Multiplex financial networks: revealing the level of interconnectedness in the banking system*

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Abstract

The network approach has been useful for the study of systemic risk; however, most of the studies have ignored the true level of interconnectedness in the financial system. In this work we show the missing part on the study of interconnectedness of the banking system. Complexity in modern financial systems has been an important subject of study as well as the so called high degree of interconnectedness between financial institutions. However, we still lack the appropriate metrics to describe such complexity and the data available in order to describe it is still scarce. In addition, most of the focus on the subject of interconnectedness has been on a single type of network: interbank (exposures) networks. In order to have a more complete view of the complexity in the Mexican banking system, we use a comprehensive set of market interactions that include transactions in the securities market, repo transactions, payment system flows, interbank loans, cross holding of securities, foreign exchange exposures and derivatives exposures. This the first attempt, to the best of our knowledge, to describe so comprehensively the complexity and interconnectedness in a banking system. By resorting to the multiplex paradigm we are able to identify the most important institutions in the whole structure, the most relevant layer (in structural terms) of the multiplex and the community structure of the Mexican banking system.

*The views expressed here are those of the authors and do not reflect the views of the Mexican central bank.

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1 Introduction

There has been a lot of recent research on financial networks for the purposes of studying systemic risk, performing stress testing or determining the relevance of financial institutions. A commonly shared view is that the financial system is highly interconnected. However, most of the previous works use the interbank unsecured market as the only source to measure interconnectedness in the banking system.

Financial networks have become recently an expanding field as it can be witnessed by the number and the importance of the recently published papers. Moreover, the term interconnectedness its now commonly mentioned by the financial authorities and in research papers. This is the result of the huge impact that the recent financial crises had and continues to have on the financial markets and the real economy.

Nevertheless, we argue that research on interconnectedness has been mostly focused (with some exceptions) in very specific types of networks: interbank direct exposures networks, CDS networks or payment systems networks. Our point and main contribution is to investigate the real face of structural complexity in banking systems.

Financial institutions interact in different markets, which can be thought of as different networks within a meta-structure which can be interpreted as a multilayer network or a multiplex network Kivela et al. [2014]. The selection of either structure depends on the specific characteristics of the system and on the different aspects under study.

This approach gives rise to a rich set of complex interactions among these layers. Each layer possesses different topological properties and the roles played by the institutions might be different depending on the strategy followed by each of them.

In this work we study 7 different layers or types of interactions among banks:

1. Transactions on the securities market (CVT layer)
2. Transactions on the repurchase agreements market (Repo layer)
3. Payment system flows (SPEI layer)
4. Exposures arising from *interbank deposits and loans* (D&L layer)
5. Exposures arising from cross holding of securities (Securities layer)
6. Exposures arising from derivatives transactions (Derivatives layer)
7. Exposures arising from foreign exchange transactions (FX layer)

It is a possibility to simple aggregate all the layers in order to study the multiplex system of interest; nevertheless, insightful information regarding interactions is lost when the multiplex structure could be lost. Given that the last 4 layers can are built under the common concept of exposure at default, we will add up such layers and we will call this new layer as the Total layer. The exposures multiplex system has been studied in Poledna et al. [2015] from the systemic risk point of view, not from the structural side. In all the following computations and results we will use the Total layer instead of all the 4 exposures layers for computational costs saving purposes.

This paper has two main goals: first, we argue that we should overcome such limited view on interconnectedness by using a multiplex approach; second, we propose to use stochastic block models (SBM) and topological data analysis (TDA), to disentangle and understand the complexity in the banking system. Some additional related objectives are:

- Propose new metrics and methods in order to characterize and understand the multiplex network of the Mexican financial system, which can be used in other jurisdictions
- Identify important players in the multiplex structure rather than only on single layers
- Study the level of interconnectedness and the complexity of the financial system

For this purpose we will use two well established approaches to study systems which are composed by more than one network, and additional complexity measures, borrowed from Stochastic Block Models (SBM) and Topological Data Analysis (TDA), which can contribute to reveal the level of interconnectedness which exists in the banking system. We argue that the complexity and interconnectedness in the financial system are multi-faced and that for successfully revealing their real face, one must consider that agents interact in financial systems in many different ways and markets.

We propose to study interconnectedness of the banking system by modelling it as a temporal, weighted multiplex system. We argue that the three aspects (time, links' weights and multiple levels of interaction) are necessary in order to get a more accurate structural view of the banking system.

The main contributions of this work are: i) we propose a comprehensive structural analysis of a multiplex banking system and we document some of the structural aspects of the Mexican multiplex banking system; ii) by resorting to some well known approaches in multiplex systems we are able to identify relevant players in the whole system, relevant layers (market interactions) and the community structure of such system. Additionally, we perform the analysis for several different dates in order to explore the dynamical aspects of the system.

The rest of the paper is organized as follows: Section 2 explores some of the related works in the field of multiplex or multilayer financial networks. Section 3 provides some important definitions for multiplex systems, stochastic block models and topological homology. Section 4 discusses in detail the data used to build the multiplexes for this study. Section 5 describes in detail the methodological aspects of the paper. Section 6 presents the results obtained by applying the multiplex approach in the Mexican banking system and in particular the use of SBM and TDA. Finally, Section 7 concludes and shows possible lines of extension to this work.

2 Related literature

A multiplex or a multilayer system is one in which the nodes have different types of interactions, each level of interaction is modeled by a single layer of the multiplex. Depending on the set of nodes and its heterogeneity, one could refer to multiplex or multilayer systems. In this work, we adopt the convention that multiplex systems are composed by the same set of homogeneous nodes and a multilayer system is such that the set of nodes could be different on each layer and there is heterogeneity on the nodes attributes.

Multiplex and multilayer systems have been comprehensively studied in the past from the theoretical point of view and applied in many different contexts Kivela et al. [2014]. Examples of applications of the multiplex paradigm range from transportation networks, infrastructure networks, social networks and more recently financial networks.

Given that at the current stage, we are interested in modeling only the banking system, we refer to the Mexican banking system model as a multiplex system or multiplex network indistinctly.

There is already some emerging literature on multiplex financial networks. For example, in Bravo-Benitez et al. [2016] the authors decompose the payment system flows multiplex into three different layers and study the individual structural properties of each layer. In Montagna and Kok [2013] an agent-based multilayer network model is used to study interbank contagion. The authors find that there is an important underestimation of contagion risk if the fact that banks interact in various markets is neglected. In Poledna et al. [2015], the authors estimate the systemic risk in the Mexican banking system by modeling it as a multiplex network of exposures. In Poledna et al. [2015] the contribution of each layer (type of exposure) to the systemic risk is also computed. Bargigli et al. [2015] and Molina-Borboa et al. [2015] study multiple layers of exposures networks, their overlapping and the link persistence in Italy and Mexico. Bookstaber and Kennet [2016] describes the US financial system as a three layers system and study how the risk is transmitted from one layer the others in ways which have not being explored before. Aldasoro and Alves [2017] study an exposures multiplex network and propose a novel approach to model solvency and liquidity contagion by separating the layers by maturity and instrument type. Finally, Musmeci et al. [2016] study difference dependence structures in financial time series by modeling correlations among financial stocks as multiplex networks.

Despite this recent surge in the literature, we argue that there are some limitations in some of the above mentioned works. The most important limitations that we observe in some works is the one on the coverage of the different types of interactions among the players as well as on the dynamical aspects of the problem.

This work departs from the above mentioned works in the literature in many different aspects. In this work, it is not our intention (yet) to study contagion or systemic risk by resorting to the multiplex paradigm. We rather attempt to investigate more deeply and in the widest possible sense the structural aspects of the interconnections in the Mexican banking system.

Our approach is more constructive: we revise some of the commonly used metrics and standard analysis on financial networks. We adapt some of the already accepted concepts and generalize the ones which are not possible to be used in the same way as in simplex financial networks.

Another line of research with which this work is related is to that of community detection in complex networks Schaub et al. [2017]. This is an important line for us as we are departing from the simplex approach, the community structure of multiplex financial systems has not being studied in the past.

One of the most relevant results for financial networks is that of the existence of a core-periphery structure on interbank networks Craig and von Peter [2014]. However, in a more general context, some of the not previously studied financial networks do not exhibit such structure, as it is shown here for the repo network. It is then necessary a more general paradigm and this is the reason why we resort to stochastic block models for multiplex systems in order to identify community structures in the Mexican multiplex banking system.

3 Definitions

The multiplex network referred in most of this paper is the multiplex banking system. For the mathematical description of our We will borrow some notation and metrics from Battiston et al. [2014] and Battiston et al. [2017].

The multiplex network $\mathcal{M}$ consists of N nodes and M layers

The whole structure can be described by the set of adjacency matrices

$$\mathcal{M} \equiv A = \{A^{[1]}, ..., A^{[M]}\}$$

where $A^{[\alpha]} = \{a_{ij}^{[\alpha]}\}$, with $a_{ij}^{[\alpha]} = 1$ if i and j performed a financial transaction in market α and $a_{ij}^{[\alpha]} = 1$ otherwise.

If the links have weights, as it is the case in many financial networks,then the system can be described by the set of weighted matrices

$$\mathcal{W} = \{W^{[1]}, ..., W^{[M]}\}$$

Then we have to move from the degree in one layer $k_i^{[\alpha]} = \sum_{i \neq j}^{N} a_{ij}^{[\alpha]}$ to the multiplex degree $\boldsymbol{k}_i = \{k_i^{[1]}, ..., k_i^{[M]}\}$

A node, i, is said to be active in a layer, α, if $k_i^{[\alpha]} > 0$. Let $b_i^{[\alpha]}$ denote the activity of a node in layer α, then $b_i^{[\alpha]} = 1$ if the node is active in layer α and 0 otherwise. The activity vector is defined as:

$$\boldsymbol{b}_i = \{b_i^{[1]}, ..., b_i^{[M]}\}$$

The total activity $B_i = \sum_{\alpha=1}^{M} b_i^{[\alpha]}$ represents the number of layers in which the node i is active.

