Growth-at-risk and macroprudential policy design

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Introduction

• Paper motivated by empirical popularity of Growth-at-Risk (GaR)

[Cecchetti (2008); Cecchetti-Li (2008); Adrian-Boyarchenko-Giannone (2019); Brandao-Marques et al. (2020), Duprey-Ueberfeldt (2020), Franta-Gambacorta (2020), Galán (2020), Aikman et al. (2021),...]

- GaR is a measure of tail downside risk for GDP growth
 - Explicit and intuitive statistical interpretation (akin to VaR)
 - Measures macropru goal in same units as GDP growth
 - Can be naturally confronted to expected growth when policy involves a trade-off with the central outlook (or with other costs translated into percentage points of GDP)
- Considers conceptual foundations for using empirical GaR approach in the assessment and design of macroprudential policies

Overview

- The benchmark formulation combines typical GaR equations (with risk determinants & policy variables as regressors) with a welfare criterion, thus providing a basis for macroprudential policy design
- In benchmark case, policy variables respond linearly to risk variables
- Optimal policy targets a constant gap between mean growth and GaR
- Extensions to richer empirically relevant formulations, confirm robustness of the insights & compatibility with other approaches

This presentation will cover:

- 1. Benchmark formulation in detail
- 2. Selected navigation over extensions & discussions

Motivation: Growth and banking crises

Figure B.1

Distribution of normalised average three-year growth

(percentages)



Sources: See Table B.1.

Benchmark formulation of the GaR approach

Let

- y : *GDP growth* over some relevant horizon (a random variable)
- \bar{y} : expected GDP growth over such horizon
- y_c : GaR at a confidence level c (=c-quantile of y with c=5% or 10%)
- Conditional forecasts for \bar{y} and y_c are given by

$$\bar{y} = \alpha + \beta x + \gamma z, \tag{1}$$

$$y_c = \alpha_c + \beta_c x + \gamma_c z, \qquad (2)$$

where

- x : unidimensional measure (determinant) of *risk*
- z: unidimensional macroprudential *policy*

[Assuming endogeneity of z is treated so that estimates of γ_c & γ measure causal impact]

• Assume

$$\beta_c < \min\{0,\beta\} \text{ and } \gamma < 0 < \gamma_c \tag{3}$$

In words:

- -x has negative impact on y_c & less so on \bar{y}
- -z has positive impact on y_c & negative on $\bar{y} \Rightarrow$ trade-off

[And ranges of variation of y & z so that $y_c < \overline{y}$ over relevant range]

• And policy maker with preferences over growth outcomes given by

$$W = \bar{y} - \frac{1}{2}w(\bar{y} - y_c)^2, \qquad (4)$$

where w > 0 measures aversion for financial instability

- Consistent with E(U) maximization under CARA preferences for GDP & normal $y \Rightarrow w$ is directly proportional to RRA at Y_0)
- Alternatively, w may reflect loss-aversion, with \bar{y} as reference point

Optimal policy rule

$$z(x) = \arg\max_{z} W(x, z) \tag{5}$$

If interior, relevant FOC is

$$\frac{\partial \bar{y}}{\partial z} - w(\bar{y} - y_c) \left(\frac{\partial \bar{y}}{\partial z} - \frac{\partial y_c}{\partial z}\right) = 0$$
(6)

$$\Rightarrow z(x) = \phi_0 + \phi_1 x = \left(\frac{\alpha - \alpha_c}{\gamma_c - \gamma} + \frac{\gamma}{w(\gamma_c - \gamma)^2}\right) + \left(\frac{\beta - \beta_c}{\gamma_c - \gamma}\right) x \quad (7)$$

- Intercept $\phi_0 \leq 0$ increases with w & policy effectiveness ($\gamma_c - \gamma$)

- Risk-responsiveness $\phi_1>0$ is
 - \ast independent of \boldsymbol{w}
 - * increasing in *impact of risk* on the gap $(\beta \beta_c)$
 - \ast decreasing in policy effectiveness

Optimal target gap property

Optimal policy features a target gap $\bar{y} - y_c$ independent of x

• FOC in (6) is equivalent to

$$\bar{y} - y_c = -\frac{1}{w} \cdot \frac{\frac{\partial \bar{y}}{\partial z}}{\frac{\partial \bar{y}}{\partial z} - \frac{\partial y_c}{\partial z}} = \frac{1}{w} \cdot \frac{(-\gamma)}{\gamma_c - \gamma} = \frac{1}{w} \cdot \frac{1}{1 + \gamma_c/(-\gamma)}$$
(8)

