Expectation Driven Business Cycles with Limited Enforcement*

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Abstract

We explore the implications of shocks to expected future productivity in a setting with limited enforcement of financial contracts. As in Lorenzoni and Walentin (2007) limited enforcement implies that to obtain external finance firms have to post collateral in terms of liquidation value of the firm.
In contrast to earlier real one-sector models, we show that a model with this type of "collateral constraint" generates an increase in stock prices in response to positive news about future productivity, as well as the standard properties of a Pigou cycle. The positive stock price response is in line with Beaudry and Portier's (2006) empirical results and the emerging standard view of expectation driven booms.

Preliminary version.

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1 Introduction

This paper is part of the growing literature following Beaudry and Portier's (2004) work on expectation driven business cycles. We explore the implications of shocks to expectations about future productivity ("news shocks") in a small DSGE model with limited enforcement of financial contracts.

With this type of financial friction it turns out that a one-sector model can generate comovement in investment, consumption, employment and stock prices in response to changes in expectations about future productivity. The contrast to the existing literature is the last part, i.e. that stock prices increase in response to positive news. This fundamental characteristic of expectation driven booms has not been successfully modelled in a real one-sector model before. Empirically, Beaudry and Portier (2004, 2006) make a strong case that stock prices increase in response to positive news shocks. Partly for this reason it seems important to get this aspect right in models of expectation driven business cycles.

The present paper is similar in its aim to Christiano, Motto and Rostagno's (2006) paper on boom-bust cycles (hereafter CMR). The limitations of their real model are stated by themselves as:

> "However, we are not successful [in] producing a rise in the price of capital in the boom phase of the cycle. In addition, we will see that it is hard to generate a boom that is much longer than one year. Finally, we will see that the model generates extreme fluctuations in the real rate of interest."[1]

CMR solve the above problems by adding a monetary dimension with sticky prices and wages to their model and imposing a Taylor rule for the interest rate. We instead address these issues in a purely real model.

A second closely related paper is Jaimovich and Rebelo (2006). They construct a real model that generates Pigou cycles neatly in a one-sector setting, but do not get a positive response of stock prices to news.[2] Another related paper is Chen and Song (2007) who explores capital reallocation in a setting with expectation chocks and a collateral constraint on entrepreneur's financing.

The model in Beaudry and Portier (2004) generates the same type of comovement between expected future productivity and current stock prices as we do. The main difference is that they use a three sector model with complimentarities between capital and the intermediate good, and a shock to the productivity of the intermediate goods sector.

The technical contribution to the news shock literature of the present paper is the analysis of limited enforcement. The first key effect of introducing limited enforcement is

[1] Christiano, Motto and Rostagno (2006), page 9.

[2] There are several recent papers exploring various mechanisms to understand expectation driven business cycles: labor market matching (Den Haan and Kaltenbrunner (2006), vintage capital (Flodén (2006)) and collateral constraints for financing wages and intermediate goods (Kobayashi, Nakajima and Inaba (2007)).

that the funds available to a firm, and thereby its investment, become a function of the discounted value of a "collateral" which is the expected next period liquidation value of the firm. This introduces an additional channel through which expectations about the future affect the economic decisions today.

Secondly, as shown in Lorenzoni and Walentin (2007), limited enforcement causes a positive time-varying *wedge* between marginal q and average q. This wedge reflects the tension between the future profitability of investment and the availability of funds today. Accordingly, the wedge, and average q, will increase with expected future productivity if current funds do not increase sufficiently, as in the case of a shock to future productivity that leaves current productivity unchanged. We illustrate this mechanism in a setup that is otherwise similar to the real model of CMR.

The paper proceeds as follows. In section 2 we set up and solve the model. In section 3 we present impulse response functions and elaborate on the intuition for the key results. Section 4 concludes.

2 The model

In this section we lay out the model. It is a DSGE model with two types of agents: consumers and entrepreneurs, each of unit mass. There are two goods, a perishable consumption good and physical capital. Transformation between consumption good and capital is subject to adjustment costs.

Markets are complete, but there is limited enforcement of financial contracts. All markets are competitive. Large parts of the model are taken from Lorenzoni and Walentin (2007), in particular the modelling of optimal financial contracts. We will therefore be slightly brief in the description of the setup and solution of the model.

Two key mechanisms from earlier papers in the expectation driven boom literature are used. Habit formation is helpful in generating an increase in consumption in response to positive news. Similarly, investment adjustment costs (i.e. as a function of I_t/I_{t-1}), as opposed to capital adjustment costs, create a tendency for investment to respond news about future productivity..

2.1 Setup

Preferences. The preferences of a consumer is described by

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(c_t-bc_{t-1})^{1-\sigma_C}}{1-\sigma_C}-\frac{\varphi_L}{1+\sigma_L}{l_t}^{1+\sigma_L}\right)\right]$$

Consumers choose consumption c, hours worked l, and save in state contingent assets. b is a habit parameter. The consumer's problem is in other words quite standard, and will be treated very briefly.

Entrepreneurs have finite lives. Each period a fraction γ of entrepreneurs dies and is replaced by an equal mass of young entrepreneurs. The first period of their life entrepreneurs are endowed with l_E units of labor. This gives new entrepreneurs positive initial wealth.

The preferences of entrepreneur i, born at date t, are described by the utility function

$$\mathbb{E}_t \left[\sum_{j=0}^{J_i} \beta_E^j c_{i,t+j}^E \right],$$

where J_i is the random duration of the entrepreneur's life. Entrepreneurs are more impatient than consumers, $\beta_E < \beta$. This assumption, together with the assumption of a finite life for entrepreneurs, guarantees the existence of a steady state where the borrowing constraint is always binding. We will discuss this assumption further below.