Two empirical facts about multiplex networks is that not all nodes have connections in all layers and the node activity is heterogeneously distributed.

One important concept is that of the overlapping degree, computed as:

$$o_i = \sum_{\alpha=1}^{M} k_i^{[\alpha]}$$

Another important concept is that of the multiplex participation coefficient:

$$P_i = \frac{M}{M-1}\left[1 - \sum_{\alpha=1}^{M}\left(\frac{k_i^{[\alpha]}}{o_i}\right)^2\right]$$

If $P_i = 1$ then all the links incidents in node i are equally distributed across layers whereas $P_i = 0$ if node i is only active in one layer.

P_i and o_i are useful to classify the nodes in multiplex hubs (high P_i and o_i); focused hubs (high o_i and low P_i); multiplex leaves (low o_i and high P_i) and focused leaves (low o_i and low P_i)

3.1 Stochastic Block Models in the context of multiplex networks

Networks have proved to be a powerful tool to describe how banks behave in financial markets. It is common to think a given network as the realization of a random graph. The ErdosRenyi model was the first one trying to describe this kind of mathematical object: Let V the set of vertex and E the set of nodes.Under the Erdos-Renyi model $\forall (i, j) \in V$

$$P(X_{ij} = 1) = \pi$$

where $X_{ij} = 1$ if i and j are connected.

Even if this model has been broadly studied and its properties such as degree distribution, connectedness and cluster coefficient can be easily computed, it does not fit real-world networks accurately. For this reason, an alternative model was proposed, in which agents are classified according to their connectivity properties and the probability of two agents to be connected depends on the cluster they belong

Stochastic Block Models were first introduced by Nowiki and Snijders [2001] and are an useful tool to uncover the latent structure in complex networks. The main hypothesis is that the attributes of agents affect the way they interact with each other. For example, it is expected that a small bank will have less activity with other institutions compared to a big bank. Agents with similar attributes are classified in the same class and the relation between agents is conditioned according to the cluster they belong. The formal definition of SBM is:

As before, let V be the set of vertex and E the set of nodes. Consider $Q = 1, 2, ..q$ classes on nodes such that Q defines a partition on E. We define the membership matrix Z as $Z_{i,q} = 1$ if $i \in q$, for $i \in E$ and $q \in Q$. In a Stochastic Block Model de distribution of $X_{i,j}$, the link between i and j, is conditioned to the membership of i in q-th class and node j in the l-th class:

$$X_{i,j} | Z_{iq} Z_{jl} = 1 \sim F_{q,l}^{i,j}$$

It is possible to use external information in order to infer the parameters and classes. This is done by adding covariates related to each vertex (i, j), but in this paper we will just use the network topology as input.

SBM are mixture models which approximates the distribution of $X_{i,j}$. In order to estimate the parameters involved in a SBM it is necessary to solve three problems: the inference of the membership of nodes to clusters, the estimation of the parameters and the selection of the number of clusters. To solve the first and second problem Maraidassou et al (2010) proposed a variational EM approach ,which is an approximation to a maximum likelihood algorithm.This strategy is computationally efficient and can be applied to different types of networks.As regards the third problem, the Integrated Classification Likelihood (ICL) criterion from Biernacky et al [2010] offers a solution by taking into account goodness of fit and sharpness of classification.

Under the variational approach , the approximation to the likelihood of SBM is given by the following expression:

$$J = \sum_{i,q} \tau_{iq} log(a_q) + \sum_{i,j;i} \sum_{q,l} \tau_{iq}\tau_{jl} log f_{ql}^{i,j}(X_{ij})$$

where a_q is the probability to belong to $q \in$Q, and τ is the variational parameter defining a distribution R_x which approximates $P(Z|X)$.

The estimation is carried out in two steps which are repeated one after the other until convergence: the Pseudo-E step and the M-Step. In the Pseudo-E step J is maximized with respect to the variational parameter τ, while in the M step the maximization is done respect to the a parameter and the parameters of f_{ql}, which we will denote by M

As it can be see, the variational approach requires the number of clusters to be known. This is a common problem when working with mixture models and the solution depends on the purpose of the analysis.If the problem is to make an accurately estimation of the density of $X_{i,j}$ a BIC criterion is a good approach, but in most cases the clusters found under this criterion do not have an intuitive interpretation. In the other hand, if the concern of the analysis is cluster identification, the ICL criterion is a better option.As our interest is to understand the underlying structure of the Financial Mexican Market by identifying the role each bank plays in it, we use the ICL criterion.

Our objective is to find the mixture model which best cluster each node, one way to do it is to look for a model M_Q with Q blocks, which maximizes the complete data likelihood and penalizes models with a high number of parameters:

Lets call θ the set of parameters of the model M_Q an N_Q the number of independent parameters the ICL criterion is defined as:

$$ICL(M_Q) = \max_{\theta} P(X, \hat{Z}|\theta) - 1/2(N_Q log(n(n-1)) - (Q-1)n)$$

Different models with different number of blocks are compared throughout this measure, and the model with the smallest value is the selected.

The algorithm here described was implemented by Leger 20016 in the R package "blockmodels", which we use to perform the estimations of the models used.

SBM models can describe uniplex as well as multiplex networks. We are interested in describing the role each bank play in the Mexican Financial System and in understanding how their activity in a given market affects their activity in others. For this reason, a multiplex network will offer a better framework to model this phenomena.In most of the research done in financial networks, the interrelations among markets are ignored leading to a poor description of the level of interconnectedness of a financial system. We believe that the approach proposed in this work would help us to understand how banks interact.

Let E the set of banks in the Financial System which participate in p different markets and $X_{i,j}$ represents the relations among bank i and bank j with $i, j \in$E.X$_{i,j}$ takes values over $\{0,1\}^p$. If $X^r_{i,j} = 1$ that means that i and j are connected in the r market. The definition of being connected can be different according to the layer. It is clear that $X_{i,j}$ follow a multivariate Bernoulli distribution. We suppose that the parameters of this distributions depends on the membership of bank i and bank j to group q and group l respectively. That is:

$$\forall x \in \{0,1\}^p \, P(X_{i,j} = x | Z_{iq} Z_{il}) = \pi_{i,j}(x)$$

3.2 Topological Data Analysis and multiplex networks

There exists some empirical studies on financial networks which reveal some of the important structural characteristics of such systems: Martinez-Jaramillo et al. [2014], Solorzano-Margain et al. [2013], Bravo-Benitez et al. [2016], Molina-Borboa et al. [2015].

Multiplex networks is a well established field with many important developments and important applications. Nevertheless this paradigm has only been used in the context of financial networks recently.

Topological Data Analysis, *TDA*, is a powerful framework for extracting insight from high-dimensional, complex data sets. TDA represents a fundamental advance in machine learning, it is a new and growing field of the applied mathematics.

These new techniques have provided important advances in the study of data in an increasingly diverse set of applications, such as contagion spread on networks D. Taylor and Mucha [2015], collective behavior in biology C. M. Topaz and Halverson [2014], viral evolution Joseph Minhow Chan and Rabadan [2013], among many others.

The basic idea behind TDA is to study the shape of a dataset through the use of different techniques developed in Algebraic Topology. Some of the most important advances in this field have being done by Carlson Carlsson [2009] and Edelsbrunner Edelsbrunner and Harer [2010], Edelsbrunner and D. [2012].

4 Data

The data used for this work consists on different types financial transaction between various types of financial institutions. The types of financial transactions being used for this study consists of the following ones:

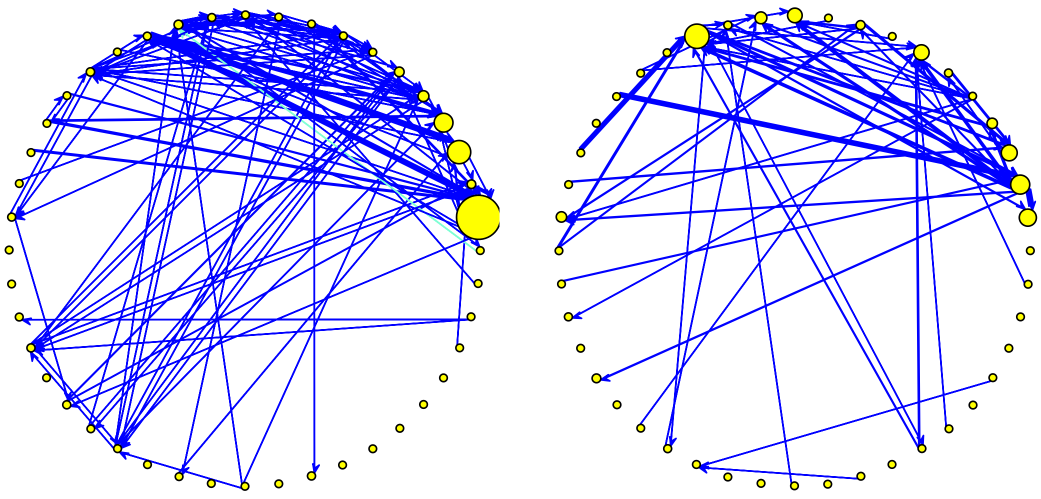
- Direct Deposits and loans, this will be called the D&L network.
- Payments flows in the Mexican payment system, this network will be referred as the SPEI network.
- Repo transactions, this will be called the Repo network.
- Foreign Exchange (FX) transactions, this will be known as the FX network.
- Cross holding of securities between banks, this will be the Xholdings network.

