+j}(i)$. Finally, investment (I_t) is subject to capital adjustment costs, $G\left(\frac{I_t}{K_t}\right)$, as in e.g. Jermann (1998). As a result, the capital stock evolves according to $K_{t+1} = (1-\delta)K_t + G\left(\frac{I_t}{K_t}\right) K_t$.

Putting everything together, the Lagrangian for the firms' optimization problem is:

$$\mathcal{L} = \max_{I_t, K_{t+1}, P_t} E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} P_t \left\{ \begin{array}{c} \left[\frac{P_{t+j}(i)}{P_{t+j}} Y_{t+j}(i) - W_{t+j} N_{t+j}(i) - \frac{\chi}{2}\left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} - 1\right)^2 - I_{t+j} \right] \\ + \kappa_{t+j} \left[K_{t+j+1} - (1-\delta) K_{t+j} - G\left(\frac{I_{t+j}}{K_{t+j}}\right) K_{t+j} \right] \end{array} \right\}$$

Firms thus face an intertemporal investment and price setting decision. The adjustment costs for capital are formulated as[4]

$$G = a1 * (\frac{I}{K})^{(1-1/\xi)} + a2$$

;The respective FOCs are given below.

3.0.1 First Order Conditions

(∂I_t)

$$\kappa_t = G'\left(\frac{I_t}{K_t}\right)^{-1}$$

(∂K_{t+1})

$$\kappa_t = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[\theta \frac{Y_{t+j}(i)}{K_{t+1}^{\theta}} + \kappa_{t+1} \left((1-\delta) + G\left(\frac{I_{t+1}}{K_{t+1}}\right) - G'\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} \right) \right]$$

[4] We also investigated the impact of alternative investment adjustment costs but the result are not reported here.

$(\partial P_t(i))$

$$\begin{aligned}
0 &= P_t \left[\begin{array}{c} \frac{1}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t - \frac{P_t(i)}{P_t}\varepsilon\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1}\frac{1}{P_t}Y_t - rmc_t\varepsilon\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1}\frac{1}{P_t}Y_t \\ -\chi\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)\frac{1}{P_{t-1}(i)} \end{array}\right]... \\
& ... + \beta\frac{\lambda_{t+1}}{\lambda_t}P_t\left[\chi\left(\frac{P_{t+1}(i)}{P_t(i)}-1\right)\frac{P_{t+1}(i)}{[P_t(i)]^2}\right] \\
0 &= (1-\varepsilon)Y_t + \varepsilon(\frac{W_t^s}{P_t}/\frac{(1-\theta)Y_t}{N_t})Y_t - \chi(\pi_t-1)\pi_t + \beta\frac{\lambda_{t+1}}{\lambda_t}\chi(\pi_{t+1}-1)\pi_{t+1}
\end{aligned}$$

The real marginal cost (rmc) equals the spot wage divided by marginal productivity of labor. Observe that when there are no costs of adjusting prices ($\chi = 0$), the spot real wage times the markup equals the marginal product of labor:

$$\frac{(1-\theta)Y_t}{N_t} = \frac{\varepsilon}{\varepsilon-1}\frac{W_t^s}{P_t}$$

$(\partial \kappa_t)$

$$K_{t+1} = (1-\delta)K_t + G\left(\frac{I_t}{K_t}\right)K_t$$

4 Equilibrium

Goods Market Clearing Condition:

$$Y_t = C_{1,t} + C_{2,t} + C_{3,t} + I_t + \frac{\chi}{2}(\pi_t - 1)^2 \tag{1}$$

Bond Market Clearing Condition:

Bonds are in zero net supply, and given that there is no government, the bond positions of active and passive shareholders must add to 0[5].

$$B_{1,t} + B_{2,t} = 0$$

Equity Market Clearing Condition:

In equilibrium the shareholders will own the entire net present value of the firm P_t^s. Therefore S_t, the share of the firm that the shareholders own, must be equal to 1 in equilibrium.

$$S_{1t} + S_{2t} = S_t = 1 \tag{2}$$

Labor Market Clearing Condition:

$$N_{1,t} + N_{2,t} + N_{3,t} = N_t$$

[5] When we allow for financial leverage in the financing of the firms capital stock, the demand for bonds must add up to the firm debt, which will be assumed to be a constant fraction of the capital stock. The debt sercive must be subtracted from the dividend flow.

$$N_t = \left[\frac{Y_t}{Z_t K_t^{\theta}} \right]^{\frac{1}{1-\theta}} \tag{3}$$

Productivity follows an exogenous process:

$$\log(Z_t) = (1 - \rho_z) \log(\overline{Z}) + \rho_z \log(Z_{t-1}) + \varepsilon_t^z$$

Monetary Policy:

$$R_t = \overline{R} + 1.5 \left(\pi_t - \overline{\pi}\right) + 0.01 \left(P_t - \overline{P}\right) \tag{4}$$

5 Calibration

In the calibration, we use values for the parameters that are standard in the literature:

Table 1: Calibration values

β	δ	θ	ξ	ρ_z	σ_z	ρ_ν	σ_ν	χ
0.99	0.02	0.30	0.5	0.95	0.01	0.95	0.25	120

The functional form and the parameters of the utility function are discussed below.

6 Some virtues of the three agent model

The model nests a variety of models studied in the literature. When the economy is vacated by active shareholders alone, the model is very similar to the standard representative agent model, analyzed in, for instance, Uhlig (2007). Alternatively, when more than one type of household is present, the model encompasses a variety of asset pricing models with heterogenous agents. For instance, when both active and passive agents are present, and the latter hold no stocks ($\alpha^B = 1$) our model is similar to that of Guvenen (2005) and, when accounting for labor decisions, to Guvenen and Kuruscu (2005). Alternatively, with $\alpha^B < 1$ our model has the flavor of agent heterogeneity as analyzed in Chien, Cole and Lustig (2007). When the economy consists of active shareholders as well as workers, our setup is very close to that of Danthine and Donaldson (2002). We incorporate the labor decision for active shareholders in all versions to maintain comparability over different models. Excluding the labor choice for these agents would make it easier to fit the asset pricing moments, as the labor choice offers shareholders another channel to smooth fluctuations in marginal utility.

Part II
Asset prices and macro allocations

We study the asset pricing implications of the model in two ways. First, we compute asset prices by combining the solution of the log-linearized model with the assumption that the distribution of the model variables is conditionally log-normal. Second, we calculate asset pricing implications of the model using second order approximations. The advantage of the first approach is that it can deliver intuition by providing fairly simple analytical asset pricing expressions[6]. The second approach, obviously, gives more precise predictions, but is less likely to provide analytical insights. Using a second order approximation instead of a first order one typically has limited effects on the macroeconomic implications of the model.

Assets are priced by the agents who hold them. We will use the stochastic discount factor of the active shareholders. The active shareholder will hold a stock position that equates expected marginal utilities of consumption over time. Passive stockholders, do not adjust stock portfolios in such an optimal way, and therefore do not affect (marginal) stock prices. We investigate bond prices based on the active shareholders' Euler Equation, as using that of the passive investors would be confounded by the presence of the intermediation margin.

7 The representative agent model: Only type 1 agents

When there are only active investors, the model reduces to a standard representative agent model. There are an enormous amount of variations on this basic representative agent framework, and our goal is not to provide a detailed overview (see e.g. Kocherlakota 1996). Rather, we here focus on the role of introducing the labor decision. To focus ideas, let us start from the textbook model by assuming that the representative shareholder has a CRRA utility function in consumption:[7] $U(C_t) = C_t^{1-\sigma}/(1-\sigma)$. For now, we restrict attention to the effects of productivity shocks and we assume that prices are fully flexible so that there is no inflation risk. Basically, the equity premium puzzle tells us that for this model to come close to having asset pricing implications that resemble the data, it requires implausibly large degrees of risk aversion. For instance, with $\sigma = 10$ this model delivers the following equity premium (EP_t^A), in annual terms:

$$\begin{aligned} EP_t^A &= -\rho_{rs,\Delta\lambda} * \sigma_{rs} * \sigma_{\Delta\lambda} \\ &= -(-1) * 0.0862 * 0.134 = 1.17\% \end{aligned}$$

where rs stands for the return on stocks, $\Delta\lambda = \lambda_{t+1} - \lambda_t$, $\lambda_t = \frac{\partial U(C_t)}{\partial C_t} = U_t^C$, and σ and ρ denote the standard deviation and correlation, respectively. The corresponding annualized Sharpe ratio (SR_t^A) is:

$$SR_t^A = -\rho_{rs,\Delta\lambda} * \sigma_{\Delta\lambda} = 0.134$$

[6]See appendix for a summary of the most important expressions used in the discussion below.

[7]In the representative agent model with only type 1 agents we drop the index referring to the agent's type to lighten notation.

While both these moments fall short of the numbers suggested by the data, the high degree of risk aversion is able to generate a significant compensation for risk (Table 2, below, compares the implications for two different degrees of risk aversion). One can juice up the risk premium further by assuming larger, more implausible numbers for the coefficient of risk aversion.

Now let us introduce the labor decision. Consider the same representative agent, but with a momentary utility function that is also a function of hours worked. In particular, let

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{\phi}}{\phi}$$

This is a frequently adopted functional form of utility. We set ϕ at 2, implying a Frisch elasticity of $1/(\phi - 1) = 1$.

The agent now has the opportunity to adjust her labor supply to smooth fluctuations in marginal utility. So following a positive productivity shock, the sharp increase in consumption, ceteris paribus, lowers marginal utility, and thereby increases the latter's volatility ($\sigma_{\Delta\lambda}$). However, the linearized FOC for labor supply,

$$\begin{aligned}(\phi - 1) * \hat{n}_t &= \hat{w}_t - \sigma * \hat{c}_t \\ \hat{n}_t &= \hat{w}_t - 10 * \hat{c}_t\end{aligned}$$

tells us that due to a large income effect, labor supply will be reduced substantially. This happens because the marginal utility of the wage is reduced. The strong reduction in working hours, in turn, will mitigate the initial expansionary effects of the productivity shock. As a result, the rise in both marginal productivity (and thus the stock return rs) and consumption (and thus volatility of marginal utility $\sigma_{\Delta\lambda}$) will be smaller. The analytical expressions above reveal that there is no hope of improving asset pricing implications by allowing for endogenous labor in a separable utility framework. Indeed, the model now produces a substantially smaller equity premium ($-(-1) * 0.0325 * 0.044 = 0.14\%$) and a Sharpe ratio (0.044).

7.1 Non-separability between labor and consumption

The above exposition of the representative agent model with endogenous labor supply and separable utility also lays bare a possible way to improve the financial moments. The introduction of non-separabilities between labor and consumption in the utility function strongly affects the model's macro and financial responses.

7.1.1 Some key analytical expressions[8]

On the financial side, one can write the Sharpe ratio as (see e.g. Uhlig 2007 and Appendix 1):

$$SR_t = \rho_{rs,\Delta c} * \eta_{cc} * \sigma_{\Delta c} - \rho_{rs,\Delta n} * \eta_{cn,n} * \sigma_{\Delta n} \tag{5}$$

[8]The analytical asset pricing expressions below are those for the conditional moments. These are most intuitive. When comparing model and data implications, we use unconditional moments. Except for a few models with unreasonably high volatilities in some dimensions, differences are fairly small.

where $\eta_{cc} = -\frac{U^{CC}*C}{U^C}$ is the relative risk aversion. The term $\eta_{cn,n}$ measures the degree of non-separabilities in the utility function:

$$\eta_{cn,n} = \frac{U^{CN} * N}{U^C} > 0 \ (complements) < 0 \ (substitutes)$$

The first term in (5) implies that the price of risk increases with the correlation between consumption and stock returns, the risk aversion, and the volatility of consumption growth. This is the traditional mechanism also at work in the representative agent model with exogenous labor and separable utility. Non-separability leads to an additional effect depending on the volatility of labor supply, the cross derivative of marginal utility with respect to hours worked, and the correlation between hours worked and the return on equity. The sign of this term depends on the cross derivative and the correlation.