Technology. Each period t entrepreneurs have access to a constant returns to scale technology described by the concave production function $A_t F(k_{i,t}, l_{i,t})$, where $k_{i,t}$ is capital installed in period $t-1$. The aggregate productivity parameter A_t follows

$$\log A_t = a_t = \rho a_{t-1} + \varepsilon_t + \eta_{t-p}$$

where ε_t and η_t are Gaussian iid shocks. Note that η is a "news" shock - it is known p periods before it affects the productivity. The basic idea behind this is that there is a time lag between any technological innovation and its broad implementation.

We study the characteristics of a model with convex investment adjustment costs of the type used by Christiano, Eichenbaum and Evans (2005), where the law of motion for capital is the following.

$$k_{i,t+1} = (1-\delta) k_{i,t} + \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \quad (1)$$

$$\text{where } S(x) = \frac{g}{2}(x-1)^2 \quad (2)$$

The timing of events is as follows. At the beginning of period t, production is realized and entrepreneur i learns if period t is his last period of activity. Then, entrepreneurs trade used capital. With this timing assumption entrepreneurs are able to liquidate all their capital on their last period of activity. Furthermore, this assumption also helps in modelling the liquidation proceedings in the event an entrepreneur defaults.

Aggregate uncertainty is described by the Markov process s_t in the finite state space $\mathcal{S}$, with transition probability $\pi(s_{t+1}|s_t)$. Individual uncertainty is described by the random variable $\chi_{i,t}$, which is equal to 1 in all the periods when entrepreneur i is active, except in the last period, when $\chi_{i,t} = 0$.

Financial contracts. Consider an entrepreneur born at time t. The entrepreneur finances his current and future investment by selling a long-term financial contract $\mathcal{C}_{i,t}$. The contract specifies a sequence of state-contingent transfers $\{d_{i,\tau}\}_{\tau=t}^{\infty}$,[3] for all the periods in which the entrepreneur is alive. The transfers are contingent both on the history of aggregate shocks and on the idiosyncratic termination shock of entrepreneur i. The choice variable $k_{i,\tau+1}$, and the transfer $d_{i,\tau}$, are set after the idiosyncratic termination shock is realized. Let q_t^m denote the price of capital and w_t the wage rate in period t. Feasibility requires that the transfers $\{d_{i,\tau}\}$ satisfy:

$$c_{i,\tau}^E + d_{i,\tau} \leq A_\tau F\left(k_{i,\tau}, l_{i,\tau}\right) - w_\tau l_{i,\tau} - q_\tau^m \left(k_{i,\tau+1} - k_{i,\tau}\left(1-\delta\right)\right), \tag{3}$$

for all the periods where the entrepreneur is active.[4]

Limited enforcement. Financial contracts are subject to limited enforcement. The entrepreneur has full control over the firm's assets. In each period, after production takes place, the entrepreneur can choose to divert part or all of the current profits and the capital stock. In this way he can capture up to a fraction $(1-\theta)$ of the firm's *liquidation value*, $v_{i,t}$, which is equal to current profits plus the resale value of the capital stock:

$$v_{i,t} = A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} + q_t^m k_{i,t}\left(1-\delta\right).$$

The only recourse outside investors have against such behavior is the liquidation of the firm. Upon liquidation, the investors can recover the remaining fraction θ of the firm's liquidation value. After liquidation the entrepreneur can start anew with initial wealth $(1-\theta)\, v_{i,t}$. That is, the only punishment for a defaulting entrepreneur is the loss of a fraction θ of the firm's liquidation value.

2.2 Optimal financial contracts

Before turning to the competitive equilibrium, we concentrate on the decision problem of a single entrepreneur. We begin by spelling out some results from consumers' optimization and introducing some preliminary definitions that will simplify the analysis. Then we give a recursive characterization of the optimal financial contract and show that, under constant returns to scale and given the notion of limited enforcement introduced above, the optimal financial contract is linear.

2.2.1 Preliminaries

Consumers. We will study equilibria where consumers always have positive consumption, $c_t > 0$. Therefore, the price of a sequence of state-contingent transfers $\{d_{i,t+s}\}_{s=0}^{\infty}$ is discounted using the consumer's discount factor, i.e. at the rate $m\left(X', X\right)$. This factor is

[3] The transfer will typically be negative in the first period (initial investment) and can be positive or negative in the following periods, corresponding to dividend payments minus new investment in the firm.

[4] In the first period of activity the constraint is:

$$c_{i,t}^E + d_{i,t} \leq A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} - q_\tau^m \left(k_{i,t+1} - k_{i,t}\left(1-\delta\right)\right) + w_t l_E,$$

with $k_{i,t} = 0$.

defined by

$$m\left(X', X\right) = \beta \frac{\lambda_{C,t+1}}{\lambda_{C,t}}$$

where λ_C denotes the marginal utility of consumption and can be written as

$$\lambda_{C,t} = (c_t - bc_{t-1})^{-\sigma_c} - b\beta E_t \left(c_{t+1} - bc_t\right)^{-\sigma_c}$$