The time series used for this study are of a formidable length and frequency for any previous study on financial networks in the past. With data from SPEI and regulatory reports available at the Mexican central banks Datawarehouse we are able to construct daily matrices from January 2005 to December 2015. This represents a 11 years time window of daily networks for which standard structural and topological metrics will be developed.

As we want to characterize the complexity in the financial system we wanted to use different market activities. For example, in the case of the Repo network, we defined a link between two banks, A and B; if bank A loans cash to bank B and receives securities as collateral. The weight for each link of such network is the whole amount lent by the bank A not considering the collateral or credit risk mitigation.

In the case of the SPEI network, there will be a link between a pair of banks, C and D, if money has been sent from bank C to bank D regardless if such money was sent by one of the bank C's clients or by the bank itself. The links's weight is the accumulated flow of payments at the end of the business day.

The cross holding of securities network is defined in the following form: bank E issues some bonds which are later bought by bank F; therefore we define a link



(a) Unsecured interbank network (b) Interbank Repo network

Figure 1: Two examples of banking networks

between F and E which represents that bank F is exposed to bank E. The weight on each link is the amount (market to marked) of bank E securities held by F.

In the case of FX networks, a link between bank G and bank H exists if bank G has sold or bought any currency from bank G and the link's weight will be the end of the day accumulated amount of FX transactions between each pair of banks.

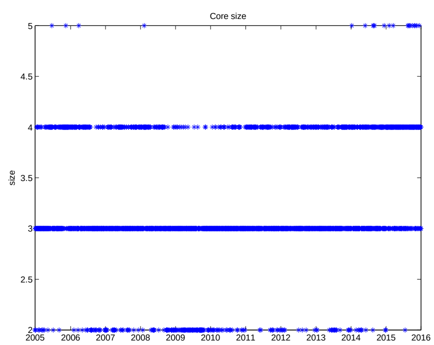
The D&L network is the classical interbank network commonly used in the literature of financial contagion and systemic risk.

4.1 The Mexican Multiplex Banking Network

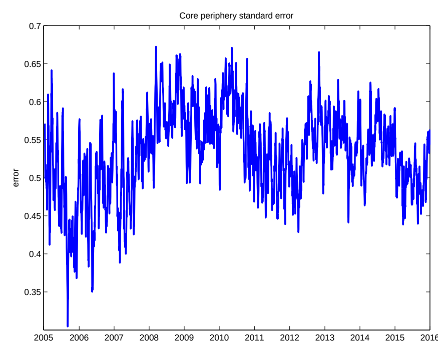
Figure 4.1 illustrates two important banking network in the Mexican financial system: the banking networks of unsecured and secured loans.

Much has been said about the structure of the banking system and a core-periphery tiered model has been proposed and found in many markets and jurisdictions around the world. Figure 4.1 shows to important metrics, useful to describe the possible tiered structure of the repo market.

Figure 4.1 shows two other relevant networks which could be more useful to describe interconnectedness. Only the visual inspection of the networks makes obvious

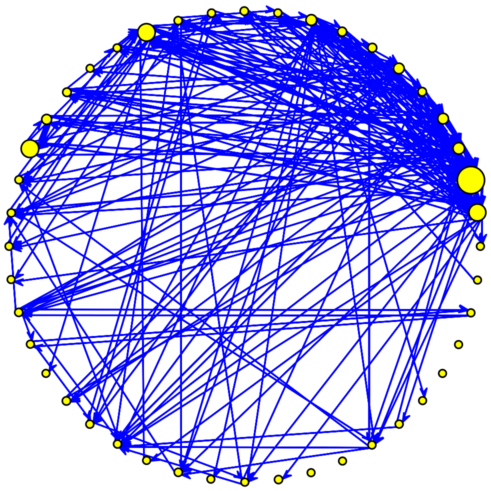


(a) Core size

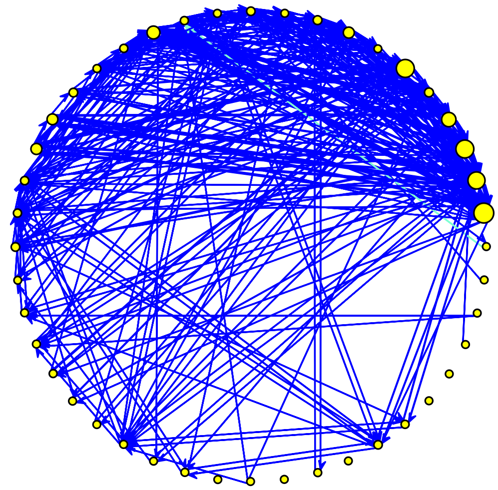


(b) Core periphery error

Figure 2: Two relevant metrics for the core-periphery models in the repo network



(a) Payment flows network



(b) Total exposures interbank network

Figure 3: The payment flows and the total exposures networks

that these are denser networks than the first two presented above.

5 Methods

There is now a good wealth of common metrics used to describe financial networks. In order to precise the type of metrics used to characterize complexity in a financial system some basic definitions are needed. Most of the structural metrics used in this paper are described in Martinez-Jaramillo et al. [2014]; the topological metrics are described in Flood et al. [2015].

5.1 Structural metrics for financial networks

Figure 4 shows some of the most basic metrics to describe the structure of a network: the number of arcs, average degree, the size of the core by using the core periphery model proposed in Craig and von Peter [2014]. This figure shows that the number of loans on a single day in the Mexican banking system in 2015 was between 110 and 145. The series shows a good deal of variability but it is not very informative

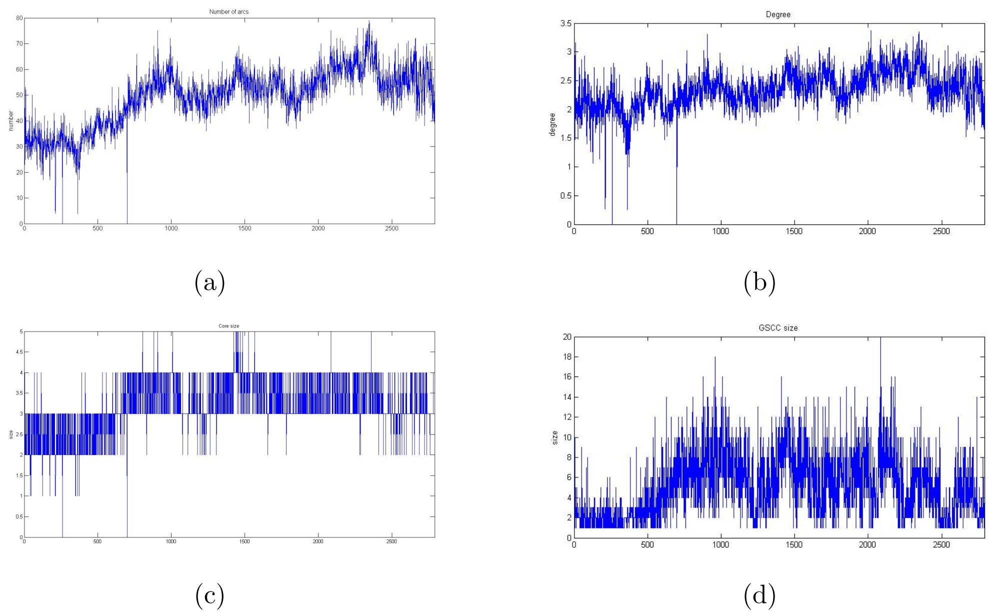
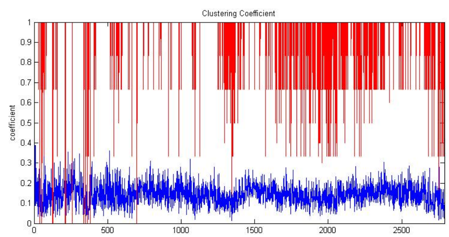


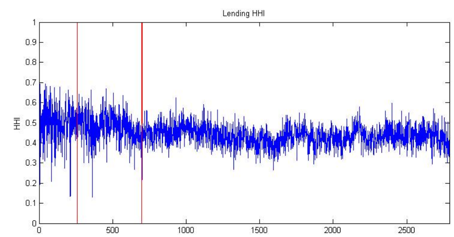
Figure 4: Number of arcs (a), average degree (b), size of the core (c) and size of the GSCC (d) for the deposits and loans network.

on the type of interactions that banks establish among themselves. The time series for the average degree are not as interesting as other in this figure like the one on the size of the core which shows that only a very small number of banks are in the core regardless of the increase on the number of participants in such market. The time series for size of the Giant Strongly Connected Component (GSCC) is the most interesting and shows an important increase of banks which are reachable in such a component of the network.

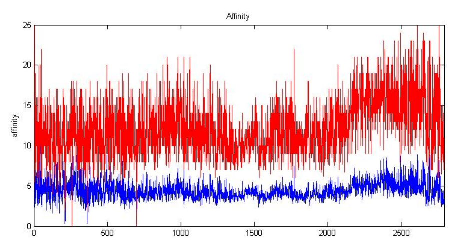
Figure 5 shows some very interesting metrics: the clustering coefficient, the Herfindahl Index, the Affinity and the Betti numbers for the 0 and 1 homologies. From this figure we can see that the average clustering coefficient and the average degree (in blue in their respective plots) show little variation despite the big changes and the long time horizon used for this study. Whereas the Betti number for the 1 homology show an important increase and important regime changes.