If hours worked are procyclical, so that the correlation $\rho_{rs,\Delta n}$ is positive, and the cross derivative is positive, then the second term has a negative effect on the price of risk. A positive cross derivative means that marginal utility of consumption increases in hours worked, in other words, consumption and labor are complements. Hence, during a recession when marginal utility is high because of a low consumption, the low hours worked will mitigate or even offset this increase in marginal utility. It is because of this stabilization of marginal utility that the price of risk will decrease.

On the macro side, the linearized FOC for labor supply can be written as:

$$(\eta_{cc} + \eta_{nc,c}) * \widehat{c}_t - (\eta_{nn} + \eta_{cn,n}) * \hat{n}_t = \widehat{w}_t \tag{6}$$

Equation (6) shows that the strength of the income and substitution effects on labor supply are also controlled by the cross derivatives of the utility function to its respective arguments. This means that introducing non-separabilities in utility does not just buy some free parameters to scale up asset pricing moments, such as the Sharpe ratio. By contrast, we impose strong discipline on the exercise by examining a careful selection of both financial and macroeconomic moments.

7.1.2 Utility functions

We will consider three different utility functions. In addition to the standard, separable utility function considered before, we also study the functional form proposed by King, Plosser and Rebelo (1988, henceforth KPR), as well as the preferences suggested by Greenwood, Hercowitz and Huffman (1988, henceforth GHH).

$$SEP : U_t = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^\phi}{\phi} \tag{7}$$

$$KPR : U_t = \frac{((C_t - hC_{t-1})(1 - \psi N_t^\phi))^{1-\sigma}}{1-\sigma} \tag{8}$$

$$GHH : U_t = \frac{(C_t - hC_{t-1} - \psi N_t^\phi (X_t - hX_{t-1)})^{1-\sigma}}{1-\sigma} \tag{9}$$

Our choice for these utility functions is motivated by the following considerations. First, they allow us to assess the effect of non-separability rigorously, as we will document below. Second, the KPR specification is consistent with a balanced growth path, which is a desirable feature for future extensions of the model, and for taking the model to the data more rigorously. Third, one can interpret the GHH utility function as one limit case of Jaimovich and Rebelo (2007) preferences, with KPR utility being on the other end of the spectrum. JR preferences are specified as $U_t = \frac{(C_t - \psi N_t^{\phi} X_t)^{1-\sigma}}{1-\sigma}$ where $X_t = C_t^{\gamma} X_{t-1}^{(1-\gamma)}$, with $0 < \gamma < 1$. The cases we checked with JR utility, typically gave intermediate results between KPR ($\gamma = 0$) and GHH ($\gamma = 1$), so we restrict our analysis to these two limit cases[9]. Finally, note that we have also introduced the possibility of specifying habits ($h > 0$) which create a different, *inter*temporal, type of non-separability.

Now how do these different functional forms of the preferences affect the financial and macroeconomic predictions of the model? To aid intuition, Table 1 below provides the implied elasticities for each of the preference specifications.

Table 2: Implied elasticities

	SEP	KPR	GHH
η_{cc}	$\frac{\sigma}{1-h}$	$\frac{\sigma}{1-h}$	$\frac{\sigma}{(1-h)(1-\psi n^{\phi})}$
$\eta_{cn,n}$	0	$(\sigma - 1) * \frac{\psi \phi n^{\phi}}{1-\psi n^{\phi}} = \frac{(\sigma-1)}{1-h}$	$\sigma \frac{\psi \phi n^{\phi}}{1-\psi n^{\phi}}$
η_{nn}	-$(\phi - 1)$	-$(\sigma \frac{\psi \phi n^{\phi}}{1-\psi n^{\phi}} + \phi - 1) = -(\frac{\sigma}{1-h} + \phi - 1)$	-$\sigma \frac{\psi \phi n^{\phi}}{1-\psi n^{\phi}} - (\phi - 1)$
$\eta_{nc,c}$	0	$-\frac{\sigma-1}{1-h}$	-$\frac{\sigma}{(1-h)(1-\psi n^{\phi})}$
$\eta_{cc} + \eta_{nc,c}$	$\frac{\sigma}{1-h}$	$\frac{1}{1-h}$	0
$\eta_{nn} + \eta_{cn,n}$	-$(\phi - 1)$	-$(\frac{\psi \phi n^{\phi}}{1-\psi n^{\phi}} + \phi - 1)$ =-$(\frac{1}{1-h} + \phi - 1)$	$-(\phi - 1)$

Note: This table assumes $C = W * N$, which implies $\frac{\psi \phi (1-n)^{\phi}}{1+\psi(1-n)^{\phi}} = \frac{1}{(1-h)\frac{n}{1-n}}$.

The analytical expressions of the previous section document how the elasticities in the first two rows of the table help to evaluate the Sharpe ratio, while the final two rows are crucial to determine the labor supply reaction.

[9] The interpretation of GHH preferences as an extreme case of JR preferences, with $\gamma \to 0$, responds to the critique the GHH preferences are inconsistent with a balanced growth path. It implies that the wealth effect on labour supply is realised only very slowly over time, but in the long run this wealth effect exactly offsets the wage effect on labour supply.

7.1.3 Baseline results

The columns of Table 2 show a set of financial and macroeconomic moments that numerous studies have aimed to match, though not necessarily all simultaneously. Each row contains a different version of the representative agent model[10]. It is useful to note that, at this point, we are mostly interested in understanding the mechanisms at work in generating certain volatilities and correlations. So rather than replicating the data moments exactly, we here aim to identify model features that direct the model in the right direction.

The first row of this table documents the performance of the textbook CRRA representative agent model in replicating salient macroeconomic and financial market features. With respect to macroeconomic statistics, the model implies that fluctuations in output and the wage bill (which coincide with the business cycle) will be largely absorbed by investment in order to smooth consumption. As a result, the model ranks the relative volatilities of output, investment and consumption right, but still misses their absolute numbers. The perfect correlation between the aggregate quantities stems from the fact that factor markets are competitive, that there are no substantial real and nominal rigidities, and that there is only one shock generating business cycles. Later on, we will abandon each of these assumptions. With respect to the financial statistics, the textbook model is unable to generate returns and volatilities observed in the data. The second row shows the typical improvement that one obtains by increasing the representative agent's risk aversion: the volatility of marginal utility increases substantially, resulting in more volatile returns, higher risk premiums and lower equilibrium rates of return (through the effect of precautionary savings). For the (annualized) risk premium and the risk free rate, we provide both the result for the conditional expressions (based on the moments of the first order approximation of the model) and the unconditional outcomes from the second order approximation.

As discussed before, the introduction of labor with separable preferences is detrimental for fitting financial statistics. Also note that increasing risk aversion is now much less effective. The agent is given the ability to costlessly stabilize marginal utility of consumption, which reduces the amount of risk he needs to take on. Moreover, the unconditional countercyclical behavior of labor due to the strong consumption-wealth effect ($\hat{n}_t = \hat{w}_t - 10 * \hat{c}_t$) is counterfactual[11].

Can labor-consumption non-separability overcome this mechanism? In principle, the cross derivative terms in Equation (5) can make a difference. But in the standard KPR utility, consumption and labor are complements (see Table 1: $\eta_{cn,n} = \frac{(\sigma-1)}{1-h} = (\sigma - 1) > 0$). In other words, agents will prefer positive comovement between consumption and labor, and such positive comovement will stabilize marginal utility and thus lower the Sharpe ratio. This follows immediately from the expressions of the SR and labor supply (Equations (5) and (6), respectively), evaluated for KPR preferences. The derivative terms are evaluated at $\eta_{cc} = 10$, $\eta_{nc,c} = -9$, $\eta_{nc,n} = 7.87$ and $\eta_{nn} = -9.74$. The volatilities of consumption

and employment growth are respectively 0.0158 and 0.002 (annualized st.dev. not shown in

[10] For all cases in this table, we keep $h = 0$.

[11] There is, however, a lively debate ongoing with respect to the conditional value of $\rho(N/Y)$. See Rios Rull, Schorfheide, Fuentes-Albero, Santaeulalia and Kryshko (2007) for an overview.

Table 3: Financial and Macro moments

	SR^A	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
SEP σ_c=4, N exo	0.072	0.41/0.40	0.41	3.76/3.78	1.04	5.65	100.19	8.97
SEP σ_c=10, N exo	0.136	1.17/1.15	1.30	3.11/3.11	1.71	8.62	107.11	21.50
SEP σ_c=4	0.038	0.11/0.11	0.115	3.95/3.97	0.57	3.03	99.31	4.78
SEP σ_c=10	0.044	0.14/0.14	0.158	3.92/3.94	0.67	3.25	99.78	7.60
KPR σ_c=4	0.074	0.42/0.41	0.41	3.76/3.78	1.04	5.69	100.17	8.88
KPR σ_c=10	0.140	1.28/1.26	1.38	3.06/3.06	1.76	9.18	108.86	21.46
GHH σ_c=4	0.108	0.87/0.86	0.84	3.46/3.47	1.42	8.14	101.47	12.18
GHH σ_c=10	0.194	2.24/2.21	2.34	2.18/2.16	2.12	11.58	114.17	26.36
Data	0.33	6.33	0.78	1	0.308	19.41	113.20	52.98

	σ_Y	σ_I	$\rho_{I,Y}$	σ_C	$\rho_{C,Y}$	σ_N	$\rho_{N,Y}$	σ_W	$\rho_{W,Y}$	$\sigma_{WN/Y}$
SEP σ_1=4, N exo	1.28	1.75	1	1.16	1	0	0	1.28	1	0
SEP σ_1=10, N exo	1.27	2.90	1	0.87	1	0	0	1.27	1	0
SEP σ_1=4	0.67	0.87	1	0.60	1	0.87	-1	1.54	1	0
SEP σ_1=10	0.44	1.06	1	0.28	1	1.20	-1	1.64	1	0
KPR σ_1=4	1.30	1.77	1	1.19	1	0.04	1	1.26	1	0
KPR σ_1=10	1.36	2.85	1	0.96	1	0.13	1	1.24	1	0
GHH σ_1=4	1.96	2.52	1	1.82	1	0.98	1	0.98	1	0
GHH σ_1=10	1.96	3.59	1	1.55	1	0.98	1	0.98	1	0
Data	2.24	4.40	0.81	0.86	0.75	1.88	0.88	0.96	0.31	3.80

Note: Labor is endogenous, unless indicated otherwise (N exo). SR^A : annualized unconditional Sharpe ratio, EP^A : annual equity premium, BP^A : annual bond premium, R^f : average risk free rate, pd : quarterly price-dividend ratio.US business cycle statistics are from Boldrin and Horvath (1995). They represent the standard deviation and the correlation with output of the HP filtered series. Financial statistics are from Lettau and Wachter (2007) and Adam et.al (2008) and Kocherlakota (1996). For the (annualized) risk premium and the risk free rate, we provide both the result for the conditional expressions (based on the moments of the first order approximation of the model) and the unconditional outcomes from the second order approximation.

the table) The SR becomes $(1 * 10 * 0.0158 - 1 * 7.87 * 0.002 = 0.14)$ and the corresponding RP is (0.0918*0.14 = 1.28%). The volatility in labor is not sufficiently large to reduce the SR by much. The small but procyclical employment response follows from the labor supply condition $(1.87 * \widehat{n} = \widehat{w} - 1 * \widehat{c}\ )$ where the wealth effect of consumption is now reduced to one, so that the impact effect on wages (0.0095) and consumption (-0.0078) result in a small positive labor supply effect. The fact that consumption and labor are complements under KPR preferences avoids the negative employment effects. However, it should also be noted that there remains a conflict between the two objectives: increasing the SR and the procyclicality of labor. As labor becomes more volatile by raising the labor elasticity $1/(\phi - 1)$, it would also tend to reduce the SR.