The static problem of labor supply is standard. The consumer's first order condition with respect to labor supply implies:

$$w(X) = \frac{\varphi_L {l_t}^{\sigma_L}}{\lambda_C(X)}$$

Entrepreneurs. An entrepreneur born at date t will choose the financial contract $\mathcal{C}_{i,t}$ to maximize his expected utility subject to feasibility, (3), to the intertemporal budget constraint:

$$\sum_{s=0}^{\infty} \prod_{r=1}^{s} \mathbb{E}_t \left[m(X_{\tau+r}, X_{\tau+r-1}) d_{i,t+s}\right] \geq 0,$$

and to the condition that future promised transfers are credible. The last condition is satisfied if, at each date, the entrepreneur prefers repayment to diversion and default. This condition is stated formally below. For a recursive formulation of the problem it is useful to define the net present market value of the firm's liabilities at date τ:

$$b_{i,\tau} = \sum_{s=0}^{\infty} \left(\mathbb{E}_\tau \left[d_{i\tau}\right] + \prod_{r=1}^{s} \mathbb{E}_\tau \left[m(X_{\tau+r}, X_{\tau+r-1}) d_{i\tau+s}\right] \right).$$

The entrepreneur's problem can be simplified by exploiting the assumption of constant returns to scale. Under constant returns to scale the liquidation value of the firm can be written as:

$$v_{i,t} = R_t k_{i,t} = \max_{l_{i,t}} \left\{ A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} + q_t^m k_{i,t} \left(1-\delta\right) \right\},$$

where R_t, the gross return on capital, is taken as given by the single entrepreneur and is a function of the prices w_t and q_t^m. Also, constant returns to scale for adjustment costs, and the presence of a competitive market for used capital, imply that there exists a price of capital, q_t^m, which is taken as given by the single entrepreneur, such that[5]:

$$q_t^m = \frac{1 - \beta_E E_t \left[\frac{\phi_{t+1}}{\phi_t} q_{t+1}^m\right] \left[S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)}. \tag{4}$$

[5] The derivation of this expression revolves around noting that investment in $t+1$ can be decreased by $S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$ units with an unchanged capital stock in $t+2$. The expression for q_t^m is identical to its counterpart in CMR except that the discount factor of the entrepreneur, instead of the consumer, is used.

Putting together the definitions above, the feasibility constraint (3) can be written as:

$$c_{i,\tau}^{E}+d_{i,\tau}+q_{\tau}^{m}k_{i,\tau+1}\leq v_{i,\tau}. \tag{5}$$

2.3 Recursive characterization of entrepreneur's problem

We study recursive competitive equilibria, where the state of the economy is captured by a vector of aggregate state variables $X_t \in \mathcal{X}$, including the exogenous state s_t, with transition probability $H\left(X_{t+1}|X_t\right)$. The vector X_t will be defined and discussed in section 2.4. For now, consider an entrepreneur, who takes as given the law of motion for X_t. The state X_t determines the wage rate, w_t, and the price of capital, q_t^m. Therefore, it also determines the gross rate of return, R_t. Let this dependence be captured by the functions $R\left(X_t\right)$ and $q^m\left(X_t\right)$.

Now we can use a recursive approach to characterize the optimal financial contract. The individual state variables for the entrepreneur are given by $v_{i,t}$, $b_{i,t}$, and $\chi_{i,t}$. Define $W\left(v,b;\chi,X\right)$ as the expected utility, in state X, of an entrepreneur who controls a firm with liquidation value v and outstanding liabilities b.[6] The expected utility W is defined at the time when production has already taken place and the idiosyncratic termination shock has been observed. Also, W is defined after the default decision has taken place, assuming that the entrepreneur does not default in the current period. For now, we will assume that the entrepreneur's problem has a solution in each state $X \in \mathcal{X}$, and the expected utility W is finite. This will be the case in the recursive equilibria we study below (see Proposition (3)).

In all periods prior to the last period of activity, i.e. for $\chi = 1$, W satisfies the Bellman equation:

$$W\left(v,b;1,X\right)=\max_{\substack{c^E,d\\k',b'(.)}} c^E+\beta_E\mathbb{E}[W\left(v',b';\chi',X'\right)|X] \tag{P}$$

$s.t.$

$$c^E+d+q^m\left(X\right)k'\leq v, \tag{6}$$

$$b=d+\mathbb{E}[m\left(X',X\right)b'\left(\chi',X'\right)|X], \tag{7}$$

$$v'\left(X'\right)=R\left(X'\right)k' \quad \forall X', \tag{8}$$

$$W(v'\left(X'\right),b'\left(\chi',X'\right);\chi',X')\geq W((1-\theta)\,v'\left(X'\right),0;\chi',X')\ \forall\chi',X', \tag{9}$$

where the conditional expectation $\mathbb{E}[.|X]$ is computed according to the transition $H\left(X'|X\right)$, with χ' independent of X'.

Problem (P) can be interpreted as follows. At each date, an entrepreneur who does not default has to decide how to allocate the current firm's resources, v, to its potential uses: payments to insiders, c^E, payment to outsiders, d, and investment in physical capital, $q^m k'$. This is captured by the feasibility constraint (6). Moreover, the entrepreneur has to satisfy the "promise keeping" constraint (7): current and future payments to outsiders

[6] For a newborn entrepreneur, v is the entrepreneur's initial labor income, and b is zero.

have to cover the current liabilities of the firm, b. The current payments are d, the future payments are captured by the market discounted value of the firm's liabilities in the following period, $b'(\chi', X')$. These liabilities are allowed to be contingent on the realization of the idiosyncratic termination shock χ' and of the aggregate state X'. Constraint (8) simply says that the liquidation value of the firm next period will be given by the total returns on the firm's installed capital k'. Finally, the no-default constraint (9) ensures that, in all future states of the world, the future liabilities b' are credible. The no-default constraint take this form, given that the entrepreneur has the option to default and start anew with a fraction $(1-\theta)\,v'$ of the firm's liquidation value and zero liabilities.