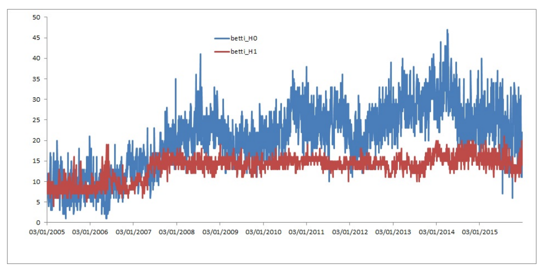
Figure 6 shows interesting and relevant metrics for the SPEI network. The core size makes evident that the SPEI network has increased in size of the core nodes; moreover, it is very interesting to see that the size is considerably larger than the size for the D&L network. The average degree does not change in time as one would





(a) (b)





(c) (d)

Figure 5: Clustering coefficient (a), HHI index (b), Affinity (c) and the Betti Numbers for the 0 and 1 homology (d) for the deposits and loans network.

like given the significant changes in such long period of study. The HHI index points to a relatively high concentration of payment amount sent between banks. Finally, the Betti number for the 1 homology show an impressive increase, showing evidence of and increase in complexity of such a network.

Figure 7 shows the evolution of some metrics for the repo network. The Completness Index show a decrease in connectivity considering the whole network, affected by the incorporation of new players on this market. We can see again that the size of the core is rather small and the size of the GSCC shows interesting components.

The clustering coefficient (Figure 8) shows an interesting picture also. The Betti numbers also show relevant dynamics not captured by the metrics shown in the previous figure.

5.2 Topological analysis of financial networks complexity

Topology is the sub-field of mathematics concerned with the study of shape. Algebraic topology offers different methods for gauging the global properties of a particular topological space by associating with it a collection of algebraic objects. One of this methods is a set of invariants known as the *Homology*.

Homology is a mathematical formalism for talking in a quantitative and unambiguous manner about how a space is connected.

Homology groups of dimension k, $H_k(X)$, provide information about properties of chains formed from simple oriented units known as *simplices*. The elements of homology groups are cycles (chains with vanishing boundary). Homology groups can be computed using the methods of linear algebra. It should be remarked that these computations can be quite time- consuming in spite of recent advances in computational techniques.

Our study analyses the complexity of different financial networks by considering its representation as a *simplicial complex*.

A simplicial complex is a topological space constructed by the union of points, line segments, triangles, and their n-dimensional counterparts. An example of this kind of structures is illustrated in figure 9. A formal definition can be found in Edelsbrunner and Harer [2010] .

Let $V_t = (v_{t,1}, ..., v_{t,n})$ denote the collection of vertices and $E_t = (e_{t,1}, ..., e_{t,r})$ the collection of edges for a given network Δ_t at time t. A simplicial complex can be constructed from V_t and E_t. There will be an edge between two vertices if there is a financial relationship between this two market participants.

We can analize the complexity of the topological structure given by a network by calculating the *Betti numbers* of the associated simplicial complex.

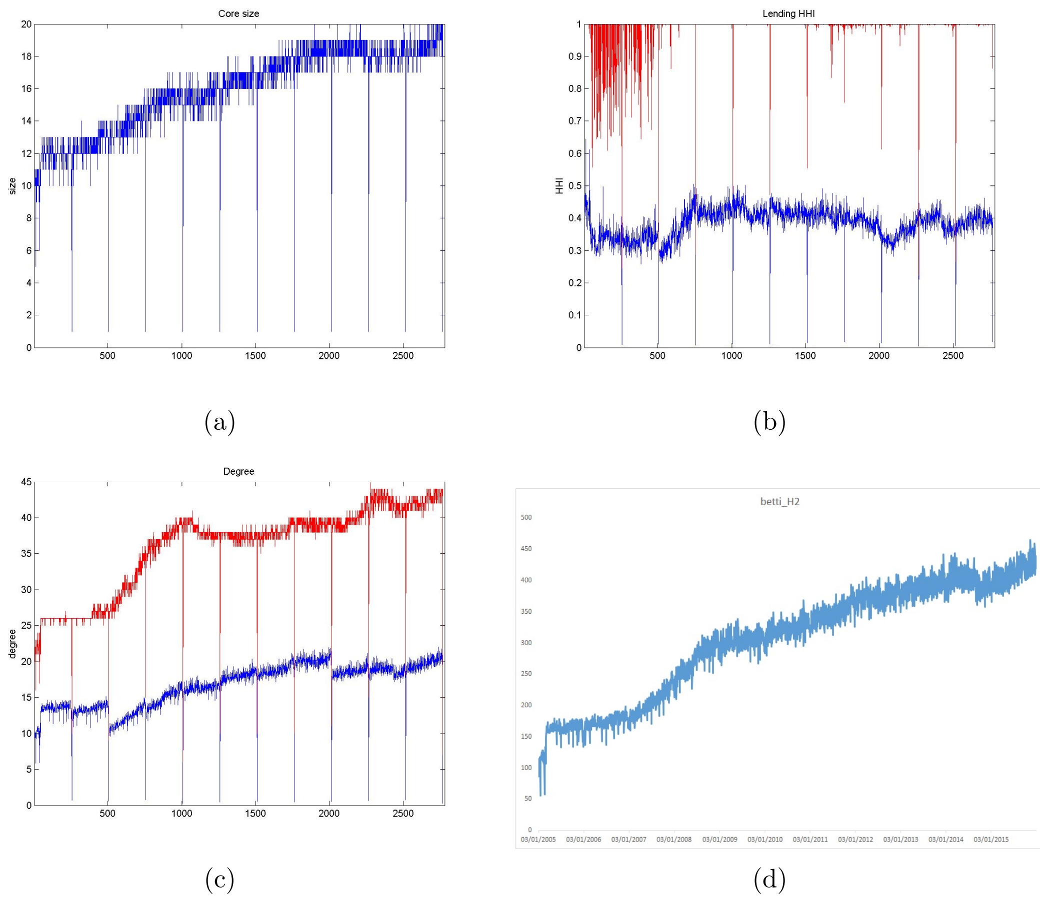


Figure 6: Core size (a), HHI index (b), Degree (c) and the Betti Numbers for the 1 homology (d) for the SPEI network.

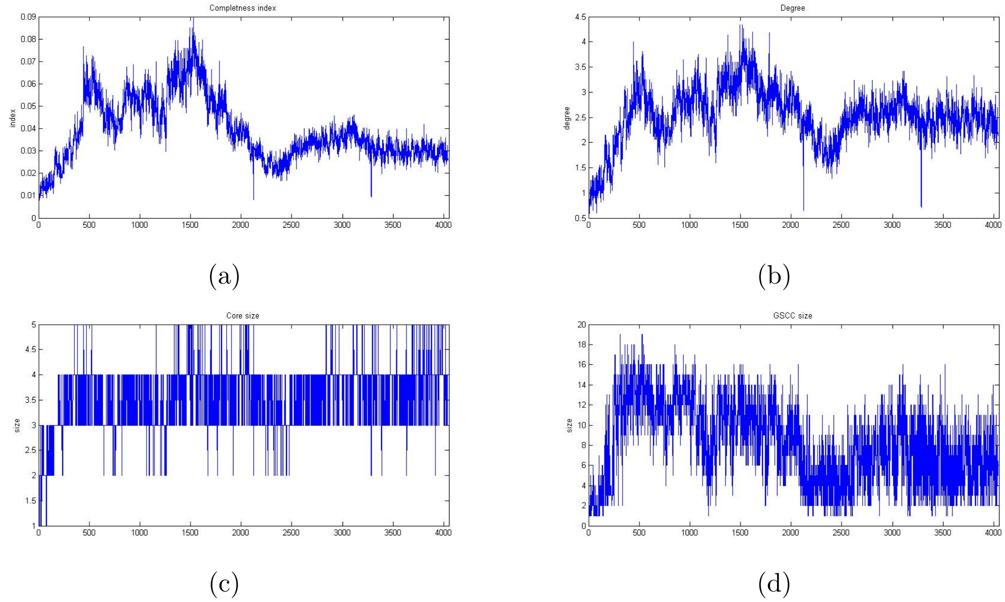


Figure 7: Completness index (a), average degree (b), size of the core (c) and size of the GSCC (d) for the repos network.

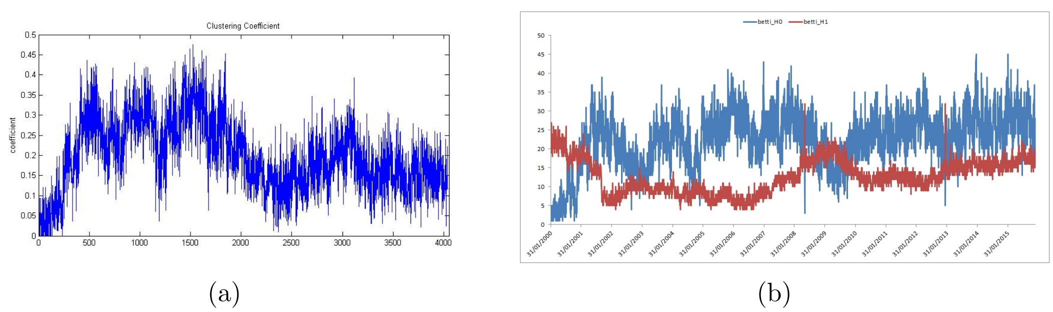


Figure 8: Clustering coefficient (a), Betti numbers for the 0 and 1 homologies (b) for the repos network.