- $-\operatorname{Decreasing}$ in preference parameter w
- Increasing in marginal growth-gap rate of transformation $\frac{-\gamma}{\gamma_c \gamma}$ [\equiv decreasing in the policy's cost-effectiveness ratio $\frac{\gamma_c}{-\gamma}$]
- Corollary:
 - Macroprudential policy **should not** target a constant GaR
 - $-\operatorname{Gap} \bar{y} y_c$ is a **more useful indicator** of stance

Graphical illustration



[risk x moves the policy frontier in parallel; expansion path is linear, with slope=1]

Explanations:

- Conditional policy frontier (for given x): pairs (y_c, \bar{y}) reached varying z
- Point $(y_c(x,0), \bar{y}(x,0))$ corresponds to z = 0. Setting z > 0 allows to reach higher y_c at cost of lower \bar{y}
- Indifference curves are convex parabolas with minima on ray $y_c = \bar{y}$
- Required compensating increases in \bar{y} for declines in y_c increase with the distance from the ray $\bar{y} = y_c$
- Term $w(\bar{y} y_c)$ in FOC (6) accounts for increasing MgC of financial instability
- Risk x moves the policy frontier in parallel (up if $\beta > \gamma \beta_c / \gamma_c$, down otherwise) but expansion path is linear, with slope=1

A framework for policy assessment & design

- 1. Estimated (1)&(2) provide positive description of risk & trade-offs
- 2. Mean & conditional policy frontiers can be depicted
- 3. Optimal policy responsiveness is $\phi_1 = \frac{\beta \beta_c}{\gamma_c \gamma}$ independently of w
- 4. If optimal rule followed, \boldsymbol{w} can be inferred from

$$w = \frac{1}{(\bar{y} - y_c)} \frac{1}{1 + \gamma_c / (-\gamma)}$$
(9)

- 5. Conditional on a value of w, policy stance could be deemed...
 - (a) *inefficient* if far away from the policy frontier (not applicable with unidimensional policy variable)
 - (b) suboptimal if far away from the expansion path (excessive distance between z & z(x) or, equivalently, $\bar{y} - y_c \&$ target gap) \Rightarrow too tight / too loose

Generalizations and further discussion (outline)

Roadmap to sections 4 and 5 in the latest draft:

- 4 Generalizing the benchmark formulation
- 4.1 Policy variable with non-linear and/or state-contingent effects \longleftarrow
- 4.2 Interactions between multiple policy tools \leftarrow
- 4.3 Policies measured with discrete variables
- 5 Further discussion
- 5.1 What if the policy variable seems to involve no trade-off?
- 5.2 Intermediate objectives and targeted policy tools
- 5.3 GaR vs. directly focusing on systemic financial crises
- 5.4 Accounting for the term structure of macroprudential policies

Policy variable with non-linear and/or state-contingent effects Consider a generalized version of (1) and (2):

$$\bar{y} = Y(x, z) \tag{10}$$
$$y_c = Y^c(x, z) \tag{11}$$

with
$$Y_x^c < \min\{0, Y_x\}, \ Y_z < 0 < Y_z^c, \ \text{and} \ Y_{zz}^c < Y_{zz} < 0$$

 $\Downarrow \text{FOC}$
 $Y(x, z) - Y^c(x, z) = \frac{1}{w} \cdot \frac{1}{1 + Y_z^c(x, z)/(-Y_z(x, z))}$ (12)

Then sign of z'(x) is the same as that of

$$S = Y_{zx} - w(Y_x - Y_x^c)(Y_z - Y_z^c) - w(Y - Y^c)(Y_{zx} - Y_{zx}^c)$$
(13)

[depends, in general, on Y_{zx} and Y_{zx}^c]

Particular cases:

1. When risk per se does not affect policy effectiveness $(Y_{zx}=Y_{zx}^c=0)$

However, having $Y_{zz}^c < Y_{zz} < 0$ means marginal cost-effectiveness declines with $z \Rightarrow$ target gap $Y - Y^c$ increases with x

2. When risk per se affects policy effectiveness Example:

$$y_c = \alpha_c + \beta_c x + \gamma_c z + \delta_c x z, \text{ with } \delta_c < 0$$

$$\Downarrow \text{ FOC}$$
(15)

$$\gamma - w(\alpha + \beta x + \gamma z - \alpha_c - \beta_c x - \gamma_c z - \delta_c x z)(\gamma - \gamma_c - \delta_c x) = 0$$
(16)

Implications:

(a) Target gap is decreasing in x:

$$\bar{y} - y_c = \frac{1}{w} \cdot \frac{1}{1 + (\gamma_c + \delta_c x)/(-\gamma)}$$
(17)

(marginal cost-effectiveness of the policy declines with x)

(b) Modified policy rule is no longer linear (or even monotonically increasing) in x:

$$z(x) = \left(\frac{\alpha - \alpha_c}{\gamma_c + \delta_c x - \gamma} + \frac{\gamma}{w(\gamma_c + \delta_c x - \gamma)^2}\right) + \frac{\beta - \beta_c}{\gamma_c + \delta_c x - \gamma} \cdot x \quad (18)$$



Optimal policy when risk diminishes policy effectiveness

This figure represents optimal policy z(x) as a function of risk variable x in a specification based on equations (1) and (15). Parameter values: $\alpha = -\alpha_c = 0.2$, $\beta = 0.1$, $\beta_c = -0.5$, $\gamma = -0.2$, $\gamma_c = 2$, $\delta_c = -5$, and w = 1.4784.

Interactions between multiple policy tools

Assume z_j with j = 1, 2, ... M

- 1. Linearity implies the existence of a **dominant policy tool** With coefficients satisfying $\gamma_i < 0 < \gamma_{cj}$:
 - Preferred policy tool j^* is that with highest marginal cost-effectivenes

$$\frac{\frac{\partial \bar{y}}{\partial z_j}}{\frac{\partial \bar{y}}{\partial z_j} - \frac{\partial y_c}{\partial z_j}} = \frac{1}{1 + \gamma_{cj}/(-\gamma_j)} > 0$$
(19)

which determines the size of the optimal target gap

• Other policy variables should remain at their lowest bound

2. Non-linearities can give rise to **optimal policy mixes**

$$y_c = \alpha_c + \beta_c x + \sum_{j=1}^M \Gamma_{cj}(z_j) \quad \text{or} \quad y_c = \alpha_c + \beta_c x + \Gamma_c(z) \quad (20)$$

E.g., with marginally decreasing effectiveness, say, $\Gamma_{cj}(z_j)$ with $\Gamma'_{cj} > 0$ & $\Gamma''_{cj} < 0$, or complementarities in $\Gamma_c(z_1, z_2, ... z_M)$

 \Rightarrow FOCs imply

$$\bar{y} - y_c = -\frac{1}{w} \frac{\frac{\partial \bar{y}}{\partial z_j}}{\frac{\partial \bar{y}}{\partial z_j} - \frac{\partial y_c}{\partial z_j}} = \frac{1}{w} \frac{1}{1 + \frac{\partial \Gamma_c}{\partial z_j}/(-\gamma)}$$
(21)

for all actively used tools, which means

- Equalization of marginal cost-effectiveness ratios
- Optimal gap would be directly proportional to such common rate

- 3. **Interactions with other policies** (e.g. monetary policy) can be analyzed within this setup...
 - w. or w/o policy coordination
 - perhaps after expanding the welfare criterion to reflect goals of other policies

Concluding remarks

- This paper explores the foundations for the design & assessment of macroprudential policies using an empirical GaR approach
- Stylized representation of equations of a typical quantile regression approach combined with a microfounded welfare criterion
- The analysis delivers:
 - Properties of the optimal policies
 - An intuitive graphical representation
 - Several generalizations & discussions
 - Several concepts for the assessment of macroprudential stance (policy frontier, target gap, marginal cost-effectiveness ratio of policy tools, optimal vs. suboptimal policies,...)
- Challenges ahead: empirical & political implementation

Thank you very much!

COMPLEMENTARY MATERIALS

Microfoundation of GaR-based welfare criterion

Let:

- $Y : \mathsf{GDP}$ over a relevant horizon
- y: (geometric) GDP growth relative to starting level Y_0

$$\Rightarrow Y = (1+y)Y_0 \tag{22}$$

1. Suppose representative risk-averse agent has CARA preferences with coefficient of *absolute* risk aversion $\lambda(Y_0)$:

$$U(Y) = -\exp(-\lambda(Y_0)Y)$$
(23)

For fixed Y_0 , her preferences can be equivalently described by

$$u(y) = -\exp(-\lambda(Y_0)Y_0y) = -\exp(-\rho_0 y),$$
(24)

where $\rho_0 = \lambda(Y_0) Y_0$ (that is, the *relative* risk aversion at Y_0)

2. Suppose $y \sim N(\bar{y}; \sigma_y^2)$. Properties of normal distribution imply $E[u(y)] = -E[\exp(-\rho_0 y)] = -\exp(-\rho_0 \bar{y} + \frac{1}{2}\rho_0^2 \sigma_y^2)$ (25)