An entrepreneur in his last period of activity will simply liquidate all capital and pay existing liabilities. Therefore, for $\chi = 0$ we have:

$$W(v, b; 0, X) = v - b.$$

As shown in Lorenzoni and Walentin (2007), the value function satisfies

$$W(v, b; \chi, X) = W(v - b, 0; \chi, X) \tag{10}$$

and the no-default condition can accordingly be written as

$$b \leq \theta v. \tag{11}$$

Equation (10) allows us to replace constraint (9) with constraint (11). The latter can be interpreted as a "collateral constraint", where the total value of the entrepreneur's liabilities are bounded from above by a fraction θ of the liquidation value of the firm. Using this replacement we note that problem (P) is linear and we obtain the following proposition.

Proposition 1 *The value function $W(.,.;\chi, X)$ is linear in its first two arguments and takes the form:*

$$\begin{aligned} W(v, b; 1, X) &= \phi(X)(v - b), \\ W(v, b; 0, X) &= v - b. \end{aligned}$$

There is an optimal policy for k', c^E, d and b' which is linear in $v - b$.

Entrepreneurial new worth, $n \equiv v - b$, represents the difference between the liquidation value of the firm and the value of the claims issued to outsiders. Proposition 1 shows that the expected utility of the entrepreneur is a linear function of the entrepreneurial net worth. The factor ϕ, which determines the marginal value of the entrepreneurial net worth, depends on current and future prices, and hence it is dependent on X.

The following proposition gives a further characterization of the optimal solution.

Proposition 2 *For a given law of motion $H(X'|X)$, let $\phi(X)$ be defined by the recursion:*

$$\phi(X) = \max\left\{\frac{\beta_E(1-\theta)\mathbb{E}\left[(\gamma+(1-\gamma)\phi(X'))R(X')|X\right]}{q^m(X)-\theta\mathbb{E}\left[m(X',X)R(X')|X\right]}, 1\right\}. \tag{12}$$

Suppose that

$$m\left(X',X\right)\phi(X) \geq \beta_E\phi\left(X'\right) \tag{13}$$

for all pairs X, X' such that $H(X'|X) > 0$. Then, the optimal policy for the individual entrepreneur involves: (i) $k' > 0$, (ii) $c^E = 0$ if $\phi(X) > 1$, and (iii) $b(1, X') = \theta v(X')$ if $m(X',X)\phi(X) > \beta_E\phi(X')$.

A central result of this proposition is point (iii), which characterizes the state pairs X, X' where it is optimal to borrow as much as possible against the revenue realized in state X' and use the proceeds to invest today.

2.4 Equilibrium

We are now in a position to define a recursive competitive equilibrium. The aggregate state is given by

$$X = (K, lag(K), B, lag(C), s),$$

where K is the aggregate capital stock and B represents the aggregate liabilities of the entrepreneurs who are not in their last period of activity.[7]

A recursive competitive equilibrium is given by a transition probability, $H(X'|X)$, such that the optimal behavior of consumers and entrepreneurs is consistent with this transition probability, and the goods market, labor market, and capital market clear. The formal definition is given in the Appendix.

A crucial property of this model is that the entrepreneur's problem is linear, and we obtain optimal policies that are linear in entrepreneurial net worth, $v_{i,t} - b_{i,t}$. Given the linearity of the optimal policies it is straightforward to aggregate the behavior of the entrepreneurial sector. We illustrate the aggregation properties of the model in the case where the collateral constraint is always binding. This is the case where the condition

$$m\left(X',X\right)\phi(X) > \beta_E\phi\left(X'\right) \tag{14}$$

holds for every pair X, X' such that $H(X'|X) > 0$. Proposition 3 below shows that, in economies with "small" productivity shocks, such an equilibrium exists. This case will be the basis for the numerical analysis in the next section.

Condition 14 implies that, in each state X, the state-contingent liabilities are set to their maximum level for each future value of X', i.e. $b'(\chi', X') = \theta v'(X')$. Therefore, the

[7] $lag(K)$ is part of the state because adjustment costs are a function of the previous period's investment. $lag(C)$ is part of the state vector because we have habit preferences in consumption.

optimal level of investment is given by:

$$k' = \frac{1}{q^m(X) - \theta \mathbb{E}[m(X', X) R(X') | X]} (v - b). \tag{15}$$

Consider an economy that enters period t with an aggregate stock of capital K_t, in the hands of old entrepreneurs. The agents who invest in period t are: a mass $(1-\gamma)$ of the old entrepreneurs, who have $v_{i,t} = R_t k_{i,t}$ and $b_{i,t} = \theta R_t k_{i,t}$, and a mass γ of newborn entrepreneurs with $v_{i,t} = w_t l_E$. Therefore, the aggregate entrepreneurial net worth of investing entrepreneurs is:

$$N_t = (1-\gamma)(1-\theta) R_t K_t + \gamma w_t l_E,$$

Using the optimal policy (15) and aggregating we obtain:

$$K_{t+1} = \frac{1}{q_t^m - \theta \mathbb{E}_t [m_{t+1} R_{t+1}]} N_t.$$

From these two equations we get the following law of motion for the aggregate capital stock

$$K_{t+1} = \frac{(1-\gamma)(1-\theta) R_t K_t + \gamma w_t l_E}{q_t^m - \theta \mathbb{E}_t [m_{t+1} R_{t+1}]}. \tag{16}$$

The next proposition shows that for a Cobb-Douglas economy with quadratic adjustment costs and bounded productivity shocks, we can construct a recursive equilibrium of this type.