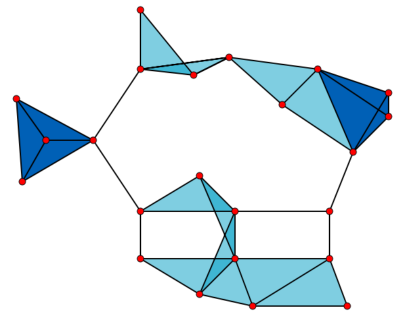


Figure 9: A simplicial complex.

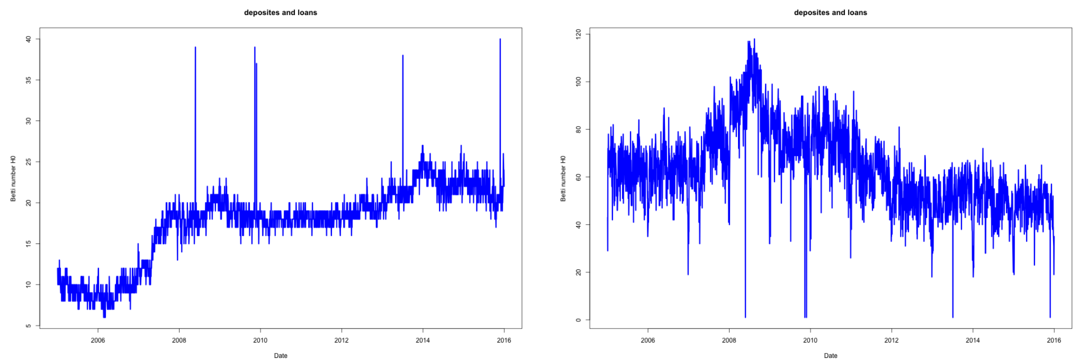
The Betti numbers are used to distinguish topological spaces based on the connectivity of n-dimensional simplicial complexes. The nth Betti number represents the rank of the nth homology group, denoted H_n, which tells us the maximum amount of cuts that must be made before separating a surface into two pieces or 0-cycles, 1-cycles, etc. We can think of the kth Betti number as the number of k-dimensional holes on a topological surface.

We will focus in the study of the 0 and 1 dimensional betti numbers, denoted by b_0 and b_1. A common interpretation of this numbers is to understand b_0 as the number of connected components and b_1 as the number of one-dimensional or "circular" holes. In the context of financial market infrastructure, an interpretation could be to consider b_0 as the number of trading blocs, and b_1 as the number of netting opportunities.

We computed the betti numbers for the networks described in previous sections, and the results provided us with a new perspective of the complexity of each network.

For the FX transaction across banks, we obtained the results shown in figure 10. It is evident the impact of new regulation over the landscape for institutional trading and settlement, specifically, the implementation of post-crisis legislation for the clearing and settlement of many transactions onto central counterparties (CCPs). We can appreciate two different trends in the right graph before and post 2008 crisis.

In the case of the payments network, the results shown in the right plot of figure 13 show us an increase in the magnitude of b_1, this could be interpreted as a constant increase in the complexity of this network. On the other hand, in Figure 14 we observe that the complexity of the repo network, despite the occurrence of important fluctuations has not increased over time.



(a) (b)

Figure 10: Betti numbers for the 0 (1) and 1 (b) homologies for the FX network.

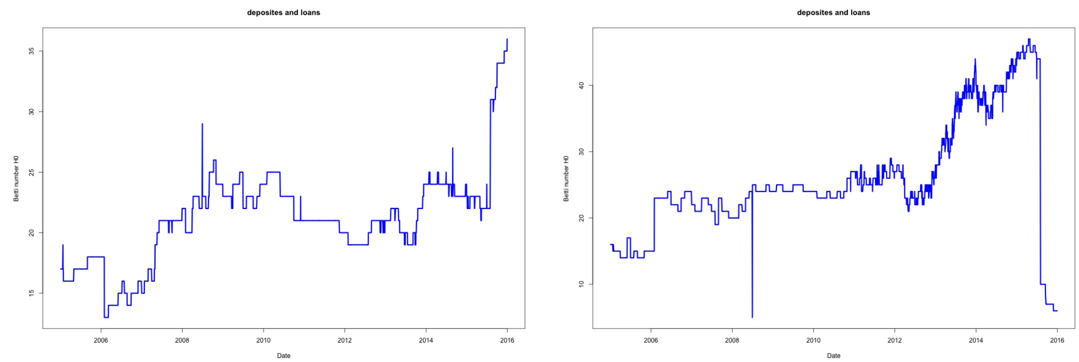
6 Results

In this section we report some of the results of the characterization of the Mexican Financial System as a multiplex network. First some general structural metrics are shown and interpreted. Second, applications of the SBM are given and the results are interpreted.

6.1 General Structural results

A first result from the application of the multiplex approach to the Mexican banking system is to determine the distribution of the overlapping degree and of the activity, metrics which were defined in Section 3. In Figure 6.1 it is possible to observe two characteristics which has been observed on other multiplex systems: an heterogeneous distribution of node activity and a power law distribution of the overlapping degree distribution. From this figure one can deduct that most of the nodes are active in many layers and that the nodes which present low activity are the minority. However, regarding the distribution of the overlapping degree, it is noticeable that a few nodes have many connections in the whole multiplex structure and the majority has only a few connections.

In addition to investigate into the community structure of the Mexican intebank multiplex system and to provide also information of the most relevant nodes. It is also desirable to establish some criteria to determine the relevance of each of the layers that compose the multiplex system. This question has been answered in Zhu



(a) (b)

Figure 11: Betti numbers for the 0 (1) and 1 (b) homologies for the derivatives network.

and Li [2014] by using the correlations among simplex networks. In Zhu and Li [2014], the authors first define the inter-simplex correlations for each node as:

$$C_{isr} = \frac{\sum_j a_{ij}^s a_{ij}^r}{\sum_j a_{ij}^s + \sum_j a_{ij}^r - \sum_j a_{ij}^s a_{ij}^r}$$

where a_{ij}^s is the interaction of banks i and j in layer s. This coefficient takes valus in $[0, 1]$. From there, the authors define the correlation among layers as:

$$C_{sr} = \frac{1}{n} \sum_i C_{isr}$$

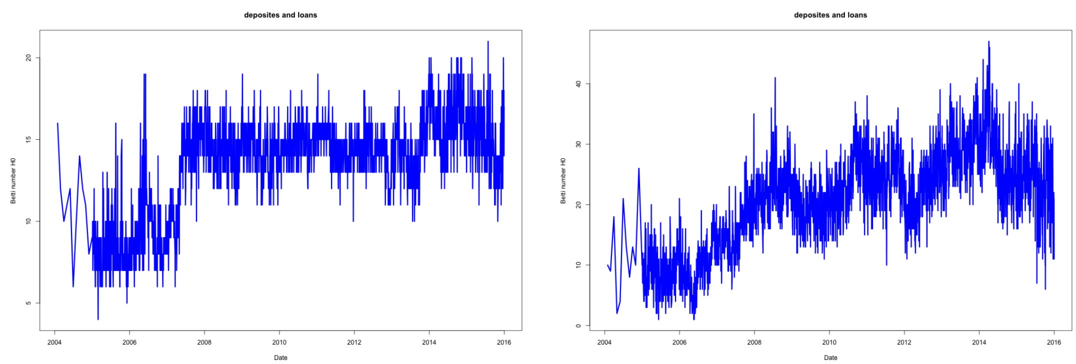
where n is the number of nodes in the multiplex system, C_{sr} takes values in $[0, 1]$.

Finally, by computing such correlations between all layers, the authors in Zhu and Li [2014] define the importance of a layer as:

$$I_s = \frac{\sum_r C_{rs}}{\sum_s \sum_r C_{rs}}$$

In the above table we can see the results of the computation of layer importance. It is noticeable that in all the evaluated periods, the total exposures layer is the one with the highest importance, followed by the payment systems network, then the CVT network follows, with the repo network located at the end of the ranking.

This result, although simple to compute provides important information on how the layers are correlated. In the next section we will find results that considers



(a) (b)

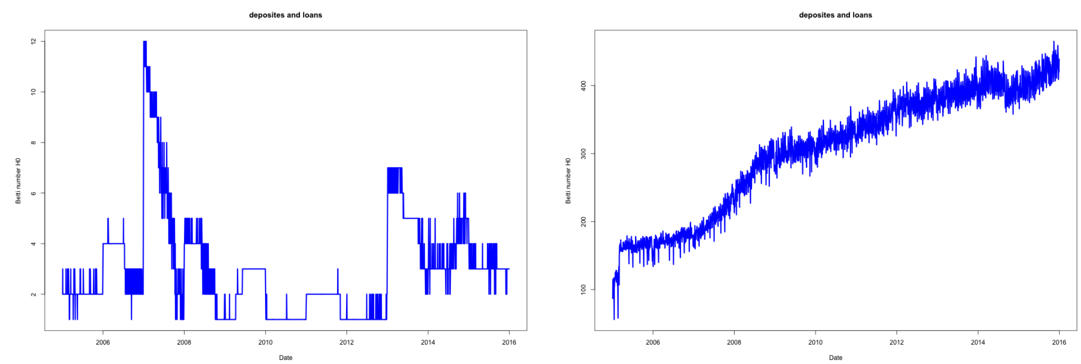
Figure 12: Betti numbers for the 0 (1) and 1 (b) homologies for the D&L network.