With GHH preferences, the consumption or wealth effect drops completely from the labor supply condition: $(1.0 * \widehat{n} = \widehat{w} - 0 * \widehat{c}\ )$ so that employment and wages move proportionally depending only on the labor supply elasticity. This allows for more volatility in the labor supply and therefore results in a higher global volatility as employment now reinforces the expansionary effects of the productivity shock. This also results in a higher SR (1*17.76*0.0246-1*15.55*0.0154=0.194) and RP (0.1158*0.194=2.24). Even though the SR is further increased, the initial conflict remains present: the consumption and labor volatility to a large extent offset each other's effect on marginal utility, as consumption and labor are still complements $(\sigma \frac{\psi \phi n^{\phi}}{1-\psi n^{\phi}} > 0)$. Increasing the volatility in employment further, will reduce the wage volatility and increase the SR, but only marginally because of the compensation between the terms in consumption and labor volatility. For instance, reducing $\phi\ = 1.25$ or increasing the Frisch elasticity to 4, increases the SR to 0.27 and the RP to 3.77. Note, however, that although the relative volatilities of employment and wages improve (higher volatility in employment and less in wages), both remain very procyclical (with a correlation for the HP filtered series of 1) and the wage share remains constant in all these outcomes.

In sum, non-separability can help by alleviating the strong income effects at work in the first order condition for labor supply. This will reduce the strong countercyclicality of employment. However, because labor and consumption are complements, their comovement will also tend to stabilize marginal utility. This, in turn, does not help the model's ability to generate significant risk premiums.

Interaction with other frictions

Wage rigidity. Following Uhlig (2007), we consider the effect of exogenous wage rigidity on the above outcomes. The prevailing wage now becomes $W_t^s = W_t = W_{t-1}^{\rho_W} * \left[(1-\theta)\frac{Y_t}{N_t}\right]^{1-\rho_W}$, where in the results we use $\rho_W = 0.8$. The intuition why rigid wages might help in generating the asset market and macro data is as follows. Wage rigidity implies that employment, at least in the short run, is no longer restricted by the labor supply first order condition. The short run impact of employment will be mainly driven by the demand for labor, which means by the marginal productivity of labor versus the wage. As the discussion before has illustrated, the labor supply condition hurts most under those preferences specifications where the income effect (IE) of consumption on marginal utility is the largest (especially the separable preferences and to a lesser extend the KPR preferences, as $IE_{SEP} > IE_{KPR} = 1 > IE_{GHH} = 0$).

With the separable utility function, the short run response of employment tends to become less negative (or even positive) with wage rigidity, since it is now driven by the positive labor productivity boost (with the actual wage rising only gradually). As a result, the short run propagation of the shock on the overall economy is much higher. The exact size and length of these effects depends, of course, on the degree of real wage rigidity (ρ_W). But as wages finally adjust, the response of employment becomes again -counterfactually- countercyclical. The increased volatility in consumption helps to improve substantially the SR (from 0.044 for the case without wage rigidity to 0.12 in the case with wage rigidity) as the volatility in consumption growth increases, while the risk premium increases even more from 0.14 to 1.33 because of the increased volatility in the return.

Under KPR preferences, wage rigidity will lead to a strong procyclical employment effect, so that the overall effect of the productivity shock is enhanced. But it has relatively minor effects on the SR (from 0.14 to 0.148) because the higher volatility in consumption is offset by an even larger increase in the labor supply.

We should not expect large effects under GHH preferences. Wage rigidity will mainly substitute for the labor supply elasticity, since $-(\phi - 1) * \hat{n}_t = \hat{w}_t$. But a high elasticity works positive on the SR, so we do not really want to reduce this elasticity. Adding wage rigidity under GHH will further stabilize wages and make them less than perfectly correlated with output. Employment will also become even more volatile and increase the propagation of the shocks. But wage rigidity does not increase the SR under GHH preferences.

Habit formation. We consider external habit only and with zero memory. For a given volatility in consumption, introducing habit formation boosts the SR because the coefficient of relative risk aversion η_{cc} increases with a factor $1/(1-h)$, as can be seen in Table 1. However, adding habit also reduces the short run expansion in consumption, as agents avoid large immediate swings in consumption (See also Lettau and Uhlig 2000). Moreover, habits will also affect the labor supply condition by raising the consumption-wealth (IE) effect. This will tend to increase employment volatility which can further offset the overall effect on the SR.

With separable preferences, introducing habit (h=0.75) with endogenous labor and flexible wages does not increase the SR or the risk premium. The extremely strong income effect reduces labor supply beyond the already strong reduction observed in the case without habit. By doing so agents are able to further reduce the volatility in consumption. This effect compensates the higher effective risk aversion coefficient in the SR and the RP. However, when habits are considered jointly with wage rigidity, the SR and RP are increased much more (to 0.143 and 1.92) because the labor supply mechanism is largely eliminated in the short run.

Under KPR and wage flexibility, habits lead to a less volatile consumption by the standard smoothing effect. However, the unfavorable effect of the labor supply condition observed in the separable case will now play a much weaker role. This happens for two reasons. On the one hand, the income effect is much less pronounced with KPR preferences. On the other hand, habits will not only affect the marginal utility of consumption, but also the marginal disutility of working, since consumption and labor are complements.[12] As immediate consumption and labor responses are fairly mild, agents will now invest a lot more in response to a productivity shock, to comply with their intertemporal smoothing objective. The resultant increase in the volatility of returns generates a positive effect on both the SR (from 0.14 without habits to 0.208 with habits) and the RP (from 1.28 to 3.49). The return must be more volatile because the effective intertemporal substitution effect on consumption is strongly reduced. The price to dividend ratio of stocks is also quite volatile in this setup. Adding wage rigidity is not particular strong in this context (SR=0.22 and RP= 3.9).

With GHH preferences and wage flexibility, the habit (h=0.4) has even larger effects and in fact is limited to some maximum value in order for marginal utility of consumption to remain positive ($\phi(1-h) > 1$ (when $C = W * N$)). For values close to this limit, the SR and the RP increase to 0.228 and 4.18. With wage rigidity, these values increase further to 0.246 and 5.19.

7.1.4 Discussion

The setup of the representative agent version of our model is similar to that of Uhlig (2007). In some dimensions, however, our results so far differ substantially. It is worthwhile mentioning that Uhlig's (2007) results, favouring the wage rigidity argument, are based on an optimization procedure aimed at matching financial and macroeconomic moments. Our current approach, by contrast, has more the flavour of a rough calibration, if that. However, there are a number of differences in parameterizations which are likely to cause substantial divergences.

One of these factors is the degree of non-separability between labor and consumption. The standard utility functions that we consider, imply that labor and consumption are complements. As can be inferred from Equation (5), this makes it harder to fit the observed market price of risk. Perhaps not surprisingly, Uhlig's (2007) optimization delivers the opposite result, viz. substitutability. Note that complementarity is not a feature particular to the utility functions we work with. Econometric evidence is in line with complementarity, more than substitutability, see e.g. the discussions in Hall (2006) and Malin (2006). Another factor is the discount factor β. Uhlig (2007) estimates a β larger than 1. While this is not impossible, it seems like an undesirable approach to arrive at a reasonable explanation for asset prices. Other factors are that our degree of real wage rigidity is far less extreme, and the fact that we do not take into account -highly persistent- memory in habit levels.

7.1.5 A fairly successful parametrization of the representative agent model

While at this point, we do not estimate the structural parameters to minimize the distance between the model and the actual moments, we here present one parametrization that does fairly well in accounting for many of the asset market facts as well as the macro regularities. In particular, consider the model with GHH preferences and σ_c=20. Note that while many

[12] This is reflected in the impulse response functions: introduction of a habit in consumption, not only implies a hump-shaped response in consumption -as always-, but additionally implies a hump in employment.

Table 4:

σ_1=10; ϕ=2	SR^Q	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
With wage rigidity								
SEP	0.120	1.33/1.01	1.05	3.29/3.63	5.71	11.03	100.19	9.80
KPR	0.148	1.47/1.44	1.56	2.92/2.96	2.04	9.97	106.75	21.24
GHH	0.194	2.25/2.22	2.34	2.13/2.16	2.11	11.63	113.76	25.72
With habit								
SEP	0.049	0.18/0.18	0.19	3.90/3.92	0.80	3.71	99.85	8.03
KPR	0.208	3.49/3.26	3.44	1.85/2.07	4.96	16.79	107.81	25.1
GHH	0.228	4.18/3.79	3.85	1.82/1.43	6.44	18.45	106.89	22.4
Combined habit & wage rig.								
SEP	0.143	1.92/1.37	1.42	2.99/3.55	7.50	13.40	100.51	11.34
KPR	0.220	5.60/3.44	3.61	1.59/2.15	7.61	18.24	106.37	22.40
GHH	0.246	5.19/3.78	3.81	0.96/2.35	11.96	21.08	105.42	19.40

σ_1=10; ϕ=2	σ_Y	σ_I	$\rho_{I/,Y}$	σ_C	$\rho_{C,Y}$	σ_N	$\rho_{N,Y}$	σ_W	$\rho_{W,Y}$	$\sigma_{WN/Y}$
With wage rigidity										
SEP	0.85	2.22	1	0.50	1	1.10	-0.27	1.56	0.74	0
KPR	1.77	2.94	0.96	1.52	0.99	0.98	0.81	1.13	0.86	0
GHH	2.59	3.59	0.96	2.39	0.99	2.02	0.97	0.81	0.79	0
With habit										
SEP	0.34	1.12	0.96	0.18	0.90	1.35	-0.95	1.68	0.96	0
KPR	1.31	4.51	0.94	0.69	0.83	0.05	0.49	1.28	1	0
GHH	1.98	4.41	0.93	1.50	0.97	0.99	1	0.99	1	0
Combined habit & wage rig.										
SEP	0.64	2.54	0.97	0.24	0.78	1.12	-0.63	1.61	0.84	0
KPR	1.46	4.05	0.96	0.89	0.95	0.41	0.67	1.23	0.97	0
GHH	2.61	3.88	0.87	2.47	0.98	2.03	0.97	0.81	0.78	0

may view this as a high risk aversion escape to the puzzle, it is nevertheless of interest because it avoids much of the detrimental endogenous labor supply effect. Recall that in the textbook model with endogenous labor, the high risk aversion solution is essentially killed by the smoothing effect of labor on marginal utility[13].