Let the production function be:

$$A_t F(k_t, l_t) = A_t k_t^{\alpha} l_t^{1-\alpha},$$

Let the unconditional mean of A_t be $\hat{A}$, and let the support of A_t be $\left[\underline{A}, \overline{A}\right]$. To show that a recursive equilibrium with always binding constraint exists we first check that there is a deterministic steady state with binding constraints. This requires that θ is not too large, inequality (A1) in the Appendix ensures that. Second, to obtain local stability of the recursive equilibrium around the deterministic steady state it is necessary to impose an additional restriction on the model parameters. This restriction is given by inequality (**??**) in the Appendix **[TBW]**. Under these two restrictions the following proposition holds.

Proposition 3 *Suppose the parameters* $\left\{\alpha, \xi, \theta, \gamma, \beta, \beta_E, \hat{A}, l_E, \sigma_L\right\}$ *satisfy conditions (A1) and (A2) in the Appendix. Then the economy with constant productivity* $A(s) = \hat{A}$ *has a deterministic steady state with* $\beta_E R > 1$*. Furthermore, there is a* $\Delta > 0$ *such that if the process* $A(s)$ *satisfies* $\overline{A} - \underline{A} < \Delta$*, then there exists a recursive competitive equilibrium where the financial constraint is always binding.*

2.5 Asset prices

We are now in a position to define the *financial value* of a representative firm. The value of the firm is simply the sum of all the claims on the firm's future profits, held by insiders and outsiders. This leads us to the following expression for the ex-dividend value of a continuing firm:

$$p_{i,t} = W\left(v_{i,t}, b_{i,t}; \chi_{i,t}, X_t\right) + b_{i,t} - d_{i,t}.$$

Where W corresponds to the net present value of the payments to the insider and $b_{i,t}$ corresponds to the net present value of the payments to outsiders.

Normalizing the financial value of the firm by the total capital invested we obtain our definition of *average q*

$$q_{i,t} \equiv \frac{p_{i,t}}{k_{i,t+1}}.$$

For continuing entrepreneurs, it is possible to show that $q_{i,t}$ is the same for all agents, and we denote it simply by q_t.

Proposition 4 *Average q is greater than or equal to marginal q,* $q_t \geq q_t^m$, *with a strict inequality if the financial constraint is binding.*

Proof. Given that $\phi_t \geq 1$ we have

$$p_{i,t} = \phi_t \left(v_{i,t} - b_{i,t}\right) + b_{i,t} - d_{i,t} \geq v_{i,t} - d_{i,t} = q_t^m k_{i,t+1}.$$

■

Notice that, absent financial constraints we have $\phi_t = 1$ and $q_t = q_t^m$. In this case the investment part of the model boils down to the Hayashi (1982) model. On the other hand, in presence of financial frictions there is a wedge between the value of the entrepreneur's claims in case of liquidation $(v_{i,t} - b_{i,t})$ and the value of the claims he holds to future profits. In other words, the fact that $\phi_t > 1$ introduces a form of mis-measurement in a fraction of the firm's current value and creates a wedge between q_t^m and q_t.

For later analysis it is convenient to define the net risk-free interest rate r^f, even if contracts in the model are state contingent. It is the inverse of the probability weighted average of the consumer's state contingent discount factor:

$$r^f\left(X\right) = \frac{1}{\mathbb{E}\left[m\left(X', X\right)\right]} - 1$$

Finally, define the external finance premium as

$$f\left(X\right) \equiv \frac{\mathbb{E}\left[m\left(X', X\right) R\left(X'\right)\right]}{q^m\left(X\right)} - 1$$

This reflects the premium that consumers ("outsiders") would be willing to pay to be able to invest directly in the physical capital of firms.

3 News Shock Dynamics

3.1 Calibration

We calibrate the model to a quarterly time period. To match an annual risk-free rate of 3% implies $\beta = 0.9925$. To satisfy (13) we set $\beta_E < \beta$, more specifically, $\beta_E = 0.99$. We let $\alpha = 0.33$, $\delta = 0.0125$ and $\rho = 0.95$ as standard RBC parameter values. We set $\varphi_L = 10$ to get a steady state value of $L = 0.30$. We follow CMR in setting the parameter values $\sigma_L = 1$ and $b = 0.63$. For the investment adjustment cost we take their parameter value $g = 15.1$. We use $\sigma_C = 1$, i.e. log utility of consumption, as a natural benchmark.

Regarding the financial side we set $\theta = 0.3$ based on Fazzari *et al* (1988) who show that firms finance 30% of their investment using external funds. Matching a 2% annual steady state finance premium, following Bernanke, Gertler and Gilchrist (2000), implies $l_E = 0.05$ and $\gamma = 0.015$.[8]

3.2 Impulse response functions

3.2.1 The empirical benchmark

Beaudry and Portier (2006) present VAR evidence in terms of impulse response functions of macro variables to a positive news shock. These IRFs are reprinted in Figure 1 below.[9] This evidence is representative of what is becoming the standard view of an expectation driven business cycle. In a very similar VAR exercise Beaudry and Portier (2006) also showed that investment responds positively to news shocks.