Network	June 2007	October 2008	October 2010	June 2015
CVT	0.572	0.422	0.455	0.469
Repo	0.359	0.360	0.312	0.360
SPEI	0.648	0.572	0.555	0.530
Total	0.780	0.645	0.678	0.640

the Total exposures network as an important layer. However, this does not mean that the exposures segment of interaction is more important for the banking system than the payment system. The only implication that we can extract is that if two banks interact on the total exposures network, such interaction is more informative than interactions in other markets. Nevertheless, the congruence of both approaches pointing to the same layer is encouraging about the application of this type of analysis.

6.2 Results on SBMs

Understanding the way banks interact is an important issue for market regulators since these interactions allow a market to be dynamic and competitive. On the other hand, it has been shown that the interconnectedness of a system play a fundamental role when analyzing the possible effects of a financial crisis.This kind of analysis helps market regulators to identify different agents groups with respect to their relationships.



(a) (b)

Figure 13: Betti numbers for the 0 (1) and 1 (b) homologies for the payments network.

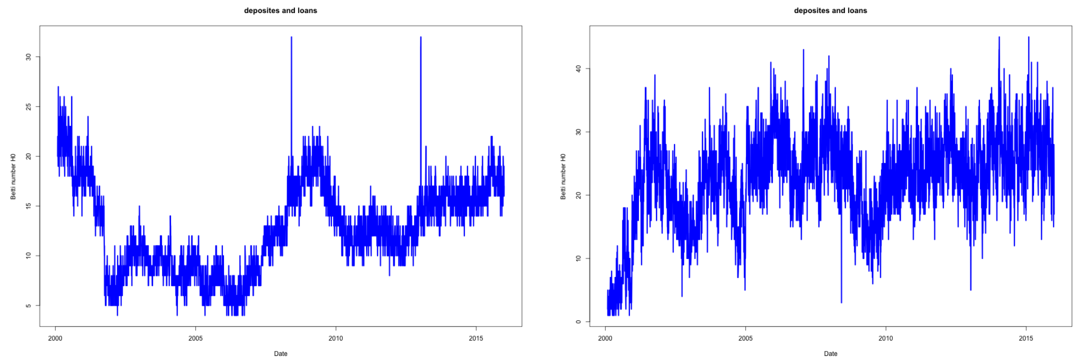
It is natural to think a market as a network where nodes represents the agents and the vertices the relationships among them. In real-world financial systems, agents interact in multiple markets and these interactions are interdependent. For example, Han and Nikolaou [2016] show that agents with strong relationships in funding markets are more likely to have a stronger position in triparty repos. A multiplex network is, then, the best way to interpret this kind of phenomena.

In this context, Multilayer Stochastic Block Models Models (SBM) can help us not just to identify connection patterns between banks in a complex system, but also to describe how the interactions in a market impact the dynamics in other markets.

In a fist stage, we analyze the relationships among banks in three different layers: SPEI, CVT and Total, without taking into account the strength of this connections. A Bernoulli Multilayer SBM was enough to uncover the underlying structure of the system.

In order to have an idea of the time component, we analyze the system in four different periods: June 2007, October 2008, June 2010 and June 2015. In all cases the banks were categorized in three groups (Figures 16 and 17). It is worth noting that the topological measures Overlapping degree and the Multiplex Participation were enough to distinguish the groups. As it can be seen, banks in group 1 can be clearly identified as focused leaves or multiplex leaves, those in group 2 can be either focused hubs or multiplex hubs, depending of the time,while group 3 contains in all cases multiplex hubs.

Figure 17 also includes information about the number of assets of each bank represented by the size of the point. Surprisingly, in some cases big banks are not



(a) (b)

Figure 14: Betti numbers for the 0 (1) and 1 (b) homologies for the Repo network.

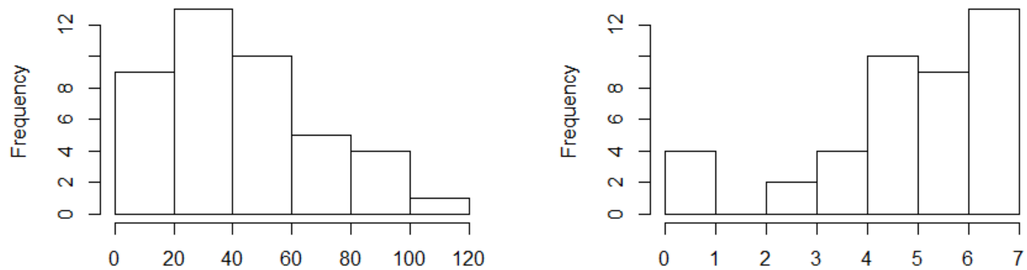
categorized in group 3 as expected.

Benoulli Multilayer SBM cluster banks according to their behavior not only in one market but in the whole structure. Figure 18 is useful to see this better: in all cases banks that belong to group 1 are those less active in all markets, group 2 is better connected in the CVT layer than the other groups while it is more diversified in the other layers and group 3 seems to be formed by banks more active in the Total and SPEI layer and less interconnected in the CVT layer.

As we have pointed out before, the classification algorithm takes into account just the connection profile of each bank, but the clusters do not have a interpretation per se because we do not impose an a priori classification. In order to have a better understanding of each cluster's features, we infer the membership of each bank from topology variables with decision trees.

The prediction variables were total degree, betweeness, closeness , eigenvalue centrality, their membership to the core , their membership to the GCSS, the correlation of the links of a node in one layer with those in other layers, as well as the participation index. The information shown in Figure 6.2 is consistent with the information shown in Figure 18: group 1 is less active in all layers, while banks in group 2 and 3 are highly connected in all layers with group 2 as the most active group in the CVT layer. It is interesting to note that in all cases just 2 variables were required to almost perfectly classified each bank.

Despite the fact that topological measures are able to identify the clusters correctly, they do not give enough information about how these groups interact with each other. Once the parameters of a multiplex SBM have been estimated, it is pos-



(a) Total Overlapping Degree

(b) Total Activity

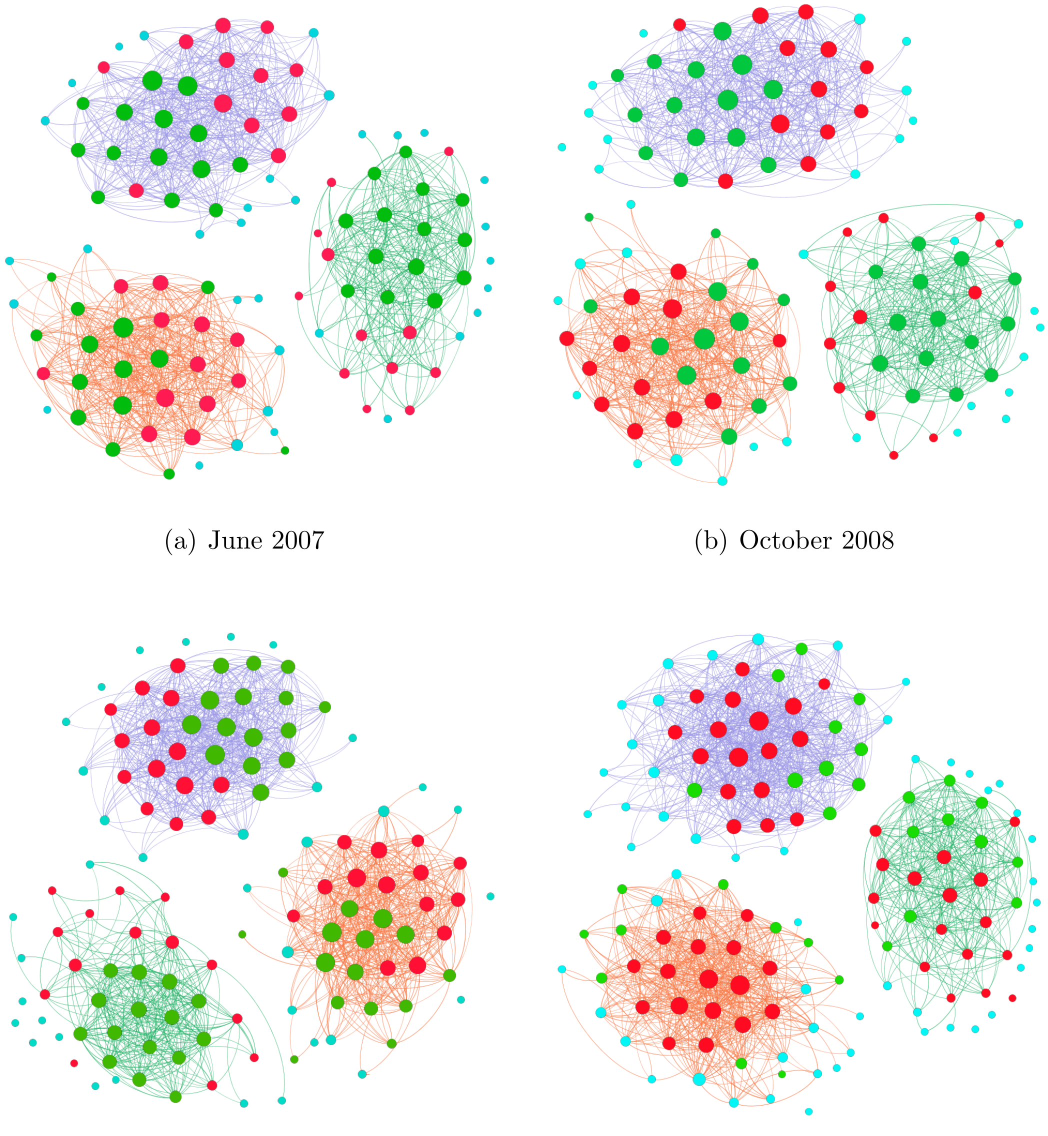
Figure 15: Overlapping degree and total activity

sible to have a clearer picture of the activity within and between groups. Figure 20 shows the probability of banks to be connected according to the group they belong to, the size of the links depends on the magnitude of this probability with no link if it is less than 0.1.

