# The Role of Debt and Equity Finance over the Business Cycle 

Francisco Covas ${ }^{1}$ Wouter J. Denhaan ${ }^{2}$<br>${ }^{1}$ Bank of Canada<br>${ }^{2}$ University of Amsterdam

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## Macroeconomics \& Agency Problems in Firm Finance

- Agency problems vary over the business cycle: Can financial frictions amplify shocks?
- Agency problems vary by firm size: Can financial frictions explain the stronger cyclical response of small firms?


## Deficiencies of Existing Models

- Linear technology in sector with frictions
- Frictions only present in investment producing sector
- Debt only source of outside finance
- No defaults, or procyclical default rate
- Models not successful in magnifying external shocks


## Empirical Objective of the Paper

- Document the cyclical behavior of firm financing sources
- Debt finance
- External equity \& retained earnings
- Analyze dependence on firm size
- US Data: all publicly listed firms
- Canadian Data: publicly listed and closely held firms


## Empirical Findings of the Paper

(1) Debt issuance is procyclical for all but the largest firms
(2) Equity issuance is procyclical for all but the largest firms
(3) Aggregate data gives an incomplete if not misleading picture

## Empirical Literature

- Choe, Masulis and Nanda (1993)
- Gross equity issuance is procyclical
- Gross debt issuance is countercyclical
- Korajczyk and Levy (2003)
- Probability of issuing equity increases in booms
- Jermann and Quadrini (2006)
- Equity payouts are procyclical
- Debt repurchases are countercyclical


## Data: Overview

- Compustat: annual data 1971-2004
- U.S. companies listed in NYSE, AMEX and Nasdaq
- Exclude financial and utilities
- Look at 10 size classes (bottom quartile, bottom tercile, top tercile, etc) using the book value of total assets


## Averages in Compustat: 1971-2004

## Bottom tercile Top tercile ${ }^{\dagger}$

| Asset growth | $23.6 \%$ | $7.9 \%$ |
| :--- | :---: | :---: |
| Sale of stock | $13.6 \%$ | $1.1 \%$ |
| Issuance of LT debt | $8.4 \%$ | $6.2 \%$ |
| Retained earnings | $0.0 \%$ | $1.9 \%$ |
| Sale of stock $>0$ | $58.4 \%$ | $59.0 \%$ |
| ${ }^{\dagger}$ By the book value of assets |  |  |

## Correlations of Sale of Stock and GDP

## Size classes

| Smaller Firms | $[0,25 \%]$ | 0.24 |
| :---: | :---: | :---: |
|  | $[0,50 \%]$ | $0.33^{* *}$ |
|  | $[0,75 \%]$ | $0.35^{* *}$ |
|  | $[0,99 \%]$ | $0.36^{* *}$ |
| Larger Firms | $[90 \%, 95 \%]$ | $0.45^{* * *}$ |
|  | $[99 \%, 99 \%]$ | 0.12 |
|  | $[99 \%, 100 \%]$ | $-0.43^{* * *}$ |

## Correlation

0.24
0.33**
$0.35^{* *}$
0.36**
[90\%, 95\%]
$0.45^{* * *}$
[95\%, 99\%]
0.12
$-0.43^{* * *}$

Figure 1: Cyclical behavior of sale of stock for different size classes


## Correlations of Issuance of LT Debt and GDP

## Size classes

## Correlation

| Smaller Firms | $[0,25 \%]$ | $0.30^{* * *}$ |
| :---: | :---: | :--- |
|  | $[0,50 \%]$ | $0.30^{* * *}$ |
|  | $[0,75 \%]$ | $0.35^{* * *}$ |
|  | $[0,95 \%]$ | $0.33^{* * *}$ |
|  | $[0,99 \%]$ | $0.31^{* * *}$ |
| Larger Firms | $[90 \%, 95 \%]$ | $0.36^{* * *}$ |
|  | $[95 \%, 99 \%]$ | 0.19 |
|  | $[99 \%, 100 \%]$ | -0.13 |

