An Early Warning System for Tail Financial Risks

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Motivation

- The financial crisis of 2007-2009 prompted significant efforts at central banks and bank regulatory agencies in designing early warning systems (EWS) in the financial sector.
- The current implementation of key Basel bank regulations is increasingly relying on banking system-wide tail risk forecasts as embedded in stress testing exercises.
- The EWS in this paper builds on the literature taking a risk management approach to the modeling and measurement of tail financial risks
- Methodological approach: rather than conducting a horse race among competing models looking for a winner, the proposed EWS exploits the potential of several competing (mis-specified) forecasting models to improve forecasting performance.

The Early Warning System (EWS) in a nutshell

- The EWS is based on real-time multi-period forecast combinations of Value-at-Risk (VaR) and Expected Shortfalls (ES) of portfolio returns of non-financial firms and banks.
- Forecast combinations include baseline (VaR,ES) forecasts conditional on a domestic risk factor, as well as (sVaR,sES) forecasts conditional on CoVaRs of the risk factor
- Focus on surveillance in real-time.
- Implemented using monthly data of the G-7 economies for the period 1975:01-2018:12 (current paper),
- On going revision: 1975:01-**2023:04**

Three novel features

1. Weight (model) selection (NEW relative to current posted paper)
   - At each forecasting date, model forecasts are included in the combination if they pass an out-of-sample backtest in a previous evaluation period.
   - The weight of selected model forecasts solve a minimum variance portfolio problem where the "return" of the portfolios are models' scoring functions.
2. Integrating stress testing into forecasting
   - The forecast combination includes forecasts conditional on risk factors (volatilities), called baseline forecasts, and forecasts conditional on the VaR of risk factors, called stress forecasts, and denoted by (sVaR,sES)
   - The sVaR and sES measures are forecasting versions of the CoVaR and CoES measures of Adrian and Brunnermeier (2016).
3. A vulnerability signal
   - ES forecasts are used as predictors of a binary (Logit) model of the probability of the occurrence of VaR violations.

Forecasting Methods

- Forecast methods are specifications of models' forecasts that vary according to the length of the estimation window and the forecast evaluation window.

- Three basic models with an aggregate risk factor (log volatility) as a predictor:
  1. simple linear model with variance independent of the risk factor;
  2. Same as the first model, except that the variance of a return has the risk factor as predictor
  3. A quantile model with the risk factor as predictor

- The choice of simple models is dictated by the goal to examine transparently the properties of the procedure. Extensions are straightforward.

- The evaluation of each model uses the FZ0 scoring function derived by Patton, Ziegel and Chen (2019),

Main results

- Good performance of out-of-sample tail financial risk forecasts evaluated by backtests for most series even up to a 12-month horizon
- Stress forecasts have a significant role in improving forecasting performance, especially prior to periods of severe financial stress.
- Vulnerability signals anticipated actual stresses in several instances.

The EWS set-up

1. Baseline and stress forecasts
2. The FZ0 scoring function
3. ”Optimal” forecast combinations
4. Constructing a vulnerability signal

Baseline forecasts (1 of 3)

Model 1

$$R^{i,j}_{t+h} = \alpha^{i,j}_h + \beta^{i,j}_h V^i_t + \sigma^{i,j}_{t+h}\eta^{i,j}_{t+h} \tag{1}$$

The baseline forecasts (projections) of the h-month-ahead expected return and (VaR_τ, ES_τ) are:

$$E_t(\hat{R}^{i,j}_{t+h}) \equiv \hat{\alpha}^{i,j}_h + \hat{\beta}^{i,j}_h V^i_t \tag{2}$$

$$VaR_\tau(\hat{R}^{i,j}_{t+h}) = E_t(\hat{R}^{i,j}_{t+h}) + \hat{\sigma}^{i,j}_{t+h} G(\tau) \tag{3}$$

$$ES_\tau(\hat{R}^{i,j}_{t+h}) = E_t(\hat{R}^{i,j}_{t+h}) - \hat{\sigma}^{i,j}_{,t+h} H(\tau) \tag{4}$$

Baseline forecasts (2 of 3)

Model 2

Model 2's projection of the h-month-ahead return is the same as that of Model 1, but the variance depends on the risk factor:

$$\sigma_{2t+h} = \exp(\phi_0 + \phi_1 V_t) \quad (5)$$

The h-month-ahead baseline (VaR, ES) forecasts of Model 2 are therefore:

$$VaR_\tau(\bar{R}_{t+h}) = E_t(\hat{R}^{i,j}_{t+h}) + \sqrt{\exp(\bar{\phi}_0 + \bar{\phi}_1 V_t)}G(\tau) \quad (6)$$

$$ES_\tau(\bar{R}_{t+h}) = E_t(\hat{R}^{i,j}_{t+h}) - \sqrt{\exp(\bar{\phi}_0 + \bar{\phi}_1 V_t)}H(\tau) \quad (7)$$