The representative agent model with $\sigma_c = 20$ produces relatively good results for the GHH preferences. The SR and risk premium arrive at the observed levels. A weakness of this calibration is that the mean risk free rate becomes negative illustrating the risk free rate puzzle: the high volatility of marginal utility (a high SR) depresses the risk free rate through the effect of precautionary savings. Note that this effect is not obscured here by the higher volatility of the short rate, which is more typical for the cases with habit and investment adjustments costs, and which tends to offset the volatility of the marginal utility on the mean risk free rate (at least in a full second order approximation approach). Another weakness is the high term premium on bonds. This model does well in explaining the high volatility in equity returns and especially in the price/dividend ratio. See Adam (2007) for a discussion of these two last statistics. This model also generates a high volatility in investment and a stable consumption. For the labor market, the results are standard: too little volatility in employment and too much in real wages. Adding more elastic labor and/or wage rigidity is necessary there.

Table 5:

	SR^Q	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
σ_1=20 no habit	0.294	4.02/3.99	4.39	-0.29/-0.36	2.55	13.71	146.54	38.36

	σ_Y	σ_I	$\rho_{I,Y}$	σ_C	$\rho_{C,Y}$	σ_N	$\rho_{N,Y}$	σ_W	$\rho_{W,Y}$	$\sigma_{WN/Y}$
σ_1=20 no habit	1.96	4.25	1	1.38	1	0.98	1	0.98	1	0

8 Alternative risk sharing arrangements in the heterogeneous agent model

In a model with heterogeneous agents and a complete market of contingent claims, one can easily show that the optimal risk sharing between agents results in a constant relative marginal utility:

[13]It is still the case that higher risk aversion creates larger premia. However, with endogenous labor one needs to go far beyond the values of risk aversion that created the puzzle in the first place. So this is not much more than a technical solution.

$$\frac{U'_{c1,t}}{U'_{c2,t}} = \frac{U'_{c1,t+1}}{U'_{c2,t+1}} = \mu \tag{10}$$

where μ depends on the relative wealth of the two agents. In our setup, there are no contingent claims and we also assume that Type 2 and Type 3 agents have no access to the stock market. We consider two risk sharing arrangements.

In one setup, Type 2 agents only have access to the bond/funds market in order to share risk with the active shareholders. In addition we assume that the access to the bond/fund market is characterized by a financial cost: the interest rate at which they trade in this market will increase with their lending (decrease with savings). This last assumption is necessary to prevent Type 2 agents from infinite borrowing in this bond/fund market. This implies that the risk sharing that can be achieved through this bond/fund market is not perfect. In bad times, Type 2 agents will dissave which tends to raise their effective interest rate so that it becomes more costly to smooth consumption. Therefore, highly risk averse Type 2 agents have a strong precautionary saving motive: by holding a net bond/fund position they can avoid the high borrowing costs during bad times. The resulting stochastic steady state is characterized by a net bond/funds position for the Type 2 agents which will be a positive function of the relative risk aversion of Type 2 agents relative to Type 1 agents and a negative function of the financial cost parameter. The bond/fund market with financial costs therefore offers only a partial risk sharing device. This setup is similar to the model considered in Guvenen, who considers only pure bondholdings and excludes infinite borrowing by imposing finite borrowing constraints.

In the second setup, Type 3 agents have no access at all to the financial market, but they will design an optimal labor contract which allows for risk sharing (See also Gomme and Greenwood (1992) for an analysis of risk sharing through the labor contract between workers and entrepreneurs in an RBC model). If we assume that the worker-firm relation is a permanent relation, these optimal labor contracts exactly reproduce the optimality condition (10) expressed above. This setup of the contract is considered in Danthine & Donaldson (2002). The shareholders guarantee a consumption level to the workers that implies a constant relative marginal utility. In this setup, the ratio between the two marginal utilities (μ) will reflect the bargaining power of the two parties in the contract arrangement. In our steady state we assume that the optimal contract will imply a wage and consumption level for the workers that is equivalent to the steady state outcome under the spot labor market. We could also assume that workers have some extra bargaining power, which would result in a consumption level above the spot market outcome. This would imply a lower level of dividends and consumption for the shareholders and at the same time a higher volatility in their dividends/consumption stream (implying a higher risk premium). A formal and rigorous representation of such a bargaining game remains to be done[14]. An alternative assumption to the permanent relations is that the worker-firm relation takes the form of one-period contract: this contract will guarantee an expected relative marginal utility level to the workers. If workers have no bargaining outcome, then this expected relative utility will be equal to the expected outcome in the spot market. Again, if workers have some

[14] A potential problem with such a setup is that firms would prefer a spot labour market and avoid engaging in the labour contract if they would have the possibility to do so

bargaining power the wage will guarantee some extra relative to the market outcome. This contract setup is similar to the one considered in Boldrin and Horvath (1995).

Under both contracts, the contract wage guarantees the required smoothness in the consumption of workers which will typically be much less volatile than the spot wage. This contract wage plays only a distributive role, and the optimal contract will imply an efficient labor supply by the workers that equalizes their marginal rate of substitution between labor and consumption to the marginal product of labor (or the spot wage). In exchange for the insurance provided by the firm, workers will offer the required labor services. The result is a countercyclical labor share and more volatile and highly procyclical profits. With one period contracts, the labor/profit share will only be affected during one period and, in absence of further shocks, it will return to the steady state level in subsequent periods.

It is clear that under both risk sharing devices, the relative risk aversion between the two agents will play a crucial role. If the risk aversion of all agents is equal, then the risk sharing through the bond/fund market or through the labor market will reproduce an outcome similar to that of the representative agent economy. Differences may arise from wealth distribution effects, in particular on labor supplies, and from financial costs. With a higher risk aversion for Type 2 or Type 3 agents relative to the Type 1 agents, their desire to smooth consumption increases and as a consequence available funds for the shareholders' consumption will become more volatile and procyclical. This mechanism will increase the Sharpe ratio and the required risk premium on stocks. The assumption on the relative risk aversion is confirmed in the data. For instance, Vissing-Jorgensen (2002) provides evidence of the lower elasticities of intertemporal substitution (or higher risk aversion) for non-stockholders. This risk aversion can not be estimated directly from the first order condition for stocks as these agents do not participate in that market. In our contract setup, the risk aversion of the Type 3 agents will shown up in the resulting wage rigidity. Wachter and Yogo (2007) propose a justification for the negative relation between risk aversion and wealth based on a non-homothetic function of two types of consumption goods (basic and luxury goods) and show how such a model also explains the observed positive relation between the share of risky assets in the portfolio and wealth.

8.1 The Guvenen model: combination of Type 1 + 2 agents

Type 2 agents have access to the bond/fund market but face a financial cost, so that the risk sharing that is provided by this market is limited. In good times, Type 2 agents will smooth consumption and accumulate additional bonds/funds. The active shareholders are the only counterparty for this bond/fund trade and their income available for consumption will therefore become more procyclical. With a higher procyclical consumption stream, the required return on stocks and the risk premium will also increase. However, with endogenous labor and separable utility functions, both agents can smooth their income stream via the labor supply as well, so that the overall impact of the productivity shock on the economy is minimized and the effect of the above mechanism on the risk premium is negligible. With exogenous labor, or with KPR preferences, where the income effects on labor supply are excluded (or strongly reduced), the consumption of the active shareholders becomes more volatile and the required risk premium increases in comparison to the outcomes under the representative agent model. The size of this effect might seem small, but if we use a similar calibration as in Guvenen (capital adjustment cost of 0.23 instead of 0.5 and a volatility of the productivity shock of 0.02 instead of 0.01), the risk premium is quickly increasing to 4.48 with a SR of 0.22. In the case of GHH preferences, labor supply tends to become more cyclical, which further enhances the impact of the productivity shock and its effect on the risk premium. In all these results, we assume that the active shareholders make up 20% of the population, and that the Type 2 agents have a risk aversion of 10, while the active shareholders have risk aversion of 4.

Table 6:

σ_1=4, σ_2= 10	SR^A	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
SEP	0.038	0.12/0.11	0.12	3.95/3.97	0.62	3.04	99.12	5.78
SEP, N exo	0.082	0.60/0.58	0.59	3.70/3.72	1.41	7.27	100.86	13.10
Guvenen calibration	0.222	4.48/4.33	4.29	1.55/1.69	3.85	20.16	106.57	26.23
KPR	0.084	0.61/0.59	0.60	3.69/3.71	1.41	7.32	100.82	12.84
GHH	0.110	1.06/1.05	1.04	3.43/3.44	1.76	9.66	101.95	16.14

σ_1=4, σ_2= 10	σ_Y	σ_I	$\rho_{I,Y}$	σ_C	$\rho_{C,Y}$	σ_{C1}	σ_{C2}	σ_N	$\rho_{N,Y}$	σ_W	$\rho_{W,Y}$	$\sigma_{WN/Y}$
SEP	0.51	0.97	1	0.39	1	0.60	0.28	1.16	-1	1.66	1	0
SEP, N exo	1.29	2.28	1	1.04	1	1.34	0.95	0	-	1.29	1	0
Guvenen calibration	2.61	2.95	1	2.53	1	3.61	2.08	0	-	2.61	1	0
KPR	1.36	2.30	1	1.12	1	1.33	1.04	0.09	1	1.27	1	0
GHH	1.94	2.96	1	1.68	1	1.84	1.67	0.97	1	0.97	1	0

8.2 The Danthine-Donaldson model: combination of Type 1 + 3 agents

We now consider the two agent setup with Types 1 and 3 agents, for each of the preference functions considered before. If the risk aversion of the two agents is the same, then the optimal risk sharing contract will result in an outcome that is very similar to the representative agent model. There is however one difference: the wealth of the two agents is different as the active shareholders hold the entire capital stock and the consumption level of the shareholders will be much higher. To neutralize the effect on the steady state labor supply, we adjust the ψ parameter that defines the relative weight of consumption and labor in the utility function. With the appropriate choice of ψ that equalizes the steady state labor supply of the two agents for a similar wage, the two agent model approximates the outcomes of the representative case if the relative risk aversion of the two agents is the same. Increasing the risk aversion of workers above that of shareholders will increase the risk premium as workers prefer a more stable wage, so that the wage share becomes countercyclical and profits and shareholders consumption more volatile.

Again, with separable preferences and endogenous labor the effect of this mechanism on the risk premium remains very limited. With exogenous labor or with KPR preferences, which imply that the endogenous impact reaction of employment to the productivity shock is minimal, we notice a significant increase in the required risk premium. The risk premium increases because the consumption of the active shareholders becomes more volatile. This increases the Sharpe ratio and at the same time delivers an increase in the volatility of the stock return and the price dividend ratio.

The model with GHH preferences also improves the outcome on the labor market statistics. The model endogenously reproduces the observed rigidity in real wages as measured by its standard deviation relative to that of output . On this aspect, the model performs as well as the representative agent model with exogenously imposed wage rigidity. Wage fluctuations are still perfectly cyclical but the model does generate a variable and countercyclical wage share.

The countercyclical wage share, as well as the higher volatility in profits and the return on equity is crucial for matching the observed behavior of these variables in the data. At the same time, these features also generate a difference between the risk premium on equity and the risk premium on long bonds, which was completely absent in the representative agent model or in the heterogenous agent model with risk sharing through the bond/fund market. In the next section, we will discuss how distribution risk, in addition to aggregate productivity risk, can further improve the model on these dimensions.