 \Rightarrow indirect utility can be equivalently described by

$$v = \bar{y} - \frac{\rho_0}{2}\sigma_y^2 \tag{26}$$

3. From the definition of GaR and the properties of the normal,

$$\Pr(y \le y_c) = c \Leftrightarrow \Phi((y_c - \bar{y})/\sigma_y) = c, \qquad (27)$$

where $\Phi(\cdot)$ is cdf of a N(0,1). Then

$$\sigma_y = \frac{y_c - \bar{y}}{\Phi^{-1}(c)} \Rightarrow v(\bar{y}, y_c; \rho_0, c) = \bar{y} - \frac{\rho_0}{2(\Phi^{-1}(c))^2} (\bar{y} - y_c)^2$$
(28)

$$\Rightarrow \text{ setting } w = \frac{\rho_0}{(\Phi^{-1}(c))^2} \text{ in (4) makes } W = v(\bar{y}, y_c; \rho_0, c)$$
(29)

Policies measured with discrete variables

Assume $z \in \{z_1, z_2, \dots z_N\}$ with $N \ge 2$

Subsequent levels of activation optimal or not depending on sign of

- Particular cases: (i) equally spaced values of z_i , (ii) binary z = 0, 1
- Insights:
 - Discreteness does not alter the indifference curves and the location of the "hypothetical" policy frontier
 - Heuristically, optimal policy is the one bringing the gap as close as possible to continuous-case target

What if the policy variable seems to involve no trade-off?

That is, $\gamma = 0$ (or even $\gamma > 0$).

- Implication that z should be increased up to making the gap=0 or to its upper bound may not be plausible / economically meaningful
- Most likely cause: estimated equation for \bar{y} does not capture relevant non-linearity (perhaps due to lack of historical experience)
- Practical solutions:
 - capture the non-linearity
 - use auxiliary calculation to impute certainty-equivalent cost to use of z in \bar{y}

Intermediate objectives and targeted policy tools

Assume M intermediate objectives I_j , each with targeted tool z_j :

$$I_j = \lambda_{0j} + \lambda_{1j} z_j \tag{30}$$

Reformulate baseline equations as

$$y_c = \alpha_c + \Gamma_c(I_1, I_2, \dots I_M) \tag{31}$$

(increasing & concave)

$$\bar{y} = \alpha + \sum_{\substack{j=1\\(-)}}^{M} \gamma_j z_j \tag{32}$$

(33)

 $\Rightarrow \text{For activated tools: } \bar{y} - y_c = \frac{1}{w} \cdot \frac{1}{1 + \frac{\partial \Gamma_c}{\partial I_j} \lambda_j / (-\gamma_j)}$

[equalization of Mg cost-effectiveness ratios]

- \bullet Optimal gap would be decreasing in w & such common ratio
- Changes in λ_{0j} may alter the optimal target & policy mix [If additively separable $\Gamma_c, \downarrow \lambda_{0j} \Rightarrow \uparrow z_{j'}$ across all $j' \& \uparrow gap$]

GaR vs. directly focusing on systemic financial crisis

Suppose:

$$y = \begin{cases} y_0 - g_H(z), & \text{w. pr. } 1 - \varepsilon(z), \\ y_0 - \Delta + g_L(z), & \text{w. pr. } \varepsilon(z), \end{cases}$$
(34)

where $y_0 \sim f(\cdot), F(\cdot)$ is a baseline stochastic growth rate, $\Delta > g_H(z) + g_L(z)$ is baseline cost of crises, $\varepsilon' < 0$ & $g'_L > 0$ but $g'_H > 0$

Then

1.
$$\bar{y} = E(y_0) - (1 - \varepsilon(z))g_H(z) - \varepsilon(z)(\Delta - g_L(z))$$

 $d\bar{y}/dz < 0 \Leftrightarrow (1 - \varepsilon(z))g'_H(z) - \varepsilon(z)g'_L(z) > -\varepsilon'(z)(\Delta - g_L(z) - g_H(z))$ (35)

2.
$$y_c$$
: $(1 - \varepsilon(z))F(y_c + g_H(z)) + \varepsilon(z)F(y_c + \Delta - g_L(z)) = c$
 $dy_c/dz > 0 \Leftrightarrow -\varepsilon'(z) [F(y_c + \Delta - g_L(z)) - F(y_c + g_H(z))] >$
 $(1 - \varepsilon(z))f(y_c + g_H(z))g'_H(z) - \varepsilon(z)f(y_c + \Delta - g_L(z))g'_L(z)$ (36)