As it can be seen, the graph is directed since it is not as easy for small banks connecting with other banks as it is for big banks. This figure sheds more light on the connection profile of each cluster. In the figures shown before, Group 2 and 3 seem to be similar, but Figure 20 show that the main difference between them is that banks in group 2 prefer to make transactions in the securities market and to have a bigger exposure with banks of the same group. Cluster 3, on the other hand, diversifies its exposures and it is less focused in its transaction in the securities market.

Another advantage of these model is that it provides a total description of the jointly distribution of the links. This information enables us to compute conditional probabilities, which help us to understand the relationship between markets. The networks showed in Figures 21 to 23 represent the probability of two banks to be connected in two markets at the same time and how it changes when there is a connection in a third market. Being connected in a layer have a positive effect in the probability of being connected in other layers, the magnitude of this effect depends, as it can be seen, on the membership of the bank.

Figure 21 illustrates that if a bank from cluster 2 and a bank from cluster 3 make transaction in the securities market, they are more likely to be exposed to each other



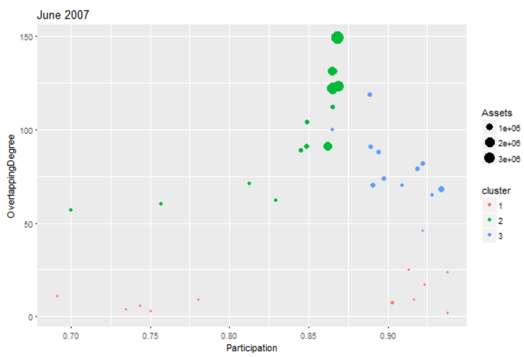
(a) June 2007

(b) October 2008

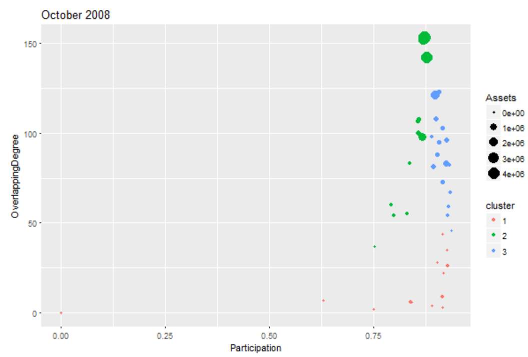
(c) June 2010

(d) June 2015

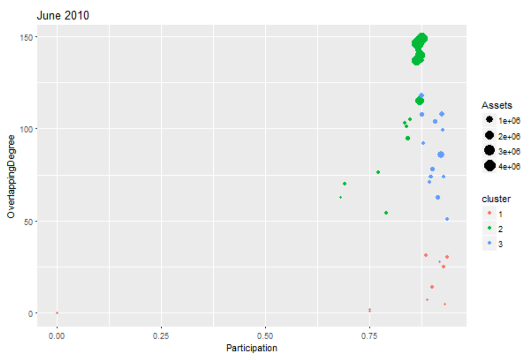
Figure 16: Clusters found using Bernoulli Multiplex SBM



(a) June 2007



(b) October 2008



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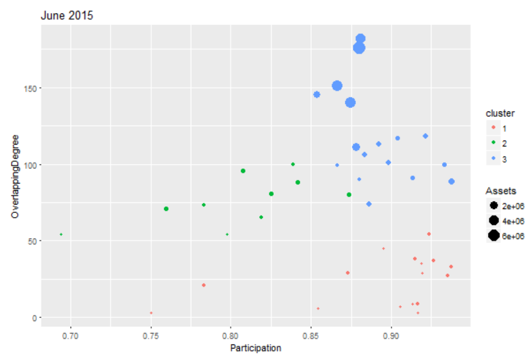

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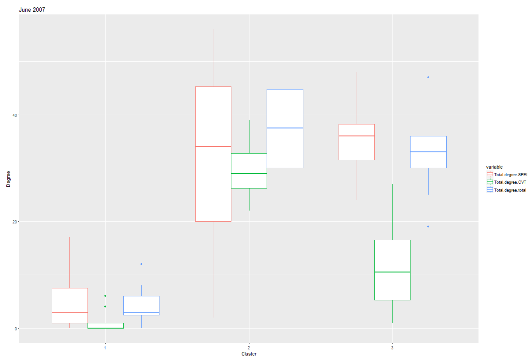
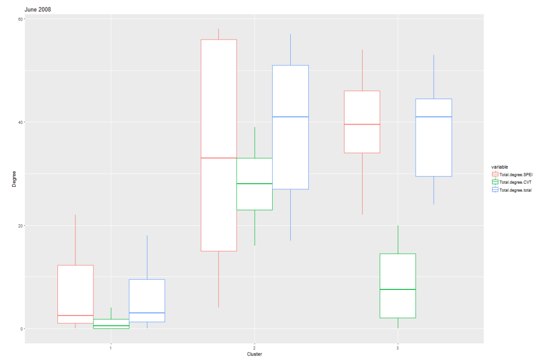
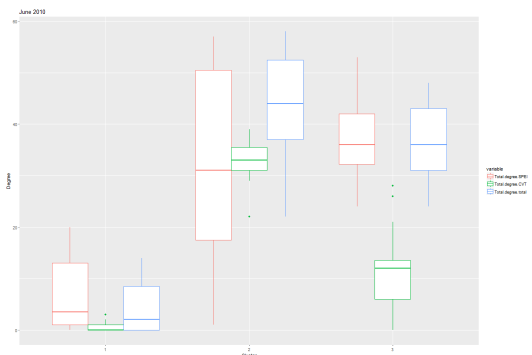
Figure 17: Clusters found using Bernoulli Multiplex SBM



(a) June 2007



(b) October 2008



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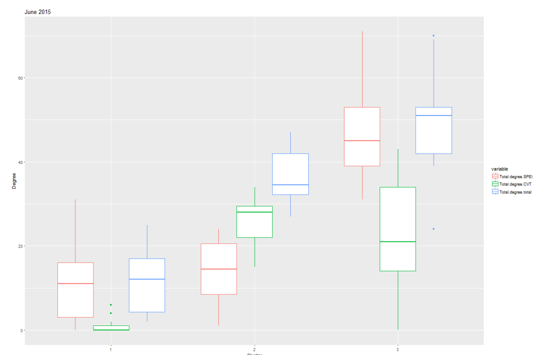
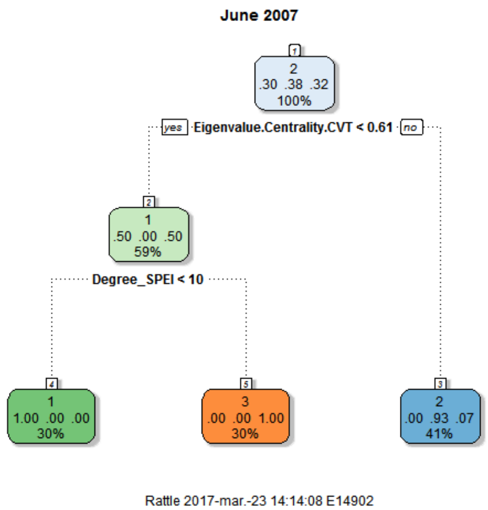
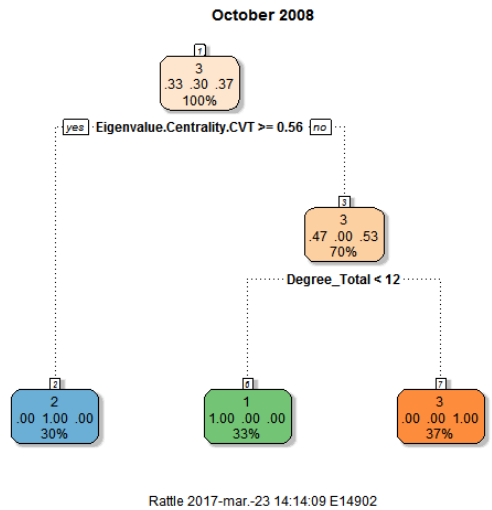


(d) June 2015

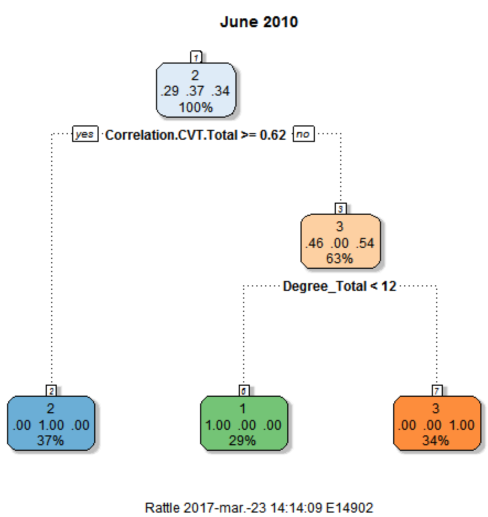
Figure 18: Clusters found using Bernoulli Multiplex SBM