Baseline forecasts (3 of 3)

Model 3 (quantile model)

$$VaR_\tau(\hat{R}^{i,j}_{t+h}) = \hat{\alpha}^{i,j}_h(\tau) + \hat{\beta}^{i,j}_h(\tau)V^i_t \quad (8)$$

Conditional h-month-ahead ES forecast:

$$ES_\tau(\hat{R}^{i,j}_{t+h}) = E_t R^{i,j}_{t+h} - \tau^{-1}\hat{\sigma}^{i,j}_{t+h} \quad (9)$$

Gourieroux and Li (2012):

$$E_t R^{i,j}_{t+h} - \tau^{-1}\hat{\sigma}^{i,j}_{t+h} = L^h_{ij}(\tau) VaR_\tau(\hat{R}^{i,j}_{t+h}) \quad (10)$$

$$L^h_{ij}(\tau) = c^h_{ij,1}(\tau) I_{(VaR_\tau(\hat{R}^{ij}_{t+h})<0)} + c^h_{ij,2}(\tau) I_{(VaR_\tau(\hat{R}^{ij}_{t+h})>0)} \quad (11)$$

$$ES_\tau(\bar{R}^{ij}_{t+h}) = [\hat{c}^h_{ij,1}(\tau) I_{VaR_\tau(\hat{R}^{ij}_{t+h})<0} + \hat{c}^h_{ij,2}(\tau) I_{VaR_\tau(\hat{R}^{ij}_{t+h}>0)}] VaR_\tau(\hat{R}^j_{ij,t+h}) \quad (12)$$

Example of in-sample estimation of the models

Table: US, full sample

	horizon h	Model 1 beta(h)	Model 1 p-value	Model 2 beta(h)	Model 2 p-value	Model 2 phi(h)	Model 2 p-value	Model 3 beta(tau,h)	Model 3 p-values
RNF	1	**-2.54**	0.00	**-0.88**	0.02	**1.88**	0.00	**-6.37**	0.00
	3	**-1.84**	0.00	**-0.85**	0.24	**1.51**	0.00	**-3.08**	0.05
	6	**-1.23**	0.15	**-0.88**	0.41	**1.38**	0.00	**-2.91**	0.01
	12	**-2.72**	0.05	**-3.74**	0.02	**1.33**	0.00	**-2.45**	0.02
RB	1	**-3.76**	0.00	**-1.91**	0.00	**1.71**	0.00	**-9.20**	0.00
	3	**-4.82**	0.00	**-2.01**	0.06	**1.29**	0.00	**-0.97**	0.67
	6	**-4.70**	0.00	**-2.37**	0.14	**1.00**	0.00	**-2.04**	0.17
	12	**-3.10**	0.13	**-2.88**	0.23	**0.84**	0.00	**-0.76**	0.61

Stress forecasts

- Stress forecasts are (VaR,ES) return forecasts conditional on CoVaRs of risk factors.
- CoVaRs of risk factors capture domestic and external tail risk shocks in reduced-form.

- (a) VaR of the risk factor V_t^i in country i;

  (b) VaR of the leave-one-out average of risk factors across countries: $V_t^{-i} \equiv \sum_{k \neq i}^{N} \frac{V_t^k}{N-1}$,
  (c) quantile levels: $\tau' \leq \tau$:

$$VaR_{\tau'}(V_t^i) = a^i(\tau') + b^i(\tau')V_{t-1}^{-i} + c^i(\tau')V_{t-1}^i \quad (13)$$

$$VaR_{\tau'}(V_t^{-i}) = a^{-i}(\tau') + b^{-i}(\tau')V_{t-1}^{-i} \quad (14)$$

.