<Figure 1 around here>

3.2.2 Theoretical impulse responses

In Figure 2 we present the impulse responses of the key variables to a positive news shock. We set the number of quarters before impact that a news shock become known to $p = 8$. From Figure 2 we see that the initial increase in consumption, investment and hours are in line with the empirical evidence as well as earlier models, e.g. CMR. The fact that the law of motion for capital in our model is derived from a financial constraint makes no qualitative difference compared to CMR's model. A key mechanism is the wedge between the cost of capital q^m and the stock price q. As can be seen from Proposition 4 this wedge is driven by the marginal value of wealth of entrepreneurs, ϕ, which in turn depend on the expected future return on investment (see equation (12)). Accordingly ϕ, and therefore also the wedge, increase following a positive news shock η. On the other hand, the price of capital q^m falls because of the "flow" adjustment cost specification. The stock price is

[8] The model is parametrized so that the labor input of entrepreneurs have negligible impact on aggregate labor supply. It is constant and accounts for one quarter of a percent ($l_E\gamma/\bar{L}$) of the steady state labor supply.

[9] Both of the identification schemes used in Beaudry and Portier (2006) are plotted in this figure.

affected by both these opposing factors, the wedge dominates, and q therefore increases. In other words, we get an increase in stock prices at the impact of a positive news shock. This is the main aspect of the data, as represented by Beaudry and Portier's VAR, which earlier real models (CMR as well as Jaimovich and Rebelo (2006)) failed to match. On the other hand, we do get the same problematic size in the interest rate swings as CMR.

With this model specification we do not manage to generate a very long expectation driven boom - increasing p much beyond 8 quarters would lead to an initial decrease in investment. But the expectation driven boom do lasts substantially longer than a year and is thereby an improvement compared to CMR.

<Figure 2 around here>

We also explore the propagation induced by limited enforcement in a setting with capital adjustment costs (i.e. "level" adjustment costs, a function of I_t/K_t). As can be seen from equation (16) investment is an increasing function of $\mathbb{E}_t\left[m_{t+1}R_{t+1}\right]$. For a model with log utility an increase in expected future productivity $\mathbb{E}_t\left[a_{t+p}\right]$ decreases $\mathbb{E}_t\left[m_{t+p}\right]$ more than it increases $\mathbb{E}_t\left[R_{t+p}\right]$. To isolate the effect of $\mathbb{E}_t\left[R_{t+p}\right]$ on investment we use a setup with risk-neutral consumers. Impulse responses for this specification are presented in Figure 3. The point of this exercise is to show that if the discount factor is constant, the increase in the value of the "collateral" induced by a news shock is sufficient to cause investment to increase already today. The propagation works through the price of capital q^m and its effect on the return on capital R. The disadvantage of this setup is that it fails to generate an increase in consumption and hours before the shock materializes. The reason for this failure is the preference specification.

<Figure 3 around here>

4 Conclusion

In this paper we have explored the effects of shocks to expectations about future productivity in a DSGE model with limited enforcement. On a qualitative level the dynamics of macro quantities are surprisingly similar to models with frictionless financial markets. Instead we note that stock prices behave very differently from earlier real models such as CMR and Jaimovich and Rebelo (2006). In our setup they respond positively to news about future productivity. This squares well with basic intuition as well as the VAR evidence in Beaudry and Portier (2004, 2006).

We have also noted that with risk-neutral consumers, news driven booms can be generated even without the type of investment ("flow") adjustment costs introduced by Christiano, Eichenbaum and Evans (2005). But, a clear deficiency of that setup is that consumption falls in response to news shocks, as consumers substitute intertemporally.

The paper is still preliminary. Many important aspects of the problem remain unexplored.

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Appendix

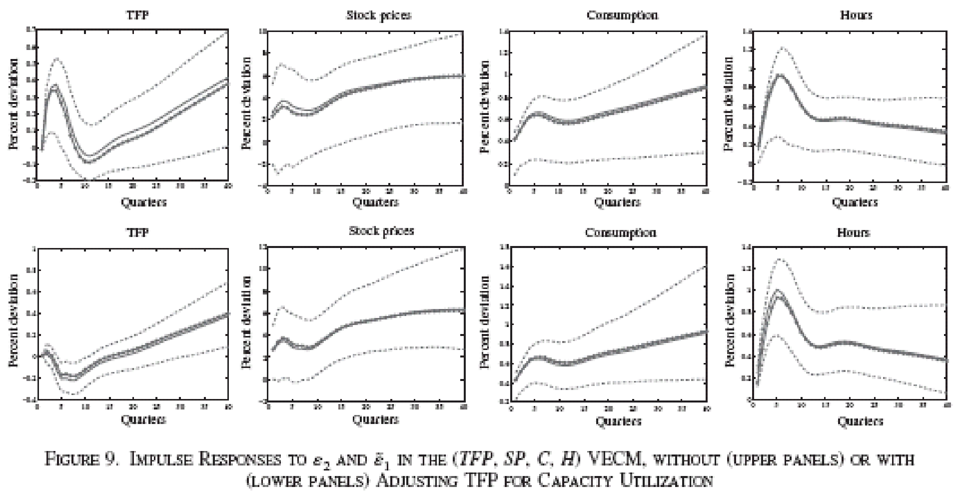

Figure 1. VAR evidence from Beaudry and Portier (2006).