Table 7:

σ_1=4, σ_3= 10	SR^A	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
SEP	0.042	0.16/0.15	0.14	3.94/3.95	0.64	3.72	99.18	3.76
SEP, N exo	0.110	1.01/0.97	0.87	3.43/3.46	1.45	9.11	100.36	9.41
KPR	0.110	1.02/0.98	0.87	3.43/3.46	1.15	9.14	100.28	9.12
GHH	0.148	1.72/1.66	1.47	2.94/2.97	1.79	11.46	102.29	13.42

σ_1=4, σ_3= 10	σ_Y	σ_I	$\rho_{I,Y}$	σ_C	$\rho_{C,Y}$	σ_{C_1}	σ_{C_3}	σ_N	$\rho_{N,Y}$	σ_W	$\rho_{W,Y}$	$\sigma_{WN/Y}$	$\rho_{WN/Y}$
SEP	0.52	1.02	1	0.40	1	0.69	0.28	1.12	-1	1.44	1	0.41	-0.44
SEP, N exo	1.31	2.42	1	1.03	1	1.77	0.71	0	-	0.83	1	1.05	-0.41
KPR	1.39	2.45	1	1.13	1	1.67	0.89	0.11	1	0.81	1	1.08	-0.40
GHH	2.06	3.24	1	1.77	1	2.34	1.52	1.03	1	0.60	1	1.06	-0.37

In the above model, we assume that the risk sharing contract is designed in a context of permanent labor-firm relationships. To illustrate to what extend the results are dependent on this assumption, we also consider the one-period labor-firm contracts discussed in Boldrin and Horvath. Under these contracts, workers are guaranteed an expected relative marginal utility only for one period. Each period, this ratio will adjust depending on the state of the economy, and in particular on the expected outcome in the spot economy if workers have no bargaining power in the contract negotiations. Therefore deviations of the wage share from steady state will only persist for one period, and the impact of the risk sharing on the volatility of consumption of the two types of agents will therefore be strongly reduced. From these results it is clear that some duration in the contracts is necessary to obtain a significant effect on the relative consumption variability. However, since profits and the return on equity remain very volatile, this will still generate a significant risk premium.

Table 8:

σ_1=4, σ_3= 10	SR^A	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
SEP	0.044	0.16/0.14	0.14	3.93/3.95	1.11	3.61	99.32	4.95
SEP, exog labor	0.120	1.13/1.09	0.87	3.30/3.38	2.90	9.66	101.12	11.12
KPR	0.106	1.00/0.98	0.86	3.47/3.63	3.97	9.62	99.73	7.39
GHH	0.134	1.59/1.43	1.38	3.12/4.17	4.17	11.84	100.50	10.2

σ_1=4, σ_3= 10	σ_Y	σ_I /$\rho_{I/Y}$	$\sigma_C/\rho_{C/Y}$	$\sigma_{C_1}/\sigma_{C_3}$	$\sigma_N/\rho_{N/Y}$	$\sigma_W/\rho_{W/Y}$	$\sigma_{WN/Y}/\rho_{WN/Y}$
SEP	0.50	0.95/1	0.39/1	0.60/0.30	1.17/-0.99	1.61/0.98	0.11/-0.51
SEP, N exo	1.27	2.69/0.97	0.95/0.99	2.21/0.53	0/-	0.64/0.90	1.96/-0.33
KPR	1.33	2.15/0.95	1.17/0.99	1.25/1.18	0.09/0.67	1.17/0.94	0.27/-0.46
GHH	1.99	2.86/0.96	1.81/0.99	1.80/1.85	1.00/1	0.91/0.94	0.25/-0.42

The next table shows how habit persistence, a higher labor supply elasticity, investment adjustment costs and a higher risk aversion for the workers further increase the SR and the RP under the Danthine & Donaldson setup. We focus on the GHH preferences. Habit and investment adjustment costs lower the interest rate sensitivity of consumption and investment and lead to a very high interest rate and return volatility. This volatility of the risk free interest rate is much higher than the one observed in nominal and real short term interest rates, but it is interesting to remember that the natural real interest rate underlying the estimated DSGE models is also very volatile. With elastic labor, the propagation of the shock through the economy is strengthened and so will the risk premiums. Habits and higher risk aversion for the worker have very similar implications for both the financial and the real statistics. In combination with habit, all these features are able to produce significant risk premiums, but with productivity shocks only it is difficult to approach the empirically observed levels. The same applies for the results on the wage share, and in all cases wages are still too procyclical.

One additional issue is the level of μ in the contract. If we assume that workers have some bargaining power, and are able to negotiate a wage that is above the outcome in the spot market (above the marginal productivity of labor), then the average profit rate in the economy is lowered so that the volatility of profits, that results from the risk sharing, increases. In the example we increase the wage share through additional bargaining power from 0.7 to 0.75 in the deterministic steady state. Note also that all these effects can be increased if we assume that active shareholders are not supplying any labor, and they earn only capital income.

Table 9:

GHH σ_1=4;σ_3=10; ϕ=2	SR^A	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
benchmark no habit	0.148	1.72/1.66	1.47	2.91/2.97	1.79	11.46	102.29	13.42
benchmark with habit	0.194	3.38/3.01	2.88	2.13/2.52	6.28	17.48	101.32	12.86
$\phi = 1.5$ no habit	0.168	2.09/2.06	1.79	2.61/2.65	1.92	12.49	103.76	15.05
$\phi = 1.5$ with habit	0.20	4.11/3.35	3.27	1.91/2.66	8.71	19.97	100.31	11.04
inv.adj.cost=2 no habit	0.142	1.58/1.32	1.17	3.00/3.29	5.52	11.32	101.70	10.47
inv adj cost=2 with habit	0.272	3.68/3.68	3.57	0.28/3.71	18.62	26.10	100.53	10.06
σ_1=4;σ_3=20; no habit	0.170	2.20/2.16	1.86	2.59/2.62	1.95	12.95	103.37	14.83
σ_1=4;σ_3=20; with habit	0.198	3.53/3.22	3.07	2.04/2.39	6.33	17.97	101.58	13.40
bargaining power	0.160	1.99/1.97	1.68	2.82/2.78	1.88	12.55	102.37	13.24
bargaining power with habit	0.206	3.77/3.41	3.25	1.88/2.27	6.36	18.37	102.01	14.45

GHH σ_1=4;σ_3=10; ϕ=2	σ_Y	σ_I /$\rho_{I,Y}$	$\sigma_C/\rho_{C,Y}$	σ_{C_1} /σ_{C_3}	$\sigma_N/\rho_{N,Y}$	$\sigma_W/\rho_{W,Y}$	$\sigma_{WN/Y}/\rho_{WN/Y}$
benchmark no habit	2.06	3.24/1	1.77/1	2.34/1.52	1.03/1	0.60/1	1.06/-0.67
benchmark with habit	1.99	3.94/0.92	1.62/0.97	1.80/1.56	0.99/1	0.69/0.86	0.58/-0.55
$\phi = 1.5$ no habit	2.46	3.53/1	2.19/1	2.63/2.00	1.64/1	0.46/1	1.39/-0.24
$\phi = 1.5$ with habit	2.50	4.58/0.85	2.23/0.96	1.97/2.36	1.67/1	0.86/0.73	0.59/-0.20
inv.adj.cost no habit	1.91	4.38/0.97	1.35/0.98	1.63/1.25	0.96/1	0.43/0.96	1.44/-0.35
inv adj cost with habit	2.00	4.41/0.99	1.44/0.99	1.37/1.50	1.00/1	0.64/0.88	0.64/-.057
σ_1=4;σ_3=20; no habit	1.99	3.40/1	1.65/1	2.52/1.27	1.00/1	0.42/1	1.82/-0.29
σ_1=4;σ_3=20; with habit	2.05	4.10/0.92	1.66/0.97	1.90/1.56	1.02/1	0.68/0.86	0.67/-0.54
bargaining power	1.97	3.24/1	1.65/1	2.32/1.45	0.98/1	0.56/1	1.32/-0.31
bargaining power with habit	1.99	4.13/0.93	1.58/0.97	1.84/1.50	0.99/1	0.64/0.86	0.82/-0.48

8.3 Distribution risk and volatility in the returns of assets

In order to further increase the risk premium and to better match the variability in the wage share and the moderate cyclicality of wages, it is very helpful to consider distribution risk in addition to aggregate productivity risk. The heterogeneous agent model offers the natural context for introducing this type of uncertainty. Note that distribution shocks are much more successful in generating risk premiums compared to the more usual mark up shocks in standard representative agent models.

The stochastic process (σ_v= 0.25 and ρ_v= 0.95) that we assume for the distribution shock remains moderate as illustrated by the standard deviation of the real wage and the wage share in the case where only distribution shocks are active. Remember that distribution shocks only change the distribution of output between the two type of agents, but do not cause any misallocation of labor as the optimal contract implies that the marginal rate of substitution between consumption and labor is still equalized to the marginal productivity of labor. Output and employment are hardly influenced by these shocks. However, shareholders are able to smooth consumption by adjusting investment expenditures, so that the effect on the SR and the RP remains moderate as well: distribution risk alone is able to produce a SR and a risk premium that is comparable to aggregate productivity risk. The high correlation between profits, or the available funds for shareholders, and investment prospects imply that shareholders can smooth consumption without large fluctuations in the short rate.

Combining productivity and distribution risk, as two uncorrelated sources of risk, leads to more significant risk premiums. However, this combination still fails on generating acyclical real wages and countercyclical wage shares. One way to improve many of these statistics is to allow for a negative correlation between the two shocks. The theoretical motivation for such a negative correlation is that the introduction of new technologies or more efficient organizations might temporarily reduce the bargaining power of the workers: it might take some time before workers recognize the increased productivity and the corresponding excess profits. New technologies might also lower temporarily the required labor input, and this might also weaken the bargaining power of the workers.

As can be judged from the table below, allowing for some negative correlation between productivity and distribution risk, improves both the financial and the real statistics. The model generates significant risk premiums for equity, which are driven by both the variability in consumption of the shareholders and the volatility in the profit stream and the return. The price dividend ratio increases and becomes more volatile as well. On the other hand, the average risk free interest rate is low (due to the strong precautionary savings effect) but remains reasonable compared to the observed value, and the standard deviation of the risk free rate also remains very low. This leaves some room to augment the model with habit persistence or investment adjustment cost without increasing the short rate volatility too much. On the real side, employment remains perfectly correlated with output; investment and shareholders consumption becomes more volatile; but aggregate consumption remains quite smooth. Wages become acyclical and the wage share displays the empirically observed variability and behaves countercyclical.

Danthine Donaldson talk about operational leverage or risk to explain the impact of a stable wage bill in the risk sharing contract and the volatility of the profits that follows from this. Financial leverage further increases this volatility of profits (after interest payments) and therefore also the risk premium on equity, but it is not affect the SR. If we allow for a realistic share of debt financing by firms of 30%, we get risk premiums on equity that approach the actually observed statistic.