All Firms $\quad[0,100 \%] \quad 0.23^{* *}$

Figure 2: Cyclical behavior of issuance of LT debt for different size classes


Figure 3: Cyclical behavior of change in liabilities for different size classes


## Correlations of Sale of Stock and Issuance of LT Debt

## Size classes

| Smaller Firms | $[0,25 \%]$ | $0.39^{* * *}$ |
| :---: | :---: | :---: |
|  | $[0,50 \%]$ | $0.39^{* * *}$ |
|  | $[0,75 \%]$ | $0.40^{* * *}$ |
|  | $[0,95 \%]$ | $0.40^{* * *}$ |
|  | $[0,99 \%]$ | 0.25 |
| Larger Firms | $[90 \%, 95 \%]$ | $0.40^{* * *}$ |
|  | $[95 \%, 99 \%]$ | 0.001 |
|  | $[99 \%, 100 \%]$ | $0.26^{* *}$ |

All Firms
[0, 100\%]
0.14

## Correlation

0.39***
$0.39^{* * *}$
$0.40^{* * *}$
$0.40^{* * *}$
0.25
[95\%, 99\%] 0.001
[99\%, 100\%] 0.26**

## One-period debt contract

$\theta$ : aggregate shock - known beginning of period
$\omega$ : idiosyncratic shock - known end of period

- One-period debt contract:
- No default if
$\left(1+r^{b}\right)(k-n)<\theta \omega k^{\alpha}$ or
- Default if
$\theta \omega k^{\alpha}<\left(1+r^{b}\right)(k-n)$
- Bankruptcy cost: $\mu \theta k^{\alpha}$


## Analytical Results I

$$
\begin{aligned}
& \alpha=1 \Rightarrow \frac{\partial \bar{\omega}}{\partial n}=0 \\
& \alpha<1 \Rightarrow \frac{\partial \bar{\omega}}{\partial n}<0
\end{aligned}
$$

Thus,

- with linear technology ( $\alpha=1$ ) size does not matter
- with nonlinear technology ( $\alpha<1$ ) size does matter


## Analytical Results II

$$
\frac{\partial \bar{\omega}}{\partial \theta}>0
$$

## In words:

- everything else equal: expected default $\uparrow$ if $\theta \uparrow$


## Intuition:

- The tradeoff between $k$ and defaults shifts when $z$ varies
- Expansion, i.e., $k \uparrow$, more attractive when $\theta \uparrow$


## Would owners like to add funds?

- Of course because of bankruptcy costs
$\Rightarrow$ internal rate of return > discount rate $\rho$
$\Rightarrow$ owner will empty his savings account, which earns $\rho$, and put it in the firm
- We will allow for additional external finance but will limit it by assuming there is an equity issuance cost


## What does this do to undesirable properties of the debt contract?

## Debt and Equity

- Both debt and equity provider have an expected return equal to $\rho$
- Debt has "bankruptcy costs" friction
- Equity has "issuance costs" friction
- Debt provider can also provide equity but buying equity doesn't alleviate the bankruptcy costs of the debt contract


## Equity Issuance Costs

- Cost of issuing $e$ is equal to $\lambda_{0} e^{2}$ (for $e>0$ )


## Adding Equity

## Equity decision

$v(x ; \theta)=\max _{e, n}\{w(n ; \theta)-e-\lambda(e)\}$
s.t. $n=x+e$

Debt Contract

$$
\begin{aligned}
& w(n ; \theta)=\max _{k, \bar{\omega}} \theta k^{\alpha} F(\bar{\omega}) \\
& \text { s.t. } \theta k^{\alpha} G(\bar{\omega})=k-n
\end{aligned}
$$