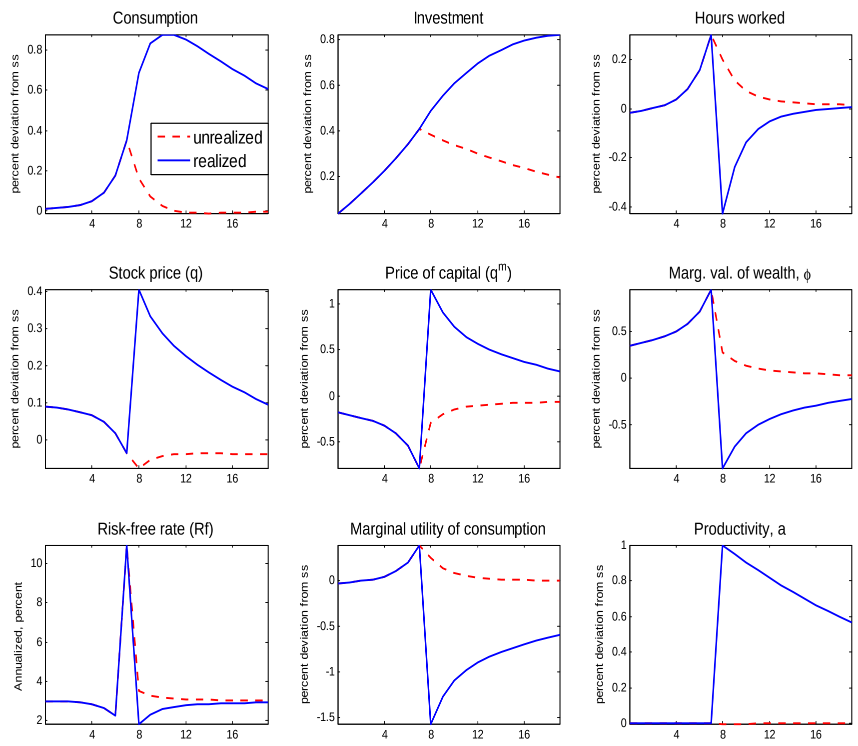

Figure 2. Impulse responses to a η shock to future TFP, $E_t\{a_{t+8}\}$.

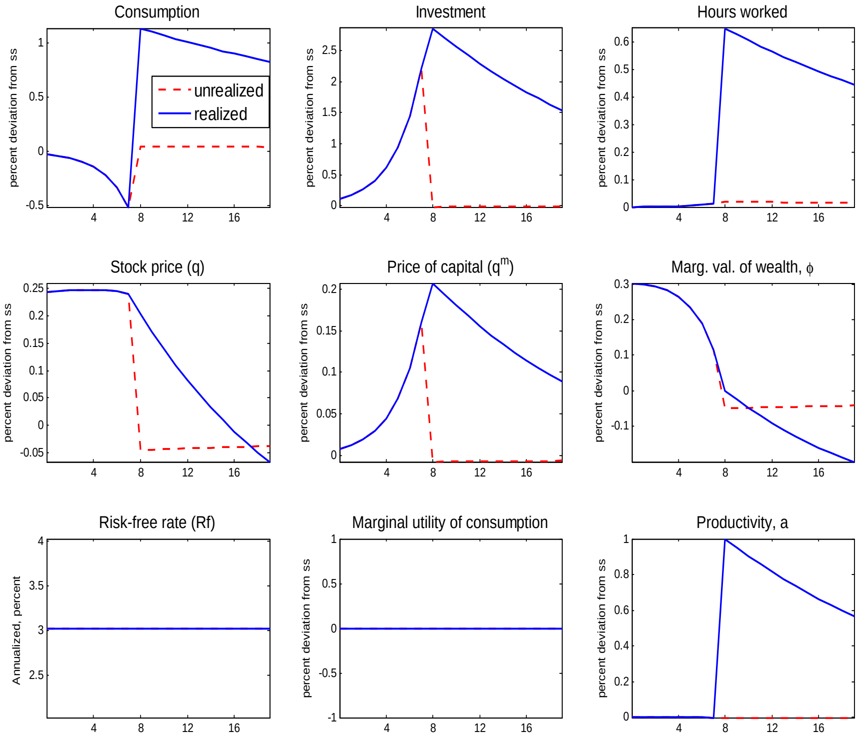

Figure 3. Impulse responses to a η shock to expected future TFP, $E_t\{a_{t+8}\}$. Setup with risk-neutrality and "level" adjustment costs.

Equations determining the equilibrium

L.O.M for capital is determined by

$$K_{t+1} = \frac{(1-\gamma)(1-\theta) R_t K_t + \gamma w_t l_E}{q_t^m - \theta E_t [m_{t+1} R_{t+1}]}$$

$$q_t^m = \frac{1 - \beta_E E_t \left[\frac{\phi_{t+1}}{\phi_t} q_{t+1}^m\right] \left[S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)}$$

$$R_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} + q_t^m (1-\delta)$$

$$w_t = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha}$$

Financial variables

$$\phi_t = \frac{\beta_E (1-\theta) E_t \left[\left(\gamma + (1-\gamma)\phi_{t+1}\right) R_{t+1}\right]}{q_t^m - \theta E_t [m_{t+1} R_{t+1}]}$$

$$q_t = \beta_E (1-\theta) E_t \left[\left\{\gamma + (1-\gamma)\phi_{t+1}\right\} R_{t+1}\right] + \theta E_t [m_{t+1} R_{t+1}]$$

$$Wedge_t = q_t - q_t^m$$

Labor market clearing

$$\frac{\varphi_L L_t^{\sigma_L}}{u'(c_t)} = w_t$$

Household's marginal utility and the state contingent market discount factor $m(X', X)$

$$\begin{aligned} u'(c_t) &= (c_t - bc_{t-1})^{-\sigma_c} - b\beta E_t (c_{t+1} - bc_t)^{-\sigma_c} \\ m_{t+1} &= \beta \frac{u'(c_{t+1})}{u'(c_t)} \end{aligned}$$