Table 10:

GHH σ_1=4;σ_3=10;	SR^A	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
(1)	0.106	0.93/0.95	0.82	3.45/3.46	1.36	8.86	100.20	7.92
(2)	0.184	2.66/2.61	2.29	2.34/2.39	2.25	14.48	103.49	15.58
(3)	0.234	4.32/4.24	3.71	1.28/1.36	2.86	18.49	106.08	19.52
(4)	0.244	7.07/6.93	3.97	1.05/1.16	2.96	28.96	135.83	58.47

GHH σ_1=4;σ_3=10;	σ_Y	σ_I /$\rho_{I,Y}$	$\sigma_C/\rho_{C,Y}$	σ_{C_1} /σ_{C_3}	$\sigma_N/\rho_{N,Y}$	$\sigma_W/\rho_{W,Y}$	$\sigma_{WN/Y}/\rho_{WN/Y}$
(1)	0.07	2.04/-0.07	0.52/0.23	1.23/1.27	0.03/1	1.01/0.14	1.99/0.03
(2)	2.01	3.73/0.83	1.81/0.96	2.57/0.87	1.00/1	1.19/0.51	2.26/-0.16
(3)	1.97	4.67/0.94	1.42/1.96	3.17/1.15	0.98/1	0.77/-0.10	2.88/-0.34
(4)	2.03	4.98/0.94	1.44/0.96	3.61/1.21	1.02/1	0.82/-0.12	3.15/-0.33

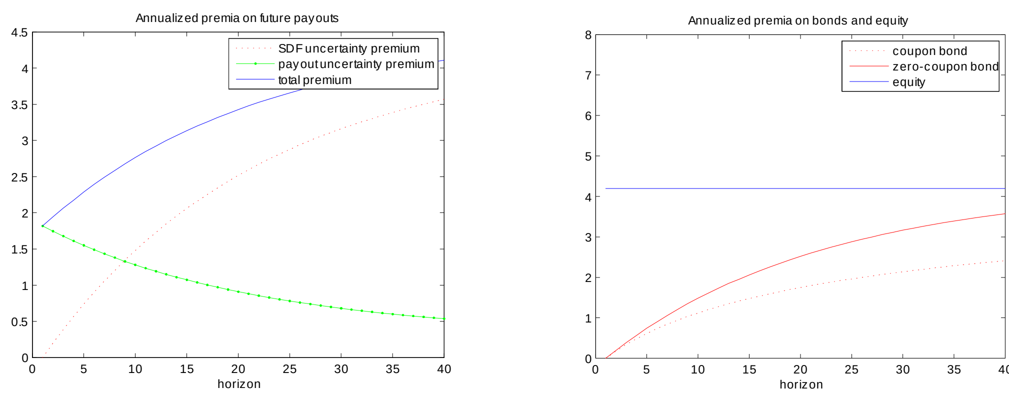
Note: (1): distribution risk only, (2) distribution & productivity risk , (3) distribution & productivity risk (ρ =-0.65), (4) including financial leverage (30%)

The model with correlated distribution and productivity shocks leads to a lower risk premium on long term bonds compared to equity, but this difference becomes really significant only after introducing financial leverage. In order to understand this phenomenon, it is helpful to decompose the risk premium related to any future income stream (d_{t+k}) in its two components: the covariance between the expected stochastic discount factor and the marginal utility of the shareholder and the covariance between the future income stream and the marginal utility (See Jermann for more detailed discussion of this decomposition and Appendix B).

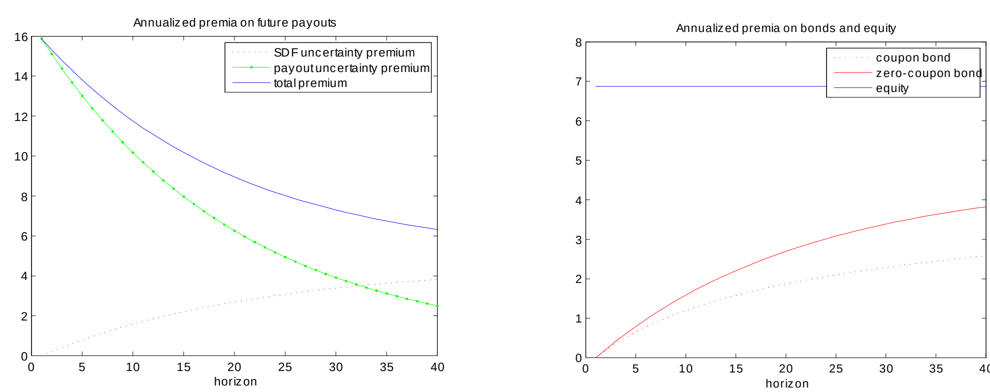
$$RP(d_{t+k}) = -cov(E_{t+1}\lambda_{t+k} - \lambda_{t+1}, \lambda_{t+1}) - cov(E_{t+1}d_{t+k}, \lambda_{t+1})$$

The term premium on a bond and the premium on equity can be written as weighted sums of future income streams, and their risk premium can also be expressed as weighted averages of the risk premium on each of these pay-outs. For a bond, the payout uncertainty is zero and the term premium is fully determined by the first covariance term. The equity premium will deviate from the term premium as far as the pay-out uncertainty delivers an important contribution to the overall risk premium on equity. An increasing role of the payout covariance is crucial for a successful joint explanation of the equity and the bond premium. Graph 1 summarizes this information for the model with negatively correlated distribution and productivity but without leverage, while graph 2 provides the same information for the model with financial leverage. Graph 1a displays the two components of the risk premium for payouts with different horizons, while 1b shows the weighted averages that determine the term premium on bonds (for different horizons expressed in quarters) and the risk premium on equity. The graph clearly illustrates the dominance of the SDF uncertainty as the major source of risk in the model without leverage. For long bonds, the term premium is of a similar magnitude than the equity risk premium. Once we allow for leverage, the payout uncertainty of the dividends becomes much more important, as a result of the financial leverage effect, and we get a much more substantial difference between the risk of bonds and equity. However, the term premium in bonds increases quickly with the duration and remains higher than what is typically observed in reality.

Graph 1: Decomposition of the term premium on bonds and the equity risk premium for the model without leverage



Graph 2: Decomposition of the term premium on bonds and the equity risk premium for the model with leverage



9 The complete model with three agents

In the complete model, we consider the following weights for the three type of agents: 60% Type 3 agents, 30% Type 2 agents and 10% of Type 1 active shareholders. Their respective risk aversion are fixed at 10-10-4. The bond holdings by the Type 2 agents will also depend on the financial cost parameter (0.0001). We allow for both aggregate productivity shocks and distribution risk, which are negatively correlated.

Table 11:

GHH σ_1=4;σ_2=10;σ_3=10	SR^A	EP^A	BP^A	R^f	σ_{R^f}	σ_{R^S}	PD	σ_{PD}
distr. & prodty risk (ρ=-0.65)	0.210	3.48/3.46	3.11	1.85/1.86	2.65	16.75	105.74	20.38
incl. financial leverage	0.220	5.80/5.68	3.35	1.58/1.68	2.74	26.21	127.12	50.99

GHH σ_1=4;σ_2=10;σ_3=10	σ_Y	$\sigma_I/\rho_{I,Y}$	$\sigma_C/\rho_{C,Y}$	$\sigma_{C_1}/\sigma_{C_2}/\sigma_{C_3}$	$\sigma_N/\rho_{N,Y}$	$\sigma_W/\rho_{W,Y}$	$\sigma_{WN/Y}/\rho$
distr. & prodty risk (ρ =-0.65)	1.98	4.44/0.96	1.45/0.98	3.10/1.86/1.18	0.99/1	0.64/0.23	2.11/-0.36
incl. financial leverage	2.03	4.65/0.96	1.46/0.97	3.62/1.92/1.24	1.02/1	0.67/0.21	2.36/-0.35

The presence of both Type 2 and Type 3 agents enhances the volatility in the consumption of the active shareholders which leads to a higher SR and RP. The effect of leverage further helps for the risk premium, but leads to an excessive volatility in the Price-Dividend ratio. The model generates endogenously the following wealth distribution: 78% of financial assets are hold by the top 10% of active shareholders, 22% is hold by the 30% bondholders and 0% is hold by the 60% of the population that does not participate at all in the financial market. This type of information can be very useful in a more sophisticated calibration exercise of the model.

10 The model with nominal rigidities: the impact of inflation risk

Up to now, we concentrated the discussion on the real model, where prices were considered as completely flexible. Now we extend the model with nominal price rigidity ($\chi = 120$) and an

explicit monetary policy reaction function. We consider directly the final model as presented in the previous section, with the same structure for the stochastic nature of the model. The role of nominal rigidities depends of course crucially on how the monetary policy behavior is specified. If we allow monetary policy to behave optimally, by an interest rate that adjust proportionally to the natural real rate implied by the model, $R_t = RN_t + 1.5\left(\pi_t - \overline{\pi}\right)$, inflation will remain zero at any moment and the risk premiums remain identical to the ones in the model without nominal rigidities. Inflation and the corresponding premium start to become interesting once we assume that monetary policy does not observe the contemporaneous natural real rate, and instead uses some simple instrument rule. For computational reason, we assume that the inflation target in this rule is equal to zero, and we also introduce a small but significant reaction to the price level in order to avoid non-stationary nominal variables in the model. This last assumption might not be neutral for the implied risk premium on long term nominal assets. To illustrate this, we run an additional simulation using a smaller reaction to the price level. We also allow for a slow moving average real interest rate in the reaction function, to avoid disturbances of the stochastic steady state outcome by a deterministically imposed real rate in the policy rule.

$$R_t = RN_{ave,t} + 1.5\left(\pi_t - \overline{\pi}\right) + 0.01\left(P_t - \overline{P}\right) \quad (11)$$

$$RN_{ave,t} = 0.99 * RN_{ave,t-1} + 0.01 * RN_t \quad (12)$$

For the chosen calibration of price adjustment costs, we get a quite high volatility of inflation of around 4% (annualized standard error) around its zero mean average. For the typical productivity shock in the model that goes together with a decline in the wage share as well, the price level drops with 0.6% on impact. As a result, the nominal short term interest rate becomes more volatile relative to the real interest rate. The properties of the real economy hardly change after introducing nominal rigidities.

Table 12:

	SR^A	EP^A	BP^A	R^f	σ_{R^f}	R^N	σ_{R^N}	σ_{R^S}	PD	σ_{PD}	σ_π
(1)	0.220	5.80/5.68	3.35	1.58/1.68	2.74	-	-	26.21	127.12	50.99	0
(2)	0.220	5.45/4.90	3.73	1.59/1.67	2.75	2.08	5.74	24.65	119.36	43.50	3.78
(3)	0.216	5.29/4.87	5.02	1.63/1.67	2.75	2.16	6.56	24.77	120.11	44.37	4.32

	σ_Y	$\sigma_I\ /\rho_{I,Y}$	$\sigma_C/\rho_{C,Y}$	$\sigma_{C_1}\ /\sigma_{C_3}/\sigma_{C_3}$	$\sigma_N/\rho_{N,Y}$	$\sigma_W/\rho_{W,Y}$	$\sigma_{WN/Y}/\rho_{WN/Y}$
(1)	2.03	4.65/0.96	1.46/0.97	3.62/1.92/1.24	1.02/1	0.67/0.21	2.36/-0.35
(2)	1.97	4.59/0.96	1.41/0.97	3.55/1.84/1.21	0.945/1	0.66/0.18	2.21/-0.37
(3)	1.93	4.51/0.95	1.39/0.97	3.49/1.81/1.25	0.94/1	0.68/0.19	2.47/-0.33

Note: (1): optimal monetary policy, (2) infl.& price level (0.01) response, (3)infl.& price level (0.001) response

The total risk premium on equity decreases: the additional term that represents the inflation risk premium ($\rho_{\pi,rrs}\sigma_\pi\sigma_{rrs}$) is negative because of the negative correlation between inflation and the return on equity. Clearly in the model with productivity and distribution risk only, equity provides some hedge against inflation risk. The presence of financial leverage does not significantly affect this result. The risk in long term nominal bonds on the other hand increases, as the correlation between inflation and return on long nominal bonds is strongly positive. The size of the term premium is obviously depending on the stabilization of the price level by the monetary policy behavior. Note that the inflation risk premium on short bonds is relatively large and of equal magnitude than the one for long bonds when the price level is more strongly stabilized.