[Satisfying (36) requires RHS is relatively small, that is, (i) $g'_H(z)$ small relative to $g'_L(z)$ (macroprudential policy is sufficiently cost-effective) and/or (ii) Δ large relative to $g_H(z) + g_L(z)$ (financial crises have a severe impact on growth outcomes)]

Accounting for the term structure of macroprudential policies

Consider an infinite horizon dynamic version of the benchmark model:

$$W_{t} = E_{t} \left\{ \sum_{s=1}^{\infty} \Lambda^{s} \left[\bar{y}_{t+s} - \frac{1}{2} w (\bar{y}_{t+s} - y_{c,t+s})^{2} \right] \right\}$$
(37)

with
$$\bar{y}_{t+s} = \alpha + \beta x_{t+s-1} + \sum_{l=1}^{\infty} \gamma_l z_{t+s-l}$$
 (38)

and
$$y_{c,t+s} = \alpha_c + \beta_c x_{t+s-1} + \sum_{l=1}^{\infty} \gamma_{c,l} z_{t-s-l}$$
 (39)

FOC:
$$\sum_{s=1}^{\infty} \Lambda^s \gamma_s - w \sum_{s=1}^{\infty} \Lambda^s (\gamma_s - \gamma_{c,s}) E_t(\bar{y}_{t+s} - y_{c,t+s}) = 0$$
(40)

$$\exists (\bar{y} - y_c)^* = \frac{1}{w} \frac{1}{1 + \left(\sum_{s=1}^{\infty} \Lambda^s \gamma_{c,s}\right) / \left[\sum_{s=1}^{\infty} \Lambda^s (-\gamma_s)\right]}$$
(41)

which can be reached in all periods setting

$$z_{t+s-1} = \frac{E_{t+s-1}(\bar{y}_{t+s} - y_{c,t+s} \mid z_{t+s-1} = 0) - (\bar{y} - y_c)^*}{\gamma_{c,1} - \gamma_1}$$
(42)

[Short-cut in one-shot formulation is a good approximation if (i) most policy effects occur within policy horizon, and (ii) $\Lambda \simeq 1$]

Growth and banking crises (i)

Table B.1

Moments of normalised per capita output growth

	1870	0-2018	1960-2018			
	Full Sample	Crisis Episodes	Full Sample	Crisis Episodes		
Mean	0.00	-0.74	0.00	-0.93		
Skewness	-0.32	-0.94	-0.17	0.20		
Excess Kurtosis	2.71	2.08	-0.01	0.34		
10 th percentile	-1.11	-1.88	-1.24	-2.16		
Number of observations	1872	207	874	97		
Median across countries of average growth	1.	99%	2.45%			
Median across countries of standard deviation of three- year growth	3.	08%	2.27%			

Sources: Maddison Project Database (2020); Baron, Verner and Xiong (2020; and authors' calculations. Notes: Data are deviations from the country mean of non-overlapping three-year average growth rates in standard deviation units. Countries are Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Czech Republic, Denmark, Egypt, Finland, France, Germany, Greece, Hong, Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Peru, the Philippines, Portugal, the Russian

Growth and banking crises (ii)

Figure B.1

Distribution of normalised average three-year growth

(percentages)



Sources: See Table B.1.

GaR-based welfare criteria without normality (i)

Table E.1

Candidate growth distributions

Average crisis growth (%)	Probability of being in a crisis regime									
	5%				10%					
	Mean	St. dev.	Skew	Excess kurtosis	Mean	St. dev.	Skew	Excess kurtosis		
0.0	2.86	2.85	-0.10	0.14	2.71	2.70	-0.15	0.18		
-2.0	2.76	2.76	-0.31	0.53	2.51	2.96	-0.42	0.54		
-4.0	2.66	2.96	-0.61	1.27	2.31	3.30	-0.75	1.08		

Notes: Unconditional moments of empirical distributions computed as the mixture of two normal distributions. The first has a mean of 3.01% and standard deviation of 2.52%; the second has a mean equal to the average crisis growth in the first column and a standard deviation of 2.78%. The probability of drawing from the second distribution is equal to 5% or 10%. All reported numbers are based on 500,000 draws. The shaded values are those that correspond to the benchmark in the data reported in Table B.1.

GaR-based welfare criteria without normality (ii)



The accuracy of the growth-at-risk approximation varying the probability of a crisis regime



GaR-based welfare criteria without normality (iii)



GaR-based welfare criteria without normality (iv)