(a) June 2007



(b) October 2008



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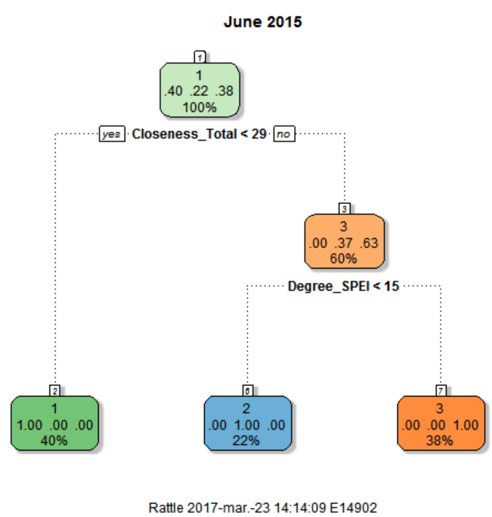
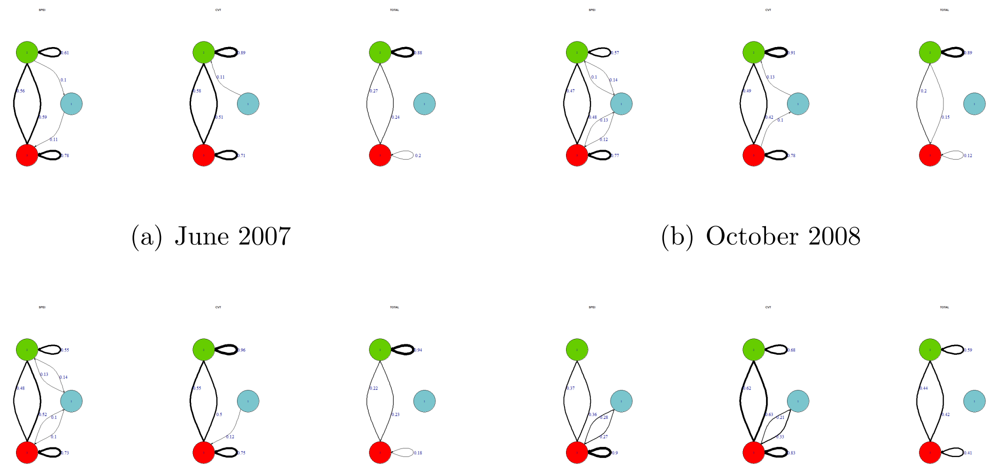


(d) June 2015

Figure 19: Decision Trees



(a) June 2007

(b) October 2008

(c) June 2010

(d) June 2015

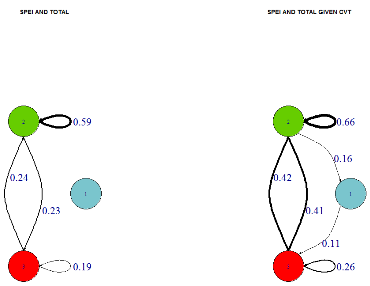
Figure 20: Marginal Distributions

and to interact more in the SPEI layer, while connections between banks from the same group increases marginally their connections in other layers. The SPEI layer has a similar behavior that the CVT layer, it strengthens the connections in the other two layers, specially between banks from group 2 and 3, but in a more subtle way.

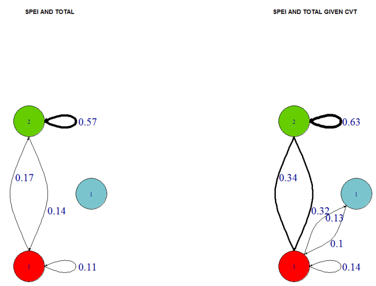
Things become completely different when we analyse the Total layer. Having a bigger exposure to a bank seems to increase the activity in other markets, in general , no matter the membership of the banks. It is really interesting that banks in group 1, which is the least active group , increases dramatically their chances to interact between its members and other groups. During June 2007 , October 2008 and June 2015 , the links that become stronger once there is a connection in the Total layer, are those between cluster 1 and cluster 3 in both directions, while in June 2010 are those between 1 and 2. In all cases except in June 2007 , keeping exposures of banks from cluster 1, help banks in cluster 1 to interact with each other.

7 Conclusion

This is the first attempt to characterize complexity and interconnectedness in a comprehensive way for a financial system.The Topological and Multiplex approaches



(a) June 2007



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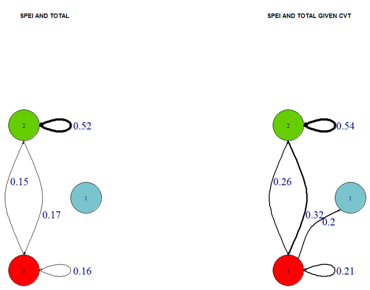

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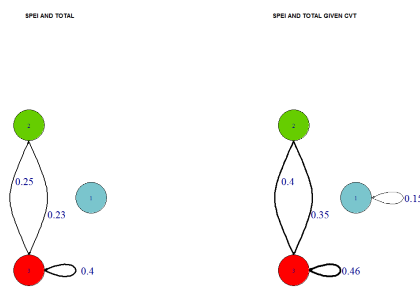

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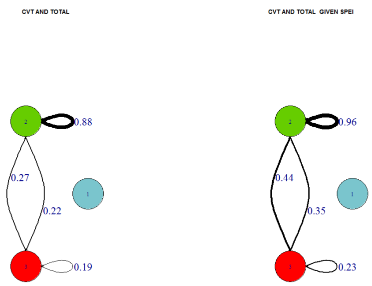
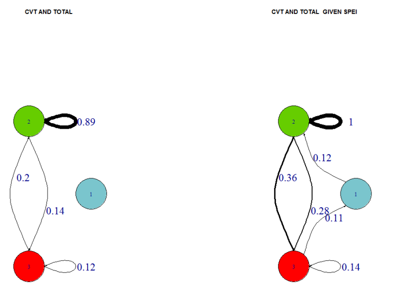
Figure 21: Joint Distributions of SPEI and TOTAL networks



(a) June 2007



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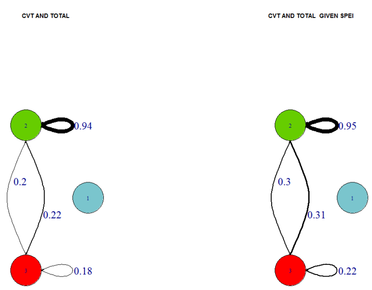

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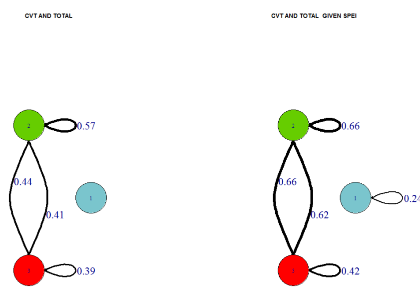

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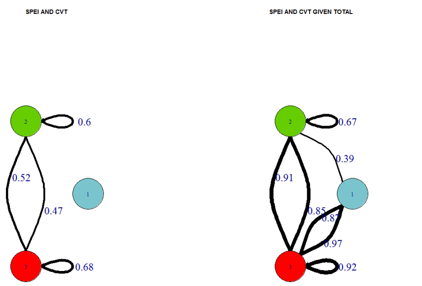
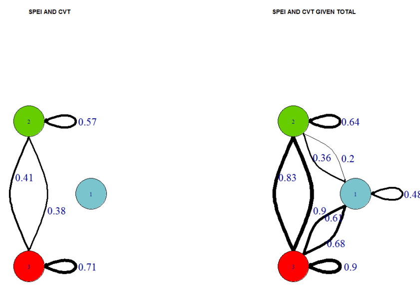
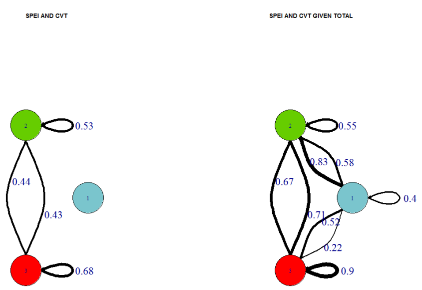
Figure 22: Joint Distributions of CVT and TOTAL networks



(a) June 2007



(b) October 2008



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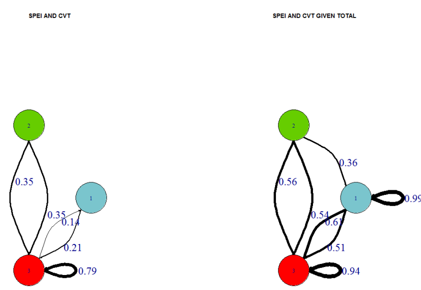

(d) June 2015

Figure 23: Joint Distributions of SPEI and CVT networks

provided the right tools to perform such analysis. However, there is still a lot to do in order to really characterize financial systems and more importantly to translate such characterization into concrete policy guidance.

Some of the main takeaways from this first approach for studying multiplex banking networks are:

- We have started to study the multiplex of the Mexican financial system
- It is possible to include more layers related to different market activities than in previous exercises
- First results: the multiplex approach deliver important information not available on a individual layer view
- This has important implications from the systemic risk point of view

Additionally, we provide some initial but useful metrics to start studying such structures and propose a concrete procedure to detect the community structure of the multiplex banking system. We also determine the most relevant layer of the multiplex in terms of node correlation and detect systemically important banks in such structure.

Nevertheless, there are still many things to be done as future work:

- Extend the multiplex analysis to more financial intermediaries
- Explore the time dimension
- Include some more sophisticated metrics into the multiplex context
- Study more in depth stochastic block models and centrality into the multiplex context
- Study how to compress some additional layers without losing information
- Apply all these results for weighted multiplex networks.

Finally, the long term goal of our work is to be able to detect structural changes on the complexity and interconnectedness of the the financial system as a whole with the main purpose of detecting scenarios which could threaten the stability of the system.

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