11 Third order simulations and the cyclical properties of the risk premium

TBA

12 Preliminary Conclusions

The objective of this research is to come up with a DSGE model that is able to fit well both on the real and the financial side of the economy. Such a model would be useful to identify the expectations of private agents about future growth, inflation and distribution variables after correcting for risk premiums. In a previous paper (De Graeve et al. 2007), we analyzed the term structure in the standard Smets and Wouters model. We learned from that exercise that the risk premiums, and the term premium in particular, generated by this standard monetary policy DSGE model were extremely small and unable to explain the observed average spread in the yield curve. Therefore, the interpretation of the yield structure obtained in that paper was purely based on the rational expectations hypothesis. This result was surprising to us because the model incorporates several features that are usually considered as potentially able to generate significant risk premiums (habit persistence, investment adjustment costs, wage rigidity, etc.). Based on this experience, we decided to extend the model by introducing agent heterogeneity. The analysis of this paper illustrates that indeed such a setup can be a useful alternative to the standard representative agent model for modelling jointly the real and the financial side of the economy.

The analysis presented here is not yet sufficient to draw strong conclusions.

First of all, we need to understand better the difference in the magnitude of the risk premium on long bonds and on equity. For the moment, the model does not differentiate enough between these two assets. This problem becomes even more important when we consider inflation risk that seems to affect mainly the term premium in nominal bonds.

Secondly, we need to consider a more general stochastic structure in the model. Preliminary exercises show that productivity and distribution risk (and inflation risk for nominal bonds) are the most important shocks for explaining the risk premium in this model. We do not expect that other sources of risk will be crucial for asset prices. But the exact specification in terms of persistence and correlation of productivity, distribution and inflation risk can be important for the outcomes.

Thirdly, we need to make a detailed analysis of the time varying nature of the risk premiums in this model by simulations with higher order approximations. This exercise will indicate whether the model is able to generate the observed cyclical profile in the risk premiums and their prediction content for expected returns, inflation and growth. Results, presented by Guvenen for instance, are very promising. Lettau and Ludvigson (2008) also explain how the cyclical behavior of aggregate versus stockholders consumption, especially during recession periods, is crucial for explaining the time variation in risk premiums. If the results are positive, it will be worthwhile to estimate the non-linear model using the particle filter.

Finally, we need to test the empirical success of explaining the observed wage rigidity by risk sharing considerations. This view on wage rigidity was popular in the late seventies, and got recently again some support from micro studies on wage dynamics and their reaction to transitory firm-specific shocks (Guiso et al. 2005).

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A Asset Pricing in Log-Linear Framework

A large body of literature on asset prices and macroeconomic dynamics uses first-order approximate solutions to derive asset prices and premiums. This literature started with Campbell (1994) and Jermann (1998). They assume that the variables are lognormally distributed, and that the first order approximation is good. The returns are derived from the Arrow-Lucas-Rubinstein asset pricing equation:

$$1 = E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}}{\Pi_{t+1}}\right]$$

with Λ and R the shadow value of wealth and the asset return, respectively.

A.1 Real Risk Free Rate

The real risk free rate is the real return on a (zero-) coupon one-period bond:

$$\begin{aligned}
1 &= E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}R^{short}_{t+1}\right] \text{ with } R^{short}_{t+1} = \frac{1}{P^{short}_t} = C^{short}_{t+1} = \overline{C^{short}} \ (Coupon) \\
0 &= \ln\beta + \ln\left(E_t\left[\exp\left(\triangle\ \lambda_{t+1} + r^{short}_{t+1}\right)\right]\right) \text{ with } \lambda = \ln\Lambda \text{ and } r^{short} = \ln R^{short} \\
0 &= \ln\beta + E_t\left[\triangle\ \lambda_{t+1} + r^{short}_{t+1}\right] + \frac{1}{2}\left[\sigma^2_{\triangle\lambda,t} + \sigma^2_{r^{short},t} + 2\rho_{\triangle\lambda,r^{short},t}\sigma_{\triangle\lambda,t}\sigma_{r^{short},t}\right] \\
\log E_t\left[R^{short}_{t+1}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda,t}\right] \text{ because } \sigma_{r^{short},t} = 0
\end{aligned}$$

The unconditional expectation then becomes:

$$r^{short} = -\ln\beta - E\left[\triangle\ \lambda_s\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda_s}\right]$$

Note that for the unconditional version of this equation $\sigma_{r^{short}} \neq 0$. In particular, $\log E\left[R^{short}_{t+1}\right] = -\ln\beta - E\left[\triangle\ \lambda_{t+1}\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda_{t+1}} + 2\rho_{\triangle\lambda_{t+1},r^{short}_{t+1}}\sigma_{\triangle\lambda_{t+1}}\sigma_{r^{short}_{t+1}}\right]$. The last term appears because although R^{short}_{t+1} is known at t and has, therefore a zero conditional variance, it is still a random variable whose unconditional variance is non-zero.

A.2 Nominal Short Rate

The nominal short rate is :

$$
\begin{aligned}
1 &= E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}^{short}}{\Pi_{t+1}}\right] \text{ with } R_{t+1}^{short} = \frac{1}{P_t^{short}} = C_{t+1}^{short} = \overline{C^{short}} \ \ (Coupon) \\
0 &= \ln\beta + \ln\left(E_t\left[\exp\left(\triangle\ \lambda_{t+1} + r_{t+1}^{short} - \pi_{t+1}\right)\right]\right) \\
\text{with } \lambda &= \ln\Lambda \text{ and } r^{short} = \ln R^{short} \text{ and } \pi = \ln\Pi \\
0 &= \ln\beta + E_t\left[\triangle\ \lambda_{t+1} + r_{t+1}^{short} - \pi_{t+1}\right] + \\
&\quad +\frac{1}{2}\left[\begin{array}{c} \sigma^2_{\triangle\lambda,t} + \sigma^2_{r^{short},t} + \sigma^2_{\pi,t} \\ +2\rho_{\triangle\lambda,r^{short},t}\sigma_{\triangle\lambda,t}\sigma_{r^{short},t} - 2\rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} - 2\rho_{\pi,r^{short},t}\sigma_{\pi,t}\sigma_{r^{short},t} \end{array}\right] \\
\log E_t\left[R_{t+1}^{short}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] + E_t\left[\pi_{t+1}\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda,t} + \sigma^2_{\pi,t}\right] \ + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \\
\text{because } \sigma_{r^{short},t} &= 0
\end{aligned}
$$

A.3 Equity Premium

The equity premium is defined as the difference between the nominal stock and the nominal risk free rate.

The stock return is:

$$
\begin{aligned}
1 &= E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}^{stock}}{\Pi_{t+1}}\right] \\
0 &= \ln\beta + E_t\left[\triangle\ \lambda_{t+1} + r_{t+1}^{stock} - \pi_{t+1}\right] + \\
&\quad +\frac{1}{2}\left[\begin{array}{c} \sigma^2_{\triangle\lambda,t} + \sigma^2_{r^{stock},t} + \sigma^2_{\pi,t} \\ +2\rho_{\triangle\lambda,r^{stock},t}\sigma_{\triangle\lambda,t}\sigma_{r^{stock},t} - 2\rho_{\pi,r^{stock},t}\sigma_{\pi,t}\sigma_{r^{stock},t} - 2\rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \end{array}\right] \\
\log E_t\left[R_{t+1}^{stock}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] + E_t\left[\pi_{t+1}\right] - \frac{1}{2}\left[\sigma^2_{\triangle\lambda,t} + \sigma^2_{\pi,t}\right] - \rho_{\triangle\lambda,r^{stock},t}\sigma_{\triangle\lambda,t}\sigma_{r^{stock},t} \\
&\quad +\rho_{\pi,r^{stock},t}\sigma_{\pi,t}\sigma_{r^{stock},t} + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t}
\end{aligned}
$$

with:

$$
R_{t+1}^{stock} = \frac{P_{t+1}^{stock} + D_{t+1}}{P_t^{stock}}
$$

So that the difference between the stock return and the short-term riskless bond, or the equity premium is:

$$
r^{stock} - r^{short} = -\rho_{\triangle\lambda_s,r^{stock}}\sigma_{\triangle\lambda_s}\sigma_{r^{stock}} + \rho_{\pi,r^{stock}}\sigma_{\pi}\sigma_{r^{stock}}
$$

The maximum equity premium (EP) is then:

$$
EP_{\max} = \sigma_{\triangle\lambda_s}\sigma_{r^{stock}} + \sigma_{\pi}\sigma_{r^{stock}}
$$

A.4 Term Premium

The term premium is the difference between the nominal short term bond and the nominal long term bond.

The long-term bond return is:

$$\begin{aligned}
1 &= E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_{t+1}^{long}}{\Pi_{t+1}}\right] \\
0 &= \ln\beta + E_t\left[\triangle\ \lambda_{t+1} + r_{t+1}^{long} - \pi_{t+1}\right] + \\
&\quad +\frac{1}{2}\left[\begin{array}{c} \sigma_{\triangle\lambda,t}^2 + \sigma_{r^{long},t}^2 + \sigma_{\pi,t}^2 \\ +2\rho_{\triangle\lambda,r^{long},t}\sigma_{\triangle\lambda,t}\sigma_{r^{long},t} - 2\rho_{\pi,r^{long},t}\sigma_{\pi,t}\sigma_{r^{long},t} - 2\rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \end{array}\right] \\
\log E_t\left[R_{t+1}^{long}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] + E_t\left[\pi_{t+1}\right] - \frac{1}{2}\left[\sigma_{\triangle\lambda,t}^2 + \sigma_{\pi,t}^2\right] \\
&\quad -\rho_{\triangle\lambda,r^{long},t}\sigma_{\triangle\lambda,t}\sigma_{r^{long},t} + \rho_{\pi,r^{long},t}\sigma_{\pi,t}\sigma_{r^{long},t} + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t}
\end{aligned}$$

with:

$$\begin{aligned}
R_{t+1}^{long} &= \frac{P_{t+1}^{long} + C_{t+1}^{long}}{P_t^{long}} \\
&= \frac{P_{t+1}^{long} + \overline{C^{long}}}{P_t^{long}}
\end{aligned}$$

So that the difference between the long-term bond return and the short-term riskless bond, or the term premium is:

$$r^{long} - r^{short} = -\rho_{\triangle\lambda_s,r^{long}}\sigma_{\triangle\lambda_s}\sigma_{r^{long}} + \rho_{\pi,r^{long}}\sigma_{\pi}\sigma_{r^{long}}$$

The maximum term premium (TP) is then:

$$TP_{\max} = \sigma_{\triangle\lambda_s}\sigma_{r^{long}} + \sigma_{\pi}\sigma_{r^{long}}$$

Note that the term premium can still be calculated as the difference between two real bonds as described in this document for real returns.