- Two stress scenarios defined by the following CoVaRs:

$$\textit{domestic} \quad co_1VaR_{\tau'}(V_t^i) = \hat{a}^i(\tau') + \hat{b}^i(\tau')V_{t-1}^{-i} + \hat{c}^i(\tau')VaR_{\tau'}(V_{t-1}^i) \quad (15)$$

$$\textit{external} \quad co_2VaR_{\tau'}(V_t^i) = \hat{a}^i(\tau') + \hat{b}^i(\tau')VaR_{\tau'}(V_{t-1}^{-i}) + \hat{c}^i(\tau')V_{t-1}^i \quad (16)$$

The FZ0 scoring function

- I use the (strictly consistent) FZ0 scoring function derived by Patton, Ziegel and Chen (2019, Proposition 1), given by:

$$FZ0(VaR_{t+h}, ES_{t+h}) \equiv -\frac{1}{\tau ES_{t+h}} I(R_{t+h} \leq VaR_{t+h})(VaR_{t+h} - R_{t+h}) + \frac{VaR_{t+h}}{ES_{t+h}} + \log(-VaR_{t+h}) - 1 \quad (17)$$

- The FZ0 scoring function applies to strictly negative values of VaR and ES, and it has a **negative orientation**: lower value indicate higher scores.
- **Backtests**: the DQ and DES tests adapted from Engle and Manganelli (1994) by Patton, Ziegler and Chen (2019).
- In priciple, other backtests can be used

”Optimal” forecast combinations (1 of 2)

Set-up

- $(VaR_m(\hat{R}_{t+h}), ES_m(\hat{R}_{t+h}))$ are the h-period ahead forecast at t of forecasting method m
- Let M be the total number of forecasting methods.
- The data range $[t-w, t]$ is the evaluation window of size w at the forecasting date t
- The data range $[t-we, t]$ is the estimation window of the forecasting models
- $f_m(t,h)$: the *FZ0* score associated with the h-month-ahead forecast of method m.

The forecasting strategy is implemented at each date in four steps described next.

"Optimal" forecast combinations (2 of 2)

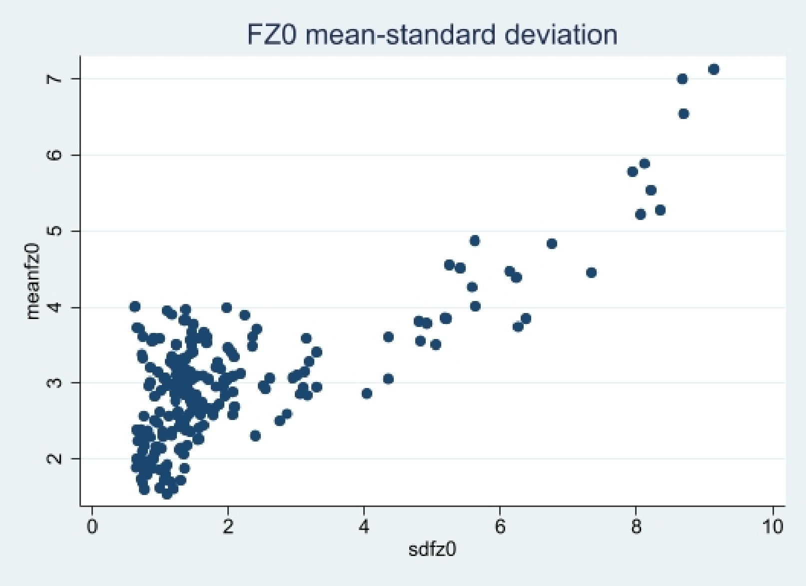
1. For each $m \in M$, dynamic VaR and ES backtests are run over data of the evaluation window $[t-w, t]$.
   - Any model for which the null hypothesis is rejected at a p-value less or equal to 0.10 is excluded from the forecast combination.
   - The set of the models included in the combination is $M' \subseteq M$.
   - if M' turns out to be empty, all models are included in the combination

2. *Minimum variance* portfolio of scoring functions.
   The $M'x1$ vector of optimal weights ω^* solves:

$$\min_{\omega} \ \omega'\Sigma^{-1}\omega, \ \ \textit{subject to} \ \ \omega'\iota = 1 \ \ \textit{and} \ \ \omega \geq 0$$

3. The h-period ahead forecast combination for VaR and ES are given by:

$$(VaR_\tau(\hat{R}_{t+h}), ES_\tau(\hat{R}_{t+h})) = (\sum_{m=1}^{M} \omega_t^{*m} VaR_m(\hat{R}_{t+h}), \sum_{m=1}^{M} \omega_t^{*m} ES_m(\hat{R}_{t+h})) \tag{18}$$


FZ0 mean-standard deviation
meanfz0
sdfz0
7
6
5
4
3
2
0
2
4
6
8
10

Construction of a forecast combination: simple example

Table: Forecast combinations' weights, three models

horizon (months)		Model 1	Model 2	Model 3	FZ0 portfolio MEAN	FZ0 portfolio SD
1	no becktest	0.55	0.45	0.00	9.20	1.47
	becktested	**0.00**	**1.00**	**0.00**	8.05	1.69
3	no becktest	0.60	0.40	0.00	18.36	1.98
	becktested	**0.00**	**1.00**	**0.00**	16.90	2.08
6	no becktest	0.63	0.37	0.00	27.64	2.14
	becktested	**0.63**	**0.37**	**0.00**	27.64	2.14
12	no becktest	0.09	0.91	0.00	37.50	2.28
	becktested	**0.00**	**0.00**	**0.00**		