## First-order condition

$$
\frac{\partial w(x+e ; \theta)}{\partial e}
$$

## Suppose $\Delta \theta>0$. What happens with $e$ ?



## Weakness becomes Strength

$$
\frac{\partial w(n+e ; \theta)}{\partial e}=\zeta
$$

$\zeta$ :Lagrange Multiplier of bank break - even constraint

$$
\zeta=\frac{-F(\overline{\bar{\omega}})}{G(\bar{\omega})} \text { with } \frac{\partial \zeta}{\partial \bar{\omega}}>0
$$

## Weakness becomes Strength



## Why is equity $\uparrow$ useful

- Consistent with observed procyclical behavior of equity
- If $\alpha<1 \Rightarrow$ increase in default rate is dampened (but not overturned)


## Key Features of Dynamic Model

- Incorporate tax advantage of debt $\Rightarrow$ firms eventually issue dividends and agency problems continue to matter
- Firms that default are replaced by firms with zero capital


## Three reasons for procyclical equity

- Shadow price of external funds is procylical with one-period debt contract
- Countercyclical equity issuance costs
- Countercyclical price or risk

Figure 4: Illustration of Equity/Dividend Policy


## Calibration: matched parameters

## Parameter

Moment
Data Model

| $\sigma_{\epsilon}$ | 0.0074 | Volatility of asset growth | 0.039 | 0.037 |
| :---: | :---: | :--- | :---: | :---: |
| $\sigma_{\omega}$ | 0.31 | Default premium | 119 bp | 105 bp |
| $\delta_{0}$ | 0.082 | Investment to assets | 0.133 | 0.134 |
| $\delta_{1}$ | -2.72 | Leverage | 0.587 | 0.532 |
| $\eta$ | 0.0975 | Fraction of dividend payers | 0.469 | 0.429 |
| $\mu$ | 0.15 | Default rate | 0.022 | 0.020 |
| $\lambda_{0}$ | 0.30 | Change in equity to assets | 0.015 | 0.011 |
| $\lambda_{1}$ | 125 | Volatility of change in equity | 0.254 | 0.221 |
| $\gamma$ | 0.138 | Volatility of retained earnings | 0.342 | 0.397 |

Figure 8: Responses of Output and Default Rate to a Positive Shock


Figure 9: Responses of Debt, Equity and RE to a Positive Shock


Figure 10: Equity Issues for Different Size Classes

Equity issues by firm size


| $\square$ | Bottom 25\% | $\square$ |
| :--- | :--- | :--- |
| $\square$ | Bottom $99 \%$ | Bottom 50\% |
|  |  |  |

Figure 11: Debt Issues for Different Size Classes


| $\square$ | Bottom 25\% | $\square$ |
| :--- | :--- | :--- |
| $\square$ | Bottom $99 \%$ | Bottom 50\% |
|  |  |  |

Figure 1: Cyclical behavior of sale of stock for the Top 1\%


Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details see the text and the data appendix.

Figure 2: Cyclical behavior of change in equity for the Top 1\%


Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details see the text and the data appendix.

Figure 3: Cyclical behavior of issues of LT debt for the Top 1\%


Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details see the text and the data appendix.

Figure 4: Cyclical behavior of change in liabilities for the Top 1\%


Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details see the text and the data appendix.

Figure 5: Sale of stock for the Top $1 \%$ in 1982


Figure 6: Default Rate on Corporate Bonds


Notes: The default rate series is from Moody's (mnemonic USMDDAIW in Datastream) and it is for all corporate bonds in the US. The plot shows the annual default rate, i.e., the number of defaults during a year divided by the number of outstanding issuers at the beginning of the year, adjusted by the number of rating withdrawals during the year.

## Summary

- Debt and equity issuance are procyclical for all but the largest firms.
- Matching the volatility of equity issues and retained earnings helps the model to amplify aggregate shocks.
- Model generates stronger cyclical response for small firms.
- Ongoing work:
- General equilibrium.
- Firm size and leverage.