A.5 Sharpe Ratio

The equity Sharpe ratio is:

$$\frac{\log E_t\left[R_{t+1}^{stock}\right] - r_{t+1}^{short}}{\sigma_{r,t}} = -\rho_{\triangle\lambda,r^{stock},t}\sigma_{\lambda,t} + \rho_{\pi,r^{stock},t}\sigma_{\pi,t}$$

So that the maximum equity Sharpe ratio is:

$$SR_{\max}^{stock} = \sigma_{\lambda,t} + \sigma_{\pi,t}$$

The bond Sharpe ratio is:

$$\frac{\log E_t\left[R_{t+1}^{long}\right] - r_{t+1}^{short}}{\sigma_{r^{long},t}} = -\rho_{\triangle\lambda,r^{long},t}\sigma_{\lambda,t} + \rho_{\pi,r^{long},t}\sigma_{\pi,t}$$

So that the maximum bond Sharpe ratio is:

$$SR_{\max}^{long} = \sigma_{\lambda,t} + \sigma_{\pi,t}$$

This implies that the maximum bond Sharpe ratio always coincides with the maximum equity Sharpe ratio. This is intuitive as the Sharpe ratio reflects the price of risk in the economy. If there is arbitrage between the two assets there is no reason why the price of risk reflected in both assets should be different. So the Sharpe ratios can only be different if the correlation between the marginal utility (valuation) and the return are different.

A.6 Inflation Risk Premium

We can either compute the inflation risk premium be subtracting the short real risk free rate from the short nominal risk free rate, or subtracting the real long rate from the nominal long rate.

So the first option is:

$$\begin{aligned}
\log E_t\left[R_{t+1}^{short,nom}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] + E_t\left[\pi_{t+1}\right] \\
&\quad -\frac{1}{2}\left[\sigma_{\triangle\lambda,t}^2 + \sigma_{\pi,t}^2\right] + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \\
-\log E_t\left[R_{t+1}^{short,\text{real}}\right] &= +\ln\beta + E_t\left[\triangle\ \lambda_{t+1}\right] + \frac{1}{2}\left[\sigma_{\triangle\lambda,t}^2\right] \\
\underbrace{\log E_t\left[R_{t+1}^{short}\right] - E_t\left[\pi_{t+1}\right] + \frac{1}{2}\left[\sigma_{\pi,t}^2\right]}_{\text{short rate in real terms}} - \log E_t\left[R_{t+1}^{short,\text{real}}\right] &= \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \\
\log E_t\left[R_{t+1}^{short}\right] - \log E_t\left[\Pi_{t+1}\right] - \log E_t\left[R_{t+1}^{short,\text{real}}\right] &= \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t}
\end{aligned}$$

and the second option is:

$$\begin{aligned}
\log E_t\left[R_{t+1}^{long,nom}\right] &= -\ln\beta - E_t\left[\triangle\ \lambda_{t+1}\right] + E_t\left[\pi_{t+1}\right] \\
&\quad -\frac{1}{2}\begin{bmatrix} \sigma_{\triangle\lambda,t}^2 + \sigma_{\pi,t}^2 \\ +2\rho_{\triangle\lambda,r^{long},t}\sigma_{\triangle\lambda,t}\sigma_{r^{long},t} \\ -2\rho_{\pi,r^{long},t}\sigma_{\pi,t}\sigma_{r^{long},t} \\ -2\rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \end{bmatrix} \\
-\log E_t\left[R_{t+1}^{long,\text{real}}\right] &= +\ln\beta + E_t\left[\triangle\ \lambda_{t+1}\right] \\
&\quad +\frac{1}{2}\begin{bmatrix} \sigma_{\triangle\lambda,t}^2 \\ +2\rho_{\triangle\lambda,r^{long},t}\sigma_{\triangle\lambda,t}\sigma_{r^{long},t} \end{bmatrix} \\
\log E_t\left[R_{t+1}^{long,nom}\right] - \log E_t\left[R_{t+1}^{long,\text{real}}\right] - E_t\left[\pi_{t+1}\right] + \frac{1}{2}\sigma_{\pi,t}^2 &= \rho_{\pi,r^{long},t}\sigma_{\pi,t}\sigma_{r^{long},t} + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t} \\
\log E_t\left[R_{t+1}^{long,nom}\right] - \log E_t\left[\Pi_{t+1}\right] - \log E_t\left[R_{t+1}^{long,\text{real}}\right] &= \rho_{\pi,r^{long},t}\sigma_{\pi,t}\sigma_{r^{long},t} + \rho_{\triangle\lambda,\pi,t}\sigma_{\triangle\lambda,t}\sigma_{\pi,t}
\end{aligned}$$

So there is "short term" inflation risk and "long term" inflation risk.

B Equity premium decomposition in payout risk, SDF risk and inflation risk.

Following Campbell (1994) and Jermann (1998) we propose the decomposition of the premium $\log\left(E_t R^{D_k}_{t,t+1}/R^1_{t,t+1}\right)$ on an asset that pays a dividend D_{t+k} (a strip) at time $t+k$ in excess of the risk free rate R^f_{t+1} as:

$$\begin{aligned}\log\left(E_t R^{D_k}_{t,t+1}/R^f_{t+1}\right) &= -cov\left(\lambda_{t+1}, E_{t+1}d_{t+k}\right) \\ &\quad -cov(\lambda_{t+1}, E_{t+1}\lambda_{t+k}-\lambda_{t+1}) \\ &\quad +cov_t\left(\lambda_{t+1}, E_{t+1}\left(p_{t+k}-p_{t+1}\right)\right)\end{aligned} \tag{13}$$

where $R^{D_k}_{t,t+1}$ is the return on the strip and p_{t+1} is the price level. The first term is the premium that arises from payout uncertainty, the second term arises from uncertainty in future marginal utility and the last term is the inflation premium.

To obtain this decomposition start from defining the one-period holding return for the strip as:

$$R^{D_{t+k}}_{t,t+1} = \frac{V_{t+1}\left[D_{t+k}\right]}{V_t\left[D_{t+k}\right]} \tag{14}$$

where $V_t\left[D_{t+k}\right] = \frac{\beta^k E_t[D_{t+k}\Lambda_{t+k}/P_{t+k}]}{\Lambda_t/P_t}$. Then,

$$\begin{aligned}E_t\left[V_{t+1}\left[D_{t+k}\right]\right] &= E_t\left[\frac{\beta^{k-1}E_{t+1}\left[\Lambda_{t+k}D_{t+k}\right]P_{t+1}}{\Lambda_{t+1}P_{t+k}}\right] \\ &= \beta^{k-1}E_t\exp\left(E_{t+1}\left[\lambda_{t+k}-\lambda_{t+1}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right) \\ &\quad *\exp\left(\frac{1}{2}V_{t+1}\left[\lambda_{t+k}+d_{t+k}-p_{t+k}\right]\right) \\ &= \beta^{k-1}\exp\left(E_t\left[\lambda_{t+k}-\lambda_{t+1}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right) \\ &\quad *\exp\left(\frac{1}{2}V_t\left\{E_{t+1}\left[\lambda_{t+k}-\lambda_{t+1}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right\}\right) \\ &\quad *\exp\left(\frac{1}{2}V_{t+1}\left[\lambda_{t+k}+d_{t+k}-p_{t+k}\right]\right)\end{aligned}$$

and

$$\begin{aligned}V_t\left[D_{t+k}\right] &= \frac{\beta^k E_t\left[\Lambda_{t+k}D_{t+k}\right]P_t}{\Lambda_t P_{t+k}} \\ &= \beta^k\exp\left(E_t\left[\lambda_{t+k}-\lambda_t+d_{t+k}-p_{t+k}+p_t\right]+\frac{1}{2}V_t\left[\lambda_{t+k}+d_{t+k}--p_{t+k}+p_t\right]\right)\end{aligned}$$

Given that the real return on a one-period bond $R_{t,t+1}\left[1_{t+1}\right]$ is risk free rate R^f_t, we have:

$$\begin{aligned} R_t^f &\equiv R_{t,t+1}\left[1_{t+1}\right] \\ &= \frac{1}{E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\right]} \\ &= \frac{1}{\beta\exp\left(E_t\left[\lambda_{t+1}-\lambda_t\right]+\frac{1}{2}V_t\left[\lambda_{t+1}\right]\right)} \\ &= \beta^{-1}\exp\left(-E_t\left[\lambda_{t+1}-\lambda_t\right]-\frac{1}{2}V_t\left[\lambda_{t+1}\right]\right) \end{aligned}$$

By replacing in equation (14) and taking the conditional expectation, we have:

$$\begin{aligned} E_tR_{t,t+1}^{D_{t+k}} &= \\ & = \beta^{-1}\exp\left(E_t\left[\lambda_{t+k}-\lambda_{t+1}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right) \\ & \quad *\exp\left(\tfrac{1}{2}V_t\left\{E_{t+1}\left[\lambda_{t+k}-\lambda_{t+1}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right\}\right)*\exp\left(\tfrac{1}{2}V_{t+1}\left[\lambda_{t+k}+d_{t+k}-p_{t+k}\right]\right) \\ & \quad *\exp\left(-E_t\left[\lambda_{t+k}-\lambda_t+d_{t+k}-p_{t+k}+p_{t+1}\right]-\tfrac{1}{2}V_t\left[\lambda_{t+k}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right) \\ & = \tfrac{1}{\beta}\exp\left(-E_t\left[\lambda_{t+1}-\lambda_t\right]-\tfrac{1}{2}V_t\left[\lambda_{t+1}\right]\right) \\ & \quad *\exp\left(\tfrac{1}{2}\left[V_t\left\{E_{t+1}\left[\lambda_{t+k}-\lambda_{t+1}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right\}+V_t\left[\lambda_{t+1}\right]\right]\right) \\ & \quad *\exp\left(\tfrac{1}{2}V_{t+1}\left[\lambda_{t+k}+d_{t+k}-p_{t+k}\right]\right) \\ & \quad *\exp\left(-\tfrac{1}{2}V_t\left[\lambda_{t+k}+d_{t+k}-p_{t+k}+p_{t+1}\right]\right) \\ &= R_t^{short}*\exp\left(-cov_t\left(\lambda_{t+1},E_{t+1}d_{t+k}\right)\right) \\ & \quad *\exp\left(-cov_t(\lambda_{t+1},E_{t+1}\lambda_{t+k}-\lambda_{t+1})\right) \\ & \quad *\exp\left(cov_t\left(\lambda_{t+1},E_{t+1}p_{t+k}-p_{t+1}\right)\right) \end{aligned}$$

Taking logs, we recover equation (13).

The return on any asset that pays a stream of payouts over k periods, $R_{t,t+1}\left[\{D_{t+k}\}_{k=1}^{\infty}\right]$, can be computed as the weighted sum of the return on k strips.

$$\begin{aligned} E_t\left\{R_{t,t+1}\left[\{D_{t+k}\}_{k=1}^{\infty}\right]\right\}/R_{t,t+1}^1 &= \\ &= \sum_{k=1}^{\infty}w_t\left[D_{t+k}\right]*\exp\left(RP\left(d_{t+k}\right)\right) \end{aligned}$$

where $w_t\left[D_{t+k}\right]=\frac{V_t[D_{t+k}]}{\sum_{k=1}^{\infty}V_t[D_{t+k}]}$. We compute the weights at the steady state:

$$\begin{aligned} \left[w_t\left[D_{t+k}\right]\right]_{SS} &= \left[\frac{\frac{\beta^kE_t[D_{t+k}\Lambda_{t+k}/P_{t+k}]}{\Lambda_t/P_t}}{\sum_{k=1}^{\infty}\frac{\beta^kE_t[D_{t+k}\Lambda_{t+k}/P_{t+k}]}{\Lambda_t/P_t}}\right]_{SS} \\ &= \frac{\beta^k}{\sum_{k=1}^{\infty}\beta^k} \\ &= \beta^{k-1}\left(1-\beta\right) \end{aligned}$$