Methods and results

Forecast combinations include the following forecasting methods:

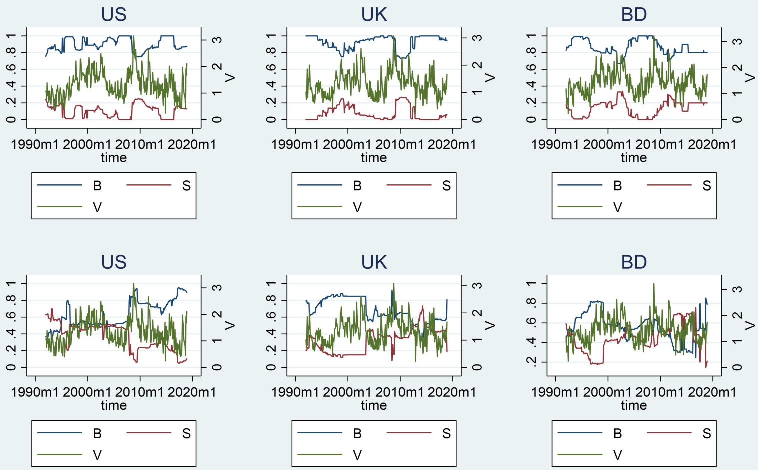
- Baseline forecasts of each model and their *Equally Weighted Combinations* (EWCs) obtained with 120-month and 84-month rolling estimation windows;
- EWC forecast combinations of the two stress test specifications Stress 1 (external) and Stress 2 (domestic) using a 84-month rolling estimation window.
- Backtest rolling evaluation window of 60 months
- A total of 10 forecasting methods

Statistics of forecast combinations' weights

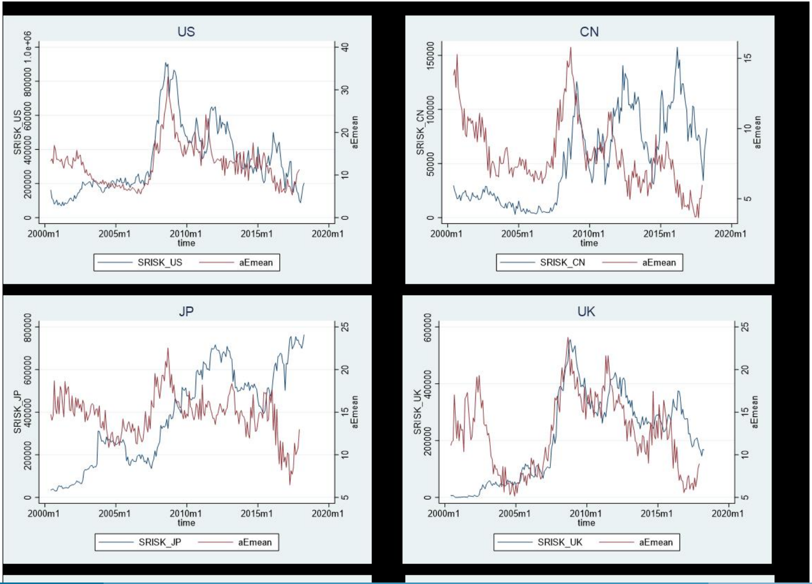
	Horizon (months)	h=1			h=12		
		Mean	Min	Max	Mean	Min	Max
US	Mod.1 (w=120)	0.11	0.02	0.28	0.04	0.00	0.19
	Mod.1 (w=84)	0.11	0.01	0.29	0.09	0.00	0.23
	Mod.2 (w=120)	0.12	0.02	0.21	0.07	0.00	0.21
	Mod.2 (w=84)	0.14	0.03	0.40	0.08	0.00	0.29
	Mod.3 (w=120)	0.10	0.01	0.23	0.07	0.00	0.29
	Mod.3 (w=84)	0.10	0.00	0.33	0.17	0.03	0.33
	EWC (w=120)	**0.11**	**0.00**	**0.18**	**0.05**	**0.00**	**0.16**
	EWC (w=84)	**0.10**	**0.00**	**0.16**	**0.06**	**0.00**	**0.20**
	Stress 1 EWC (w=84)	**0.03**	**0.00**	**0.16**	**0.19**	**0.03**	**0.34**
	Stress 2 EWC (w=84)	**0.09**	**0.00**	**0.32**	**0.19**	**0.00**	**0.54**
G7 averages	**Baseline**	**0.90**	**0.69**	**1.00**	**0.69**	**0.34**	**0.94**
	Stress	**0.10**	**0.00**	**0.31**	**0.31**	**0.06**	**0.66**

RB: (B)aseline and (S)tress weights vs risk factor (V)

top panel H=1, bottom panel h=12



Comparisons with SRISK (Engle and Brownlee, 2017)



The vulnerability index (VI)

- VI is a signal of the probability of VaR violations (p=0.10)
- The Logit model: $P(I_{t+h}) = \Lambda(X_{t-l}\beta))$ Prediction: $\hat{P}(I_{t+h}) \equiv E_t\Lambda(X_{t-l}\hat{\beta}))$ where $I_{t+h} = 1$ if $R_{t+h} < \hat{V}aR(R_{t+h})$, 0 otherwise, and X_{t-l}: vector of predictors (ES)
- Define the threshold $P \in (0,1)$. The standard ROC *confusion matrix* is:

	$R_{t+h} < \hat{V}aR(R_{t+h})$	$R_{t+h} \geq \hat{V}aR(R_{t+h})$
$\hat{P}(I_{t+h}) - P \geq 0$	$a_{11}(P)$	$a_{10}(P)$
$\hat{P}(I_{t+h}) - P < 0$	$a_{01}(P))$	$a_{00}(P))$

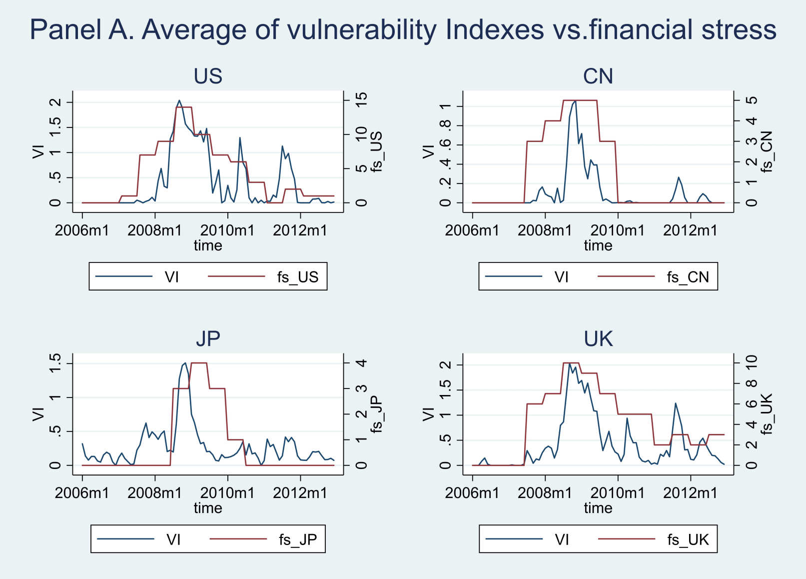
- $P^* = argmin \;\; a_{01}(P) + a_{01}(P)$ (minimization of the sum of forecast errors)
- The vulnerability index is defined by:

$$VI(R_{t+h}) = \max\{0, \hat{P}(I_{t+h}) - P^*\}$$

AUROCs of the Logit model

		RNF			RB		
	h (months)	Mean	Min	Max	Mean	Min	Max
US	1	0.82	0.78	0.85	0.78	0.75	0.80
	3	0.86	0.82	0.90	0.84	0.81	0.86
	6	0.86	0.81	0.89	0.85	0.81	0.87
	12	0.81	0.50	0.88	0.86	0.83	0.88
G-7 average	1	0.83	0.79	0.86	0.82	0.79	0.85
	3	0.86	0.81	0.89	0.85	0.80	0.88
	6	0.84	0.79	0.87	0.83	0.79	0.87
	12	0.82	0.75	0.87	0.83	0.79	0.86

Vulnerability Index (VI) for banks vs. Romer(2017) stress index



Conclusion

- This paper formulates an EWS based on forecast combinations of (VaR,ES) pairs for indicators of tail financial risk in the non-financial and banking sectors
- The EWS exploits backtesting for model selection in forecast combinations and integrates stress testing scenarios into forecasting
- The implementation on data for the G7 countries shows that the proposed EWS is promising in delivering timely early warning signals for tail risks.
- The proposed methodology can be easily and usefully expanded in several directions exploiting its flexibility.