Risk-free interest rate

$$r_t^f = \frac{1}{E_t m_{t+1}} - 1$$

Output

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Entrepreneurial consumption

$$C_t^E = \gamma N_t = \gamma (1-\theta) R_t K_t$$

Investment adjustment costs

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \\ \text{where } S(x) &= \frac{g}{2}(x-1)^2 \end{aligned}$$

Goods market clearing

$$Y_t = C_t + I_t + C_t^E$$

Technology

$$a_t = \rho a_{t-1} + \varepsilon_t + \eta_{t-p}$$

Definition of Recursive Competitive Equilibrium

A recursive competitive equilibrium, with linear policies for the entrepreneurs, is given by:

(i) a transition probability $H(X'|X)$, where $X = \{K, lag(K), B, lag(C), s\}$;

(ii) pricing functions $R(X), m(X', X), q^m(X), w(X)$;

(iii) policy functions for the entrepreneur $c^E(v, b, \chi, X), k'(v, b, \chi, X), d(v, b, \chi, X)$ and $b'(\chi', X'; v, b, \chi, X)$, that are linear in $v - b$; and[10]

(iv) policy functions for the consumer $c(X)$ and $l(X)$

which satisfy the following conditions:

(a) the policies in (iii) are optimal for problem (P) in section 2.3, given the transition H;

(b) the policies in (iv) are optimal for the consumer's problem outlined in section 2.2.1, given the transition H;

(c) the functions $R(X), m(X', X), q^m(X)$ and $w(X)$ satisfy the following equations (these conditions embed market clearing in the used capital market and in the labor market):

$$\begin{aligned}
R(X) &= A(s) F_1(K, L) + q^m(X)(1-\delta), \\
m(X'|X) &= \beta \frac{\lambda_c(X')}{\lambda_c(X)} \\
\text{where } \lambda_c(X) &= \left(\frac{1}{C - b * lag(C)} - b\beta E_t \left\{ \frac{1}{C' - bC} \right\} \right) \\
q^m(X) &= \frac{1 - \beta_E E_t \left[\frac{\phi(X')}{\phi(X)} q^m(X') \right] \left[S'\left(\frac{I'}{I}\right) \left(\frac{I'}{I}\right)^2 \right]}{1 - S\left(\frac{I}{lag(I)}\right) - S'\left(\frac{I}{lag(I)}\right)\left(\frac{I}{lag(I)}\right)} \\
\text{where } \phi(X) &= \max \left\{ \frac{\beta_E (1-\theta) \mathbb{E}\left[(\gamma + (1-\gamma)\phi(X')) R(X')|X\right]}{q^m(X) - \theta \mathbb{E}\left[m(X', X) R(X')|X\right]}, 1 \right\} \\
V &= R(X) K, \\
w(X) &= A(s) F_2(K, L);
\end{aligned}$$

(d) the following inequality is satisfied (this condition ensures market clearing in the consumption goods market, with $c_t > 0$)

$$\begin{aligned}
& A(s) F(K, L) - S\left(\frac{I}{lag(I)}\right) + \\
- \quad & \gamma c^E(R(X) K, B, 0, X) - (1-\gamma) c^E(R(X) K, B, 1, X) + \\
- \quad & \gamma d(R(X) K, B, 0, X) - (1-\gamma) d(R(X) K, B, 0, X) > 0
\end{aligned}$$

(e) the transition for s' is consistent with $\pi(s'|s)$; the transition probabilities for K' and B' are consistent with the following:

$$\begin{aligned}
K' &= k'(R(X) K, B, 1, X) \text{ with probability } 1, \\
B' &= (1-\gamma) b'(1, \{K', B', s'\}; V, B, 1, X) - \gamma w(X) l_E \text{ with probability } \pi(s'|s).
\end{aligned}$$

[10]The first two arguments of the b' function reflect the state contingent nature of the optimal contract chosen in state (v, b, χ, X).

The restriction to policy functions that are linear in $v - b$ is justified, given Proposition (1).

Proof of Proposition 3

Part I. Deterministic steady state

Consider the case of a deterministic steady state. Let productivity be constant $A_t = \bar{A}$, and let upper "bars" denote steady state values. We have $\bar{q}^m = 1$ The steady state capital stock $\bar{K}$ and gross return $\bar{R}$ can be found as the solution of:

$$\begin{aligned}\left(1-\beta\theta\bar{R}\right)\bar{K} &= (1-\gamma)(1-\theta)\bar{R}\bar{K}+\gamma\hat{w}L_E \\ \bar{R} &= \bar{A}F_K\left(\bar{K},\bar{L}\right)+1-\delta\end{aligned}$$

It is straightforward to show that $\bar{K}$ is an increasing function of θ, that as $\theta \to 0$ also $\bar{K} \to 0$ and that there exists a $\theta^* < 1$ such that $\beta_E\bar{R} = 1$. The marginal utility of entrepreneurial wealth, $\bar{\phi}$, satisfies

$$\frac{\bar{\phi}}{\gamma+(1-\gamma)\bar{\phi}} = \frac{(1-\theta)\beta_E\bar{R}}{1-\theta\beta\bar{R}}.$$

If the following condition is satisfied

$$\beta_E\bar{R} > 1,$$

then $\phi > 1$ and both (**??**) and (**??**) are satisfied. Given the discussion above this condition is satisfied as long as

$$\theta < \theta^*. \tag{A1}$$

Part II. Stochastic steady state

[TBW]

$$\tag{A2